INTERNATIONAL STANDARD

ISO 20501

Second edition
2019-03

Fine ceramics (advanced ceramics, advanced technical ceramics) — Weibull statistics for strength data

Céramiques techniques — Analyse statistique de Weibull des données de résistance à la rupture

Reference number
ISO 20501:2019(E)

Contents

Page

Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 206, *Fine ceramics*.

This second edition cancels and replaces the first edition (ISO 20501:2003), which has been technically revised. It also incorporates the Technical Corrigendum ISO 20501:2003/Cor.1:2009.

The main changes compared to the previous edition are as follows:

— the terms and definitions in Clause 3 have been updated and modified;

— a method to treat a higher number of specimens ($N > 120$) has been introduced for method A: maximum likelihood parameter estimators for single flaw populations;

— in Annex D, example codes have been added for calculating the maximum likelihood parameters of the Weibull distribution with modern analysis software.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Measurements of the strength at failure are taken for one of two reasons: either for a comparison of the relative quality of two materials regarding fracture strength, or the prediction of the probability of failure for a structure of interest. This document permits estimates of the distribution parameters which are needed for either. In addition, this document encourages the integration of mechanical property data and fractographic analysis.

Fine ceramics (advanced ceramics, advanced technical ceramics) — Weibull statistics for strength data

1 Scope

This document covers the reporting of uniaxial strength data and the estimation of probability distribution parameters for advanced ceramics which fail in a brittle fashion. The failure strength of advanced ceramics is treated as a continuous random variable. Typically, a number of test specimens with well-defined geometry are brought to failure under well-defined isothermal loading conditions. The load at which each specimen fails is recorded. The resulting failure stresses are used to obtain parameter estimates associated with the underlying population distribution.

This document is restricted to the assumption that the distribution underlying the failure strengths is the two-parameter Weibull distribution with size scaling. Furthermore, this document is restricted to test specimens (tensile, flexural, pressurized ring, etc.) that are primarily subjected to uniaxial stress states. Subclauses 6.4 and 6.5 outline methods of correcting for bias errors in the estimated Weibull parameters, and to calculate confidence bounds on those estimates from data sets where all failures originate from a single flaw population (i.e. a single failure mode). In samples where failures originate from multiple independent flaw populations (e.g. competing failure modes), the methods outlined in 6.4 and 6.5 for bias correction and confidence bounds are not applicable.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

NOTE See also Reference [1].

3.1 Defect populations

3.1.1
flaw
inhomogeneity, discontinuity or (defect) feature in a material, which acts as stress concentrator due to a mechanical load and has therefore a certain risk of mechanical failure

Note 1 to entry: The flaw becomes critical if it acts as fracture origin in a failed specimen.

3.1.2
censored data
strength measurements (i.e. a sample) containing suspended observations such as that produced by multiple competing or concurrent flaw populations

Note 1 to entry: Consider a sample where fractography clearly established the existence of three concurrent flaw distributions (although this discussion is applicable to a sample with any number of concurrent flaw distributions). The three concurrent flaw distributions are referred to here as distributions A, B, and C. Based on fractographic analyses, each specimen strength is assigned to a flaw distribution that initiated failure. In estimating parameters that characterize the strength distribution associated with flaw distribution A, all specimens (and not just those that failed from type-A flaws) shall be incorporated in the analysis to ensure efficiency and accuracy of the resulting parameter estimates. The strength of a specimen that failed by a type-B (or type-C) flaw is treated as a *right censored* observation relative to the A flaw distribution. Failure due to a type-B (or type-C) flaw restricts, or censors, the information concerning type-A flaws in a specimen by suspending the test before failure occurs by a type-A flaw[2]. The strength from the most severe type-A flaw in those specimens that failed from type-B (or type-C) flaws is higher than (and thus to the *right* of) the observed strength. However, no information is provided regarding the magnitude of that difference. Censored data analysis techniques incorporated in this document utilize this incomplete information to provide efficient and relatively unbiased estimates of the distribution parameters.

3.1.3
competing failure modes
distinguishably different types of fracture initiation events that result from concurrent (competing) flaw distributions

3.1.4
compound flaw distribution
any form of multiple flaw distribution that is neither pure concurrent, nor pure exclusive

Note 1 to entry: A simple example is where every specimen contains the flaw distribution A, while some fraction of the specimens also contains a second independent flaw distribution B.

3.1.5
concurrent flaw distribution
competing flaw distribution
type of multiple flaw distribution in a homogeneous material where every specimen of that material contains representative flaws from each independent flaw population

Note 1 to entry: Within a given specimen, all flaw populations are then present concurrently and are competing to each other to cause failure.

3.1.6
exclusive flaw distribution
mixture flaw distribution
type of multiple flaw distribution created by mixing and randomizing specimens from two or more versions of a material where each version contains a different single flaw population

Note 1 to entry: Thus, each specimen contains flaws exclusively from a single distribution, but the total data set reflects more than one type of strength-controlling flaw.

3.1.7
extraneous flaw
strength-controlling flaw observed in some fraction of test specimens that cannot be present in the component being designed

Note 1 to entry: An example is machining flaws in ground bend specimens that will not be present in as-sintered components of the same material.

3.2 Mechanical testing

3.2.1
effective gauge section
that portion of the test specimen geometry included within the limits of integration (volume, area or edge length) of the Weibull distribution function

Note 1 to entry: In tensile specimens, the integration may be restricted to the uniformly stressed central gauge section, or it may be extended to include transition and shank regions.

3.2.2
fractography
analysis and characterization of patterns generated on the fracture surface of a test specimen

Note 1 to entry: Fractography can be used to determine the nature and location of the critical fracture origin causing catastrophic failure in an advanced ceramic test specimen or component.

3.3 Statistical terms

3.3.1
confidence interval
interval within which one would expect to find the true population parameter

Note 1 to entry: Confidence intervals are functionally dependent on the type of estimator utilized and the sample size. The level of expectation is associated with a given confidence level. When confidence bounds are compared to the parameter estimate one can quantify the uncertainty associated with a point estimate of a population parameter.

3.3.2
confidence level
probability that the true population parameter falls within a specified confidence interval

3.3.3
estimator
function for calculating an estimate of a given quantity based on observed data

Note 1 to entry: The resulting value for a given sample may be an estimate of a distribution parameter (a point estimate) associated with the underlying population, e.g. the arithmetic average of a sample is an estimator of the distribution mean.

3.3.4
population
collection of data or items under consideration

3.3.5
probability density function
pdf
function $f(x)$ for the continuous random variable X if

$$f(x) \geq 0 \tag{1}$$

and

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{2}$$

Note 1 to entry: The probability that the random variable X assumes a value between a and b is given by

$$Pr(a < X < b) = \int_{a}^{b} f(x)dx \tag{3}$$

3.3.6
cumulative distribution function
function $F(x)$ describing the probability that a continuous random variable X takes a value less than or equal to a number x

Note 1 to entry: Therefore, the cumulative distribution function (cdf) is related to the probability density function $f(x)$ by

$$F(x) = Pr(-\infty < X < x) = \int_{-\infty}^{x} f(x')dx' \tag{4}$$

Differentiating Formula (4) with respect to x shows that the pdf is simple the derivative of the cdf:

$$f(x) = \frac{dF(x)}{dx} \tag{5}$$

Note 2 to entry: According to 3.3.5, $F(x)$ is a monotonically increasing function in the range between 0 and 1.

3.3.7
ranking estimator
function that estimates the probability of failure to a particular strength measurement within a ranked sample

3.3.8
sample
collection of measurements or observations taken from a specified population

3.3.9
statistical bias
type of consistent numerical offset in an estimate relative to the true underlying value, inherent to most estimates

3.3.10
unbiased estimator
estimator that has been corrected for statistical bias error

3.4 Weibull distributions

3.4.1
Weibull distribution
continuous distribution function which can be used to describe empirical data from measurements where continuous random variable x has a two-parameter Weibull distribution if the probability density function is given by

$$f(x) = \left(\frac{m}{\beta}\right)\left(\frac{x}{\beta}\right)^{m-1} \exp\left[-\left(\frac{x}{\beta}\right)^{m}\right] \text{when } x \geq 0 \tag{6}$$

or

$$f(x) = 0 \text{ when } x < 0 \tag{7}$$

and the cumulative distribution function is given by

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{m}\right] \text{when } x \geq 0 \tag{8}$$

or

$$F(x) = 0 \text{ when } x < 0 \tag{9}$$

where

m is the Weibull modulus (or the shape parameter) (>0);

β is the Weibull scale parameter (>0).

Note 1 to entry: The random variable representing uniaxial tensile strength of an advanced ceramic will assume only positive values. If the random variable representing uniaxial tensile strength of an advanced ceramic is characterized by Formulae (6) to (9), then the probability that this advanced ceramic will fail under an applied uniaxial tensile stress σ is given by the cumulative distribution function.

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_\theta}\right)^m\right] \text{ when } \sigma \geq 0 \tag{10}$$

$$P_f = 0 \text{ when } \sigma < 0 \tag{11}$$

where

P_f is the probability of failure;

σ_θ is the Weibull characteristic strength.

Note 2 to entry: The Weibull characteristic strength is dependent on the uniaxial test specimen (tensile, flexural, or pressurized ring) and will change with specimen geometry. In addition, the Weibull characteristic strength has units of stress, and has to be reported using SI-units of Pa, or adequately in MPa or GPa.

Note 3 to entry: An alternative expression for the probability of failure is given by

$$P_f = 1 - \exp\left[-\frac{1}{V_0}\int_V \left(\frac{\sigma}{\sigma_0}\right)^m dV\right] \text{ when } \sigma > 0 \tag{12}$$

$$P_f = 0 \text{ when } \sigma \leq 0 \tag{13}$$

The integration in the exponential is performed over all tensile regions of the specimen volume (V) if the strength-controlling flaws are randomly distributed through the volume of the material, or over all tensile regions of the specimen area if flaws are restricted to the specimen surface. The integration is sometimes carried out over an effective gauge section instead of over the total volume or area. In Formula (12), σ_0 is the Weibull material scale parameter and can be described as the Weibull characteristic strength of a specimen with unit volume or area loaded in uniform uniaxial tension. For a given specimen geometry, Formulae (10) and (12) can be combined, to yield an expression relating σ_0 and σ_θ (this means: $\sigma_\theta V_0^{1/m} = \sigma_0$). Further discussion related to this issue can be found in Annex A.

4 Symbols

A specimen area

b gauge section dimension, base of bend test specimen

d gauge section dimension, depth of bend test specimen

$f(x)$ probability density function

$F(x)$ cumulative distribution function

L likelihood function

L_i length of the inner load span for a bend test specimen

L_o length of the outer load span for a bend test specimen

m Weibull modulus

$\hat{m}$ estimate of the Weibull modulus

$\hat{m}_U$ unbiased estimate of the Weibull modulus

N number of specimens in a sample

P_f probability of failure

q intermediate quantity defined in 6.5.1, used in calculation of confidence bounds

r number of specimens that failed from the flaw population for which the Weibull estimators are being calculated

t intermediate quantity defined by Formula (22), used in calculation of confidence bounds

UF unbiasing factor

V tensile loaded region of specimen volume

V_0 unit size volume

V_{eff} effective volume

x realization of a random variable X

X random variable

β Weibull scale parameter

σ uniaxial tensile stress

$\hat{\sigma}$ estimate of mean strength

σ_j maximum stress in the j th test specimen at failure

σ_0 Weibull material scale parameter (strength relative to unit size) defined in Formula (12)

$\hat{\sigma}_0$ estimate of the WeibuII material scale parameter

σ_θ Weibull characteristic strength (associated with a test specimen) defined in Formula (10)

$\hat{\sigma}_\theta$ estimate of the Weibull characteristic strength

5 Significance and use

5.1 This document enables the experimentalist to estimate Weibull distribution parameters from failure data. These parameters permit a description of the statistical nature of fracture of fine ceramic materials for a variety of purposes, particularly as a measure of reliability as it relates to strength data

