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Optics and photonics — Preparation of drawings for optical elements and systems —

Part 12:
Aspheric surfaces

Optique et photonique — Préparation des dessins pour éléments et systèmes optiques —

Partie 12: Surfaces asphériques

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Contents

Page

Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 1, *Fundamental standards*.

This third edition cancels and replaces the second edition (ISO 10110-12:2007), which has been technically revised. It also incorporates the Amendment ISO 10110-12:2007/Amd.1:2013.

The main changes compared to the previous edition are as follows:

a) The document has been updated with respect to surface form tolerances as described in ISO 10110-5.

b) The reference to the new part ISO 10110-19 has been added.

c) The document has been restructured.

d) A few surface descriptions have been added.

A list of all the parts in the ISO 10110 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Optics and photonics — Preparation of drawings for optical elements and systems —

Part 12:
Aspheric surfaces

1 Scope

This document specifies rules for presentation of aspheric surfaces and surfaces with low order symmetry such as cylinders and toroids in the ISO 10110 series, which standardizes drawing indications for optical elements and systems. It also specifies sign conventions and coordinate systems.

This document does not apply to off-axis aspheric and discontinuous surfaces such as Fresnel surfaces or gratings.

NOTE For off-axis aspheric and non-symmetric surfaces, see ISO 10110-19.

This document does not specify the method by which conformity with the specifications is tested.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1101:2017, *Geometrical product specifications (GPS) — Geometrical tolerancing — Tolerances of form, orientation, location and run-out*

ISO 10110-1, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 1: General*

ISO 10110-5, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 5: Surface form tolerances*

ISO 10110-6, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 6: Centring tolerances*

ISO 10110-7, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 7: Surface imperfections*

ISO 10110-8, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 8: Surface texture*

3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Mathematical description of aspheric surfaces

4.1 Coordinate system

Aspheric surfaces are described in a right-handed, orthogonal coordinate system in which the Z axis is the optical axis.

Unless otherwise specified, the Z axis is in the plane of the drawing and runs from left to right; if only one cross section is drawn, the Y axis is in the plane of the drawing and is oriented upwards.

The origin of the coordinate system is at the vertex of the aspheric surface (see Figure 1).

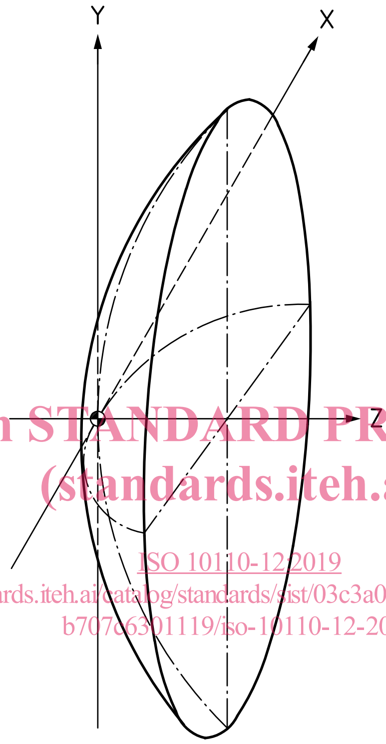

Figure 1 — Coordinate system

4.2 Sign conventions

As shown later in this document, the various types of aspheric surfaces are given by mathematical formulae. In the drawings the chosen formula and the corresponding constants and coefficients are specified. To achieve unambiguous indications of the surfaces, sign conventions for the constants and coefficients shall be introduced.

The sign of the radius of curvature is positive if the centre of curvature is to the right of the vertex, and negative if the centre of curvature is to the left of the vertex.

The sagitta of a point of the aspheric surface is positive if this point is to the right of the vertex (XY plane) and negative if it is to the left of the vertex (XY plane).

NOTE 1 In this case, "left" and "right" presume Z is increasing from left to right. When the Z axis is reversed as a result of a reflection (a 180-degree rotation about the Y axis), the sign convention for radius and sagitta is also reversed. This is discussed further in 4.3.

NOTE 2 This is the default sign convention, assuming no coordinate system according to ISO 10110-1:2019, 5.3 has been defined for the surface of interest. See ISO 10110-1 for more information about defining local coordinate systems.

4.3 Surface descriptions

4.3.1 General

The phrase “aspheric surfaces” is commonly used in optics to describe rotationally invariant surfaces such as are described below in 4.3.2. Surface descriptions for surfaces which are not rotationally invariant such as cylindrical surfaces are described in 4.3.3. More complex optical surfaces can be described using the methods given in ISO 10110-19.

4.3.2 Surface description — Rotationally invariant ($h^2 = x^2 + y^2$)

4.3.2.1 Aspheric surface described by a conic section and a power series

The aspheric surface description consists of a conic part and a power series where the axis of rotation is the Z axis.

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}+\sum_{i=2}^{n}A_{2i}h^{2i} \quad (1)$$

where

z is the sagitta;

h is surface height perpendicular to Z-axis ($h \geq 0$);

R is the radius of curvature of the base sphere;

κ is the conic constant;

A_i is the aspheric coefficient.

Where the basic conic formula behaves as follow:

$\kappa > 0$	oblate ellipse;
$\kappa = 0$	circle;
$-1 < \kappa < 0$	prolate ellipse;
$\kappa = -1$	parabola;
$\kappa < -1$	hyperbola.

NOTE 1 The formula of second order can also be used without the power series.

NOTE 2 In three dimensions, the conic formula shapes are called ellipsoid, sphere, paraboloid, and hyperboloid.

For an example drawing, see Figure 4.

An extended version with the complete power series of this description is described in Formula (2).

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}+\sum_{i=1}^{n}A_i h^i \tag{2}$$

In a special version this formula describes an axicon:

$$z(h)=A_1 h \tag{3}$$

Care should be taken that the signs of the power series named formulae $z(h)$ are in accordance with the conventions defined in 4.1 and 4.2.

In the case where the direction of the Z axis is reversed, but the lens stays unchanged, the signs of the radius of curvature and of the aspheric coefficients shall be changed. The signs of the conic constants remain unchanged.

4.3.2.2 Aspheric surface described by a conic section and orthogonal polynomials

4.3.2.2.1 Orthonormal in slope aspheres with spherical base

A surface of higher order can also be generated by combining a spherical surface with a polynomial of the following kind, which has orthonormal derivatives.

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-\left(\frac{h}{R}\right)^2}\right]}+\frac{w^2\left[1-w^2\right]}{\sqrt{1-\left(\frac{h}{R}\right)^2}}\sum_{m=0}^{n}B_m Q_m^{\mathrm{bfs}}\left(w^2\right) \tag{4}$$

where

R is the radius of curvature of the base sphere;

h is the surface height;

h_0 marks the upper limit of h; and

w (normalized surface height) is defined as $w=\frac{h}{h_0}$;

B_m is the coefficient; and

Q_m^{bfs} is the polynomial term.

The description z is valid for $0 \le h \le h_0$ only. The formula for the polynomial terms is

$$Q_{m+1}^{\mathrm{bfs}}\left(w^2\right)=\left[P_{m+1}\left(w^2\right)-g_m Q_m^{\mathrm{bfs}}\left(w^2\right)-k_{m-1}Q_{m-1}^{\mathrm{bfs}}\left(w^2\right)\right]/\,l_{m+1} \tag{5}$$

starting with

$$Q_0^{\mathrm{bfs}}\left(w^2\right)=1 \tag{6}$$

$$Q_1^{\mathrm{bfs}}\left(w^2\right)=\frac{1}{\sqrt{19}}\left(13-16w^2\right) \tag{7}$$

$$Q_2^{\text{bfs}}\left(w^2\right)=\sqrt{\frac{2}{95}}\left[29-4w^2\left(25-19w^2\right)\right] \tag{8}$$

$$P_{m+1}\left(w^2\right)=\left(2-4w^2\right)P_m\left(w^2\right)-P_{m-1}\left(w^2\right) \tag{9}$$

starting with

$$P_0\left(w^2\right)=2 \tag{10}$$

$$P_1\left(w^2\right)=6-8w^2 \tag{11}$$

These auxiliary polynomials have to be solved in the order given here and are valid for $m \geq 2$.

$$k_{m-2}=-m(m-1)/2l_{m-2} \tag{12}$$

$$g_{m-1}=-(1+g_{m-2}k_{m-2})/l_{m-1} \tag{13}$$

$$l_m=\left[m(m+1)+3-g_{m-1}^2-k_{m-2}^2\right]^{1/2} \tag{14}$$

starting with

$$g_0=-\frac{1}{2} \tag{15}$$

$$l_0=2 \tag{16}$$

$$l_1=\frac{1}{2}\sqrt{19} \tag{17}$$

NOTE 1 $Q_2^{\text{bfs}}\left(w^2\right)$ is given above to be used as a check of the recursion algorithm provided in Formulae (5) through (17). See also Annex B.

NOTE 2 "bfs" is an abbreviation for "best fit sphere", which matches the sag of the aspherical surface at the vertex and h_0.

For an example drawing, see Figure 5.

4.3.2.2.2 Orthonormal in slope aspheres with conic base

It is also possible to generate a surface by combining a conical surface with a polynomial of the same kind as in Formula (4). This kind is also an orthonormal set of polynomials.

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}+\frac{w^2\left[1-w^2\right]}{\sqrt{1-\left(\frac{h}{R}\right)^2}}\sum_{m=0}^{n}B_m Q_m^{\text{bfs}}\left(w^2\right) \tag{18}$$

where

R is the radius of curvature of the base sphere;

h is the surface height;

h_0 marks the upper limit of h;

w (normalized surface height) is defined as $w = \frac{h}{h_0}$;

κ is the conic constant;

B_m are the coefficients; and

$Q_\mathrm{m}^\mathrm{bfs}$ are the polynomial terms.

4.3.2.2.3 Orthonormal in amplitude aspheres

A surface of higher order can also be generated by combining a conical surface with a polynomial of the following kind, which has orthonormal amplitudes.

$$z(h) = \frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]} + w^4 \sum_{m=0}^{n} C_m Q_m^\mathrm{con}\left(w^2\right) \tag{19}$$

where

R is the radius of curvature of the base sphere;

h is the surface height;

h_0 marks the upper limit of h; and

w (normalized surface height) is defined as $w = \frac{h}{h_0}$;

κ is the conic constant;

C_m are the coefficients; and

$Q_\mathrm{m}^\mathrm{con}$ are the polynomial terms.

The formula for the polynomial terms is

$$Q_m^\mathrm{con}\left(w^2\right) = T_m\left(2w^2 - 1\right) \tag{20}$$

starting with

$$Q_0^\mathrm{con}\left(w^2\right) = 1 \tag{21}$$

$$Q_1^\mathrm{con}\left(w^2\right) = -\left(5 - 6w^2\right) \tag{22}$$

$$Q_2^\mathrm{con}\left(w^2\right) = 15 - 14w^2\left(3 - 2w^2\right) \tag{23}$$

$$T_m\left(w^2\right)=\left[\left(b(m)+c(m)w^2\right)T_{m-1}\left(w^2\right)-d(m)T_{m-2}\left(w^2\right)\right]/a(m) \tag{24}$$

starting with

$$T_0\left(w^2\right)=1 \tag{25}$$

$$T_1\left(w^2\right)=3w^2-2 \tag{26}$$

These auxiliary polynomials have to be solved in the order given here and are valid for $m \geq 2$.

$$a(m)=2m(m+4)(2m+2) \tag{27}$$

$$b(m)=-32m-48 \tag{28}$$

$$c(m)=(2m+2)(2m+3)(2m+4) \tag{29}$$

$$d(m)=2(m-1)(m+3)(2m+4) \tag{30}$$

NOTE 1 $Q_0^{\text{con}}\left(w^2\right)$, $Q_1^{\text{con}}\left(w^2\right)$ and $Q_2^{\text{con}}\left(w^2\right)$ are given above to be used as a check of the recursion algorithm provided in Formulae (20) through (30). See also Annex C.

NOTE 2 The polynomial form given here is identical to Zernike radial polynomial expansion $R_{2n}^{q}\left(w^2\right)$ of order $q = 4$ correlated to ISO/TR 14999-2 as $w^4 Q_m^{con}\left(w^2\right)=R_{2n}^{4}\left(w^2\right)$ with $n = m + 2$, and $m = 0, 1, 2, 3, 4, 5, 6 ...$

NOTE 3 Instead of using the recursion algorithm above, for lower orders of m ($m \leq 8$), the polynomials can easily be computed using the formula $Q_m^{con}\left(w^2\right)=\sum_{s=0}^{m}(-1)^s \frac{(2m+4-s)!}{s!(m+4-s)!(m-s)!} w^{2(m-s)}$.

For an example drawing, see Figure 6.

4.3.3 Surface description — Rotationally variant

4.3.3.1 Centred quadrics

In the coordinate system given in 4.1, the formulae of the surfaces of second order which fall within the scope of this document are derived from the canonical forms

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad \text{for centered quadrics} \tag{31}$$

where

a, b are real or imaginary constants;

c is a real constant.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+2z=0 \qquad \text{for parabolic surfaces} \tag{32}$$

where a, b are real or imaginary constants, and can be written as

$$z=f(x,y)=\frac{\dfrac{x^2}{R_X}+\dfrac{y^2}{R_Y}}{1+\sqrt{1-(1+\kappa_X)\left(\dfrac{x}{R_X}\right)^2-(1+\kappa_Y)\left(\dfrac{y}{R_Y}\right)^2}} \tag{33}$$

where

R_X is the radius of curvature in the XZ plane;

R_Y is the radius of curvature in the YZ plane;

κ_X, κ_Y are conic constants.

Using curvatures $C_X = 1/R_X$ and $C_Y = 1/R_Y$ instead of radii yields

$$z=f(x,y)=\frac{x^2C_X+y^2C_Y}{1+\sqrt{1-(1+\kappa_X)(xC_X)^2-(1+\kappa_Y)(yC_Y)^2}} \tag{34}$$

If the surface according to Formula (33) or (34) is intersected with the XZ plane ($y = 0$) or the YZ plane ($x = 0$), then, depending on the value of κ_Y (or κ_X), intersection lines of the following types are produced:

$\kappa > 0$ oblate ellipse;

$\kappa = 0$ circle;

$-1 < \kappa < 0$ prolate ellipse;

$\kappa = -1$ parabola;

$\kappa < -1$ hyperbola.

4.3.3.2 Cylinders

Formulae (35) and (36) describe a cylinder (due to κ_U not necessarily of circular cross section). For $u = x$, the cylinder vertex line is parallel to the Y axis which is perpendicular to the XZ plane. For $u = y$ the cylinder vertex line is parallel to the X axis which is perpendicular to the YZ plane.

Using radii:

For $R_X = \infty$ or $R_Y = \infty$ Formula (33) gives

$$z=f(u)=\frac{u^2}{R_U\left[1+\sqrt{1-(1+\kappa_U)\left(\dfrac{u}{R_U}\right)^2}\right]} \tag{35}$$

