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**Vesolje - Ugotavljanje položaja z uporabo GNSS za cestne inteligentne transportne sisteme (ITS) - Matematični model za napake PVT**

Space - Use of GNSS-based positioning for road Intelligent Transport System (ITS) - Mathematical PVT error model

Mathematisches PVT-Fehlermodell

Espace - Utilisation du positionnement GNSS pour les systèmes de transport routier intelligents (ITS) - Modèle d'erreur mathématique PVT

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**FINAL DRAFT**  
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**Space - Use of GNSS-based positioning for road Intelligent  
Transport System (ITS) - Mathematical PVT error model**

Espace - Utilisation du positionnement GNSS pour les  
systèmes de transport routier intelligents (ITS) -  
Modèle d'erreur mathématique PVT

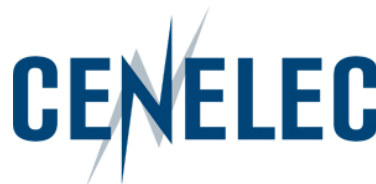
Mathematisches PVT-Fehlermodell

This draft Technical Report is submitted to CEN members for Vote. It has been drawn up by the Technical Committee CEN/CLC/JTC 5.

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<b>Contents</b>	<b>Page</b>
European foreword.....	3
1 Scope .....	4
2 Normative references .....	4
3 Terms and definitions .....	4
4 List of acronyms.....	4
5 Approach .....	5
6 Input data.....	7
7 Model .....	10
7.1 Introduction .....	10
7.2 Discussion about models.....	11
7.2.1 Laplace-steps and Cauchy-random walk .....	11
7.2.2 ARMA models.....	11
7.3 AR model .....	16
7.3.1 Model choice .....	16
7.3.2 Model parameters.....	19
8 Analysis tools .....	29
8.1 General .....	29
8.2 State of the art on data set similarity assessment.....	30
8.3 Validation tool description.....	34
8.3.1 GP-START project context.....	34
8.3.2 Validation tool overview .....	34
8.3.3 Used data set similarity metrics .....	35
8.3.4 Data set similarity decision process.....	42
8.3.5 Data set similarity results.....	44
9 Conclusions .....	47
Annex A (normative) Neural network .....	49
A.1 Neuron network concept.....	49
A.1.1 General.....	49
A.1.2 Neural network weight estimation.....	51
Bibliography.....	57

## European foreword

This document (FprCEN/TR 17447:2019) has been prepared by Technical Committee CEN-CENELEC/TC 5 “Space”, the secretariat of which is held by DIN.

This document is currently submitted to the Vote on TR.

**iTeh STANDARD PREVIEW**  
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SIST-TP CEN/TR 17447:2020  
<https://standards.iteh.ai/catalog/standards/sist/24b609b1-4370-4d22-b567-0ac178c92faa/sist-tp-cen-tr-17447-2020>

## 1 Scope

This document is written in the frame of WP1.3 of GP-START project. It discusses several models to provide synthetic data for PVT tracks and the ways to analyse and compare the tracks to ensure these are similar to the reality.

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

EN 16803-1:2016, *Space — Use of GNSS-based positioning for road Intelligent Transport Systems (ITS) — Part 1: Definitions and system engineering procedures for the establishment and assessment of performances*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in EN 16803-1:2016 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <http://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

## 4 List of acronyms

ANOVA	Analysis of variance
AR	Autoregressive
ARMA	Autoregressive moving average
CDF	Cumulated distribution function
CET	Central european time
DFT	Direct Fourier transform
DOP	Dilution of precision
FFT	Fast Fourier transform
GNSS	Global navigation satellite system
GPS	Global positioning system
HDOP	Horizontal dilution of precision
HPE	Horizontal position error
IGS	International GNSS service
ITS	Intelligent transport systems
KS	Kolmogorov–Smirnov
MFNN	Multilayer feedforward neural networks
NED	Northeast down
NN	Neural network

OBU	On board unit
PVT	Position velocity time
SBAS	Satellite based augmentation system
SVM	Support vector machine
UTC	Universal time coordiante
VDOP	Vertical dilution of precision
WGS84	World geodetic system 1984
WP	Work package

## 5 Approach

The objective of the WP1.3 model the PVT tracks similar enough to the reality to be used for application validation. The approach used is to twofold: to find a model and to build the tools to compare the simulated data to the real data.

The following figure shows the overall logic of the work to be undertaken in WP 1.3:

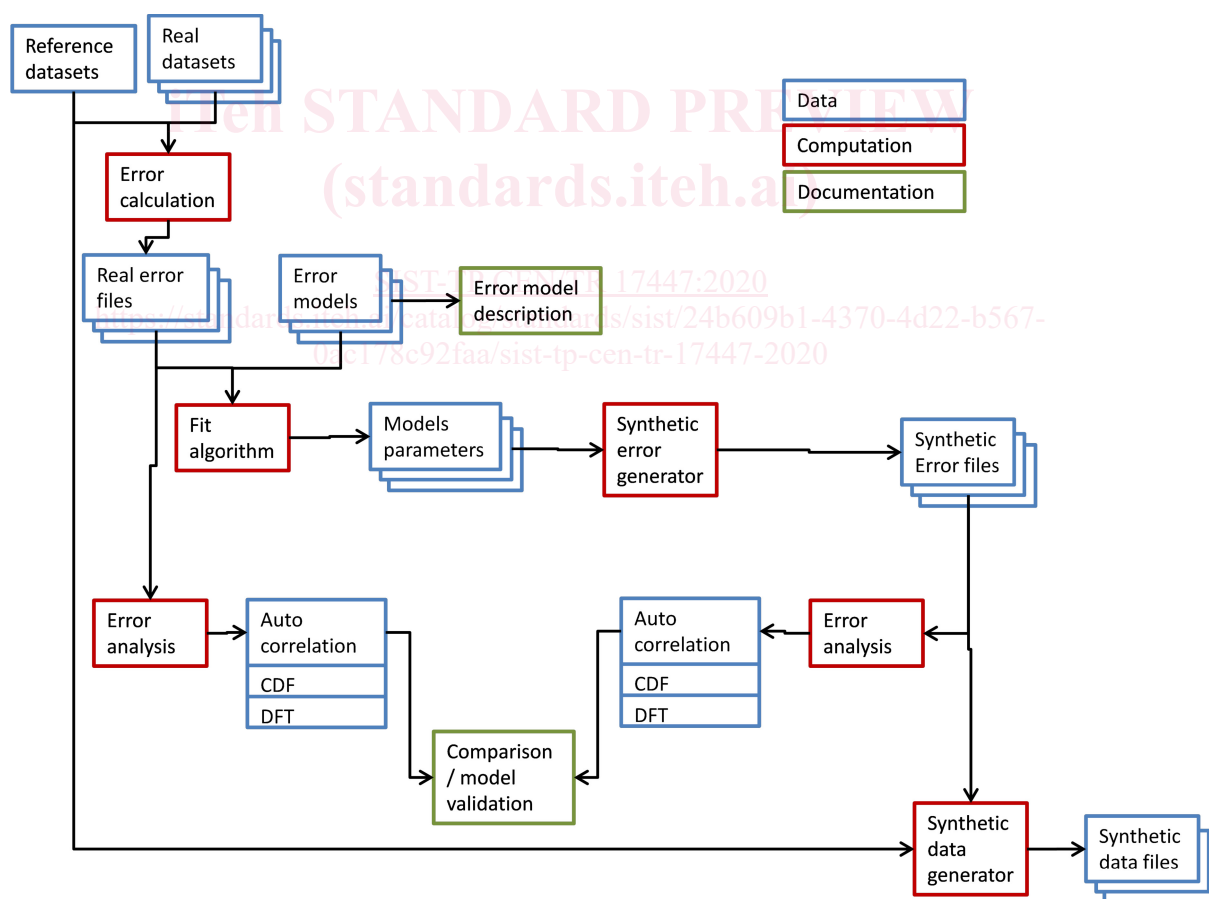


Figure 1 — WP1.3 logic

The blue boxes represent the raw data that can be stored in files. The red boxes are the computations that need to be carried out to get new data. Finally, the green boxes correspond to the outputs of this work package that need to be documented.

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In some more detail:

- Reference data set: These are the reference trajectories computed with extra sensors and is expected to have errors at centimetric level or better.
- Real data sets: These are the trajectories computed by the receiver under test.
- Error calculation: The error calculation computes the difference between the position computed by the receiver and the one from the reference trajectory. Note that this step will be used to define the parameters to be modelled. For instance in the Ifsttar paper [RD2], the model was based on the absolute error and its angle. But one could use the position in XYZ (WGS84 coordinates). Since the behaviour in the vertical axis is usually different, one could use a NED reference system or change the reference to an axis in the direction of the movement another across the direction of the movement and the third would be the vertical.
- Real error files: These files should contain the errors for each receiver trajectory.
- Error models: The error models will be described in the documentation and should depict the error of the position depending on the previous errors, random variables (which can follow any distribution) and parameters of the distributions.
- fit algorithm: The fit algorithm should calculate the parameters that define the models.
- Model parameters: Storage of the models parameters.
- Synthetic error generation: Once the parameters are found, by starting the random variables with different seed, one will get synthetic errors that should be equivalent to the real ones.
- Error analysis: As can be on the figure, the error analysis box appears twice: for the real errors and for the synthetic ones. These analysis will be compared to validate the models. So far, the analysis used have been the CDF (cumulative distribution function) and autocorrelation. As far as the CDF will be representative for the error distribution in the position domain, in the time domain, autocorrelation may not be enough to observe the behaviour of the error. It is suggested to compute the DFT (discrete Fourier transform) to analyse the error in the frequency domain and observe if there are errors at particular frequencies. Autocorrelation analysis will be kept for comparison with previous models.
- Comparison: the Error analysis of real and synthetic data are compared to validate the models.

Synthetic data generator: this step adds the synthetic errors with the reference trajectory to create a new data set that can be used for the sensitivity analysis.



## 6 Input data

The input data to observe and models has been provided by Q-Free. It contains a header and then the data with each column separated by a space. The header contains the list of fields

```
# SAVE Position Export
#
# Equipment: 1, seagull_datalogger_1
# Export Date: Tue Dec 13 13:58:20 CET 2016
#
# Space separated field list
#
# Equipment id (6 is reference)
# Drive id
# Leg id
# Date yyyy-mm-dd (UTC)
# Time hh:mm:ss.fff (UTC)
# Gps week
# Gps time of week, seconds
# OBU Timestamp, seconds
# GNSS Good flag (1 is good, 0 is bad)
# GNSS Fix Type
# Latitude
# Longitude
# Ellipsoid height (meters)
# Geoid height, height over sea level (meters)
# Speed over ground (m/s)
# Course over ground (deg)
# Yaw (deg/second)
# Pitch (deg/second)
# Roll (deg/second)
# Horizontal pos error (meter)
# Vertical pos error (meter)
# sN, st.dev of position, in north direction (meters)
# sE, st.dev of position, in east direction (meters)
# sD, st.dev of position, in down direction (meters)
# HDOP
# PDOP
# NED n speed (m/s)
# NED e speed (m/s)
# NED d speed (m/s)
# Number of satellites used in solution
# Average signal quality
# SBAS Active
# rN integrity, north (m)
# rE integrity, east (m)
# rD integrity, down (m)
#
```

**FprCEN/TR 17447:2019 (E)**

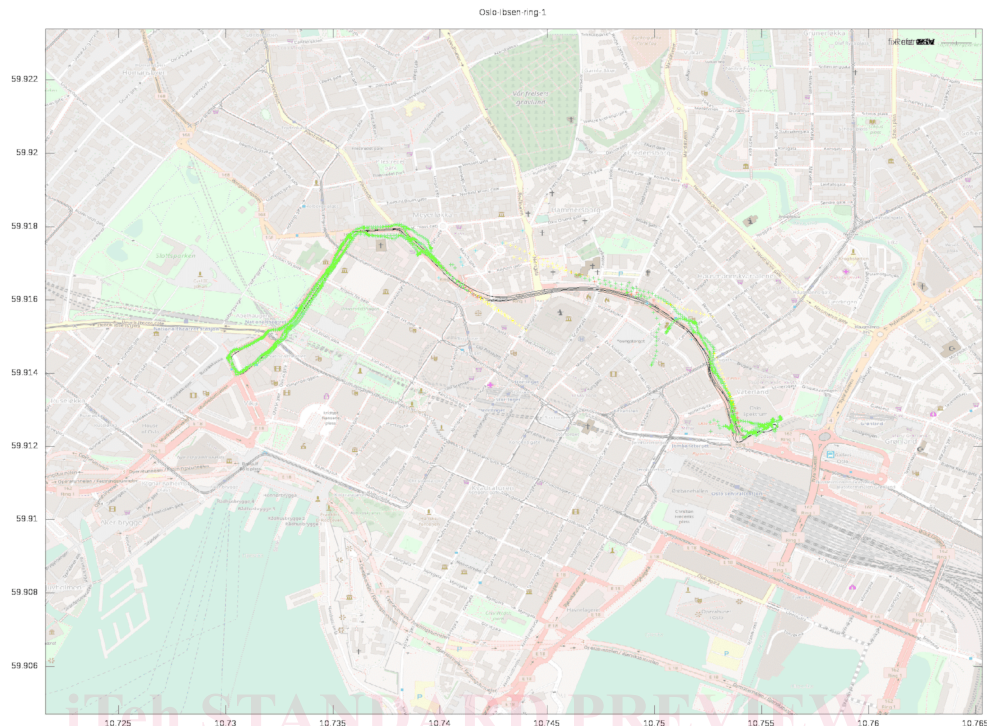
To do the analyses and modelling of the PVT error the outputs computed by the receiver in a specific proprietary way are not used. For instance, the horizontal and vertical position errors are calculated by the receiver but one does not know how. Therefore, all the positions in latitude, longitude and height are compared to the receiver #6 which is the reference. The other fields used are:

- Equipment Id and leg to separate the data sets;
- GNSS good flag and type fix to use only the positions with 4 (four) or more satellites that do not use additional sensors (i.e.: type of fix 4 or 6).

For more details, the GNSS fix types are:

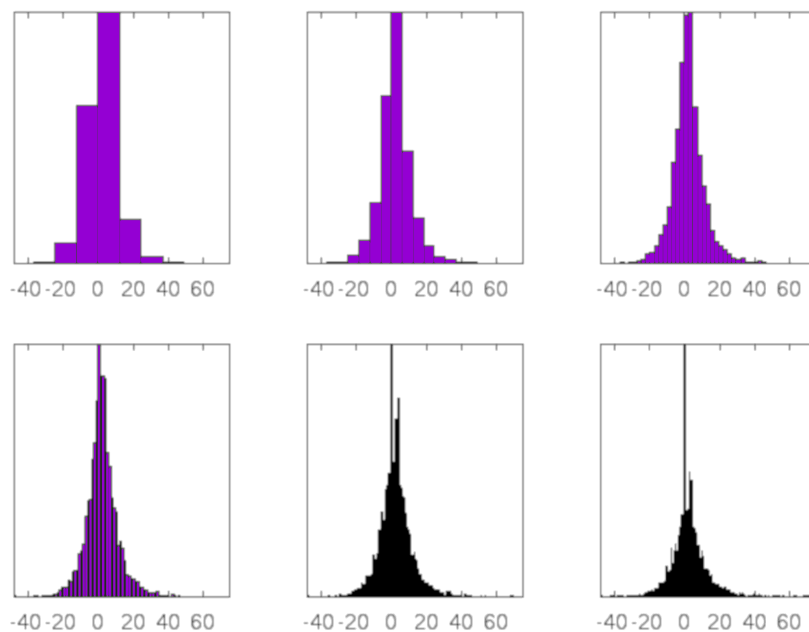
- NO\_FIX = 0, → red
- ONE\_SV = 1, → purple
- TWO\_SV = 2, → blue
- THREE\_SV = 3, → orange
- FOUR\_PLUS\_SV = 4, → green
- LS\_2D = 5, → cyan
- LS\_3D = 6, → dark green
- DEAD\_RECKONING = 7, → yellow
- PSR\_DIFF = 8, → brown
- SENSOR\_FUSION = 9 → dark purple

The following figure shows the behaviour one can expect of a receiver (Reference is black) with a tunnel in the middle of the path:



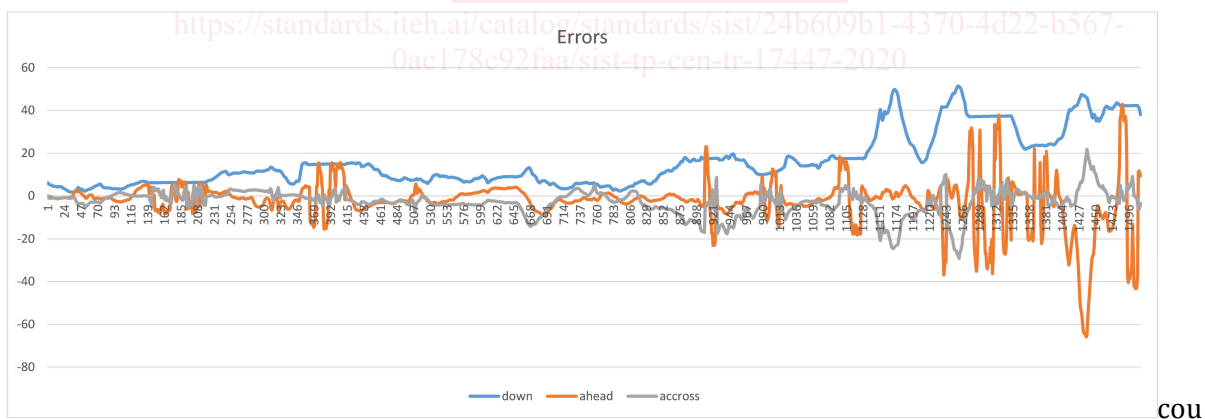
**Figure 2 — Oslo input data plotted**

Some other issues have been observed in the input data. The following figure shows the histograms of the error a data set from Frankfurt. The 6 (six) plots show the same data with different histogram bin widths:



**Figure 3 — Frankfurt errors histograms**

As it will be seen in the following clauses, the chosen model tends to a Gaussian distribution for data sets big enough. However, the errors observed in Frankfurt data set, are not likely to follow a normal distribution. When one observes the errors in the time domain, one can see that the error distribution changes during the test. This is, the test starts in small error environment (an almost open-sky) and ends in a great error environment (urban canyon).



**Figure 4 — Frankfurt errors vs. time**

## 7 Model

### 7.1 Introduction

Clause 7 presents the model chosen to create synthetic data representative of real PVT environments. The first subclause discusses the previous model made by Ifsttar and Q-Free and the ARMA models proposed in this document.

The second subclause presents the AR model chosen to fit the real data and the way its parameters are calculated.

## 7.2 Discussion about models

### 7.2.1 Laplace-steps and Cauchy-random walk

A previous model discussed is the one by Ifsttar and Q-Free that have already worked on the data sets to be studied (See RD17). The following analysis intends to analyse which are the error characteristics to be simulated and the limits of the model.

The model is twofold: on the one hand the absolute value of the horizontal error is calculated and on the other the angle between the reference heading and the error direction. The radius (absolute error) is calculated as follows:

$$R[n+1] = \begin{cases} R[n] & \text{with probability } (1-p) \\ L_r & \text{with probability } p \end{cases} + C_r[n]$$

$p$  is the probability of a Bernoulli trial that in case of failure keeps the radius and in case of success will draw a new radius from a Laplace distribution (positive values only) plus a Cauchy noise that accumulates every time a step takes place.

NOTE On the radius calculation:

- Depending on the probability of the Bernoulli trial one can observe arcs of circle which is not a GNSS positioning known behaviour.
- The probability of the Bernoulli trial allows mastering the correlation of the process since one can manage the proportion of strongly correlated samples and those which are not.
- The Laplace distribution allows managing the mean and variance of the error.
- When one adds a cumulative error, the variances add up: This means that the cumulative error will tend to infinity with time. On top of that the Cauchy distribution has undefined variance. So in the model a minimum and maximum values need to be identified to avoid divergence.

Finally the error is filtered by an autoregressive filter.

The angle is calculated with a random walk:

$$\varnothing[n+1] = \varnothing[n] + S_{\varnothing}[n]$$

The noise chosen is a Cauchy distribution truncated with a min and a max values similarly to the Cauchy distribution for the radius.

### 7.2.2 ARMA models

#### 7.2.2.1 Introduction

The previous subclause has already discussed about an autoregressive filter. The idea of this subclause is to present the characteristics of these models without the complex inputs (Laplace, Cauchy, random walk...) of the previous subclause.

To do so, the input noise of the processes shown below will be a normal noise and the differences in the outputs will entirely be due to the ARMA filter.

### 7.2.2.2 Definition

ARMA stands for autoregressive moving average. The autoregressive part means that it depends on the previous values of the process and a weighting of the previous inputs.

An ARMA process can be defined as follows:

$$X_n = a_1 X_{n-1} + \dots + b_0 N_n + b_1 N_{n-1} + b_2 N_{n-2} + \dots$$

where

X can be the position in 1 (one) axis or the radius of the error;

N is a noise;

a and b are real numbers;

n is the index of the series.

One can use the Z transform to obtain:

$$X = \frac{B(z)}{A(z)} N$$

where

A and B are the polynomials with a and b coefficients;

The zeroes of B are the points at which there is no frequency response;

The zeroes of A are the points at which there are peaks in the response.

As a brief reminder, the roots of A need to be in the unit circle for the system to be stable. The closer to the unit circle the bigger the impact of the root on the output.

### 7.2.2.3 Examples

#### 7.2.2.3.1 General

Subclause 7.2.2.3 shows a few examples of the outputs that can be obtained with an ARMA process.

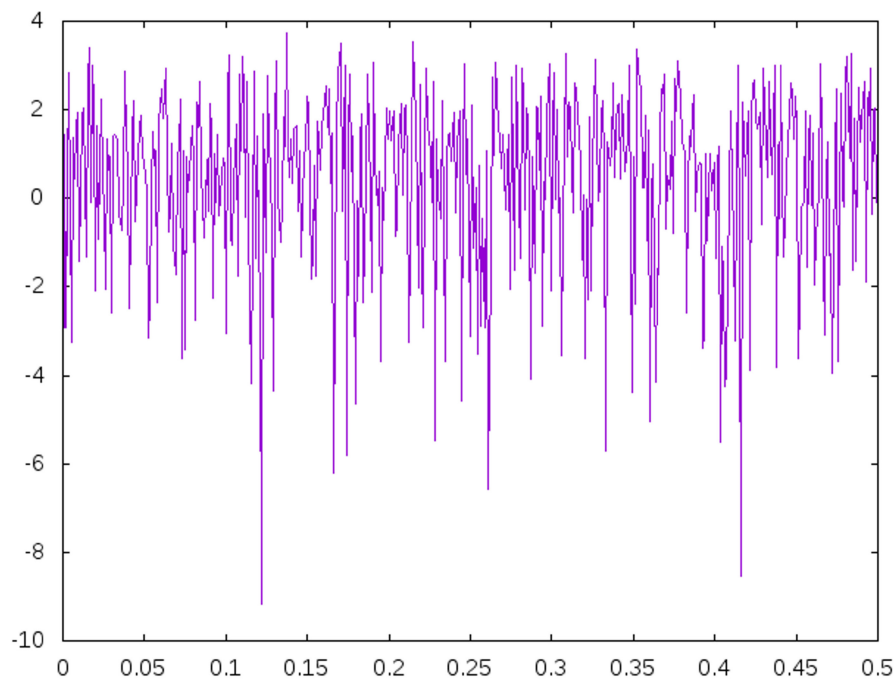
#### 7.2.2.3.2 Gaussian noise

A Gaussian noise is hardly an ARMA process since it does not depend on the past information, but it is important to see the response of a system when there is no filtering yet.

Its formula is the following:

$$X_n = b_0 N_n$$

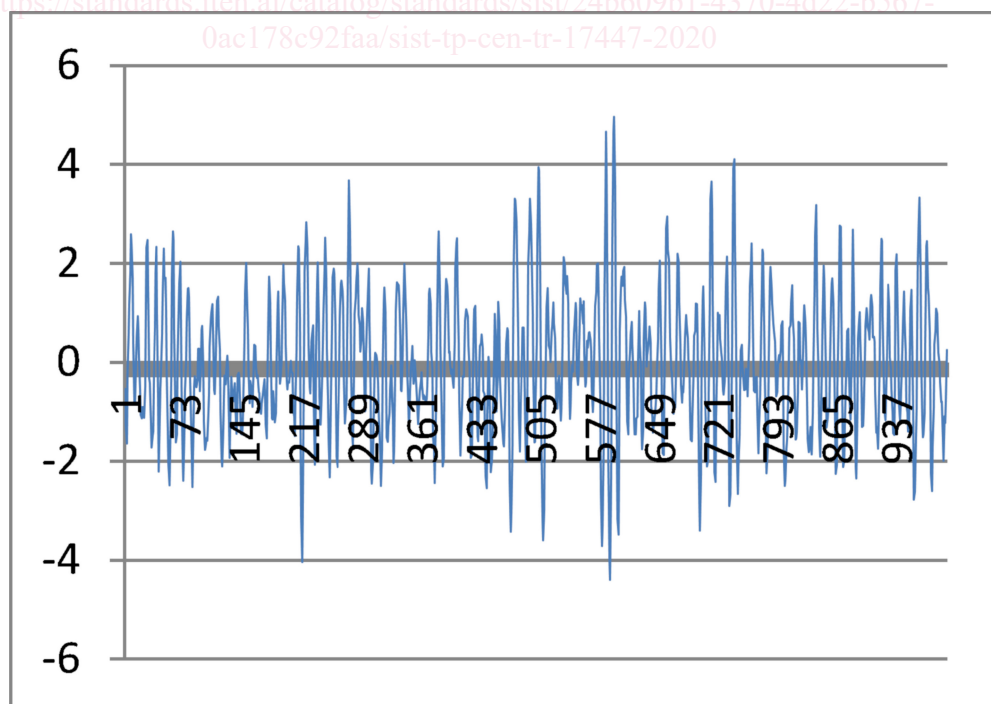
The outputs shown are the direct Fourier transform (DFT) and the time series. These two plots should illustrate the outputs of the model.



**Figure 5 — DFT of a white noise**

It is important to see that there is no specific resonance point: all the frequencies react just the same. Note that the mean is not centred at zero. The frequencies observed go up to 0,5 Hz since the GNSS positions are updated every second.

In the time domain, one observes the normal distribution behaviour.



**Figure 6 — White noise against time**