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Geometrical product specifications (GPS) — Filtration —

Part 29:
Linear profile filters: wavelets

Spécification géométrique des produits (GPS) — Filtrage —

Partie 29: Filtres de profil linéaires: ondelettes

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, Dimensional and geometrical product specification and verification, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This second edition cancels and replaces the first edition (ISO 16610-29:2015), which has been technically revised.

The main changes compared to the previous edition are as follows:

— The terminology and requirements around wavelets have been clarified and expanded to cover biorthogonal wavelets more fully.

— The requirements for cubic prediction wavelets are set out in Annex A.

— The requirements for cubic b-spline wavelets are given in Annex B.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specification (GPS) standard and is to be regarded as a general GPS standard (see ISO 14638). It influences chain links C and F of the chains of standards on profile and areal surface texture.

The ISO GPS matrix model given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this document, unless otherwise indicated.

For more detailed information on the relation of this document to other standards and the GPS matrix model, see Annex D.

This document develops the terminology and concepts for wavelets.

Geometrical product specifications (GPS) — Filtration —

Part 29:
Linear profile filters: wavelets

1 Scope

This document specifies biorthogonal wavelets for profiles and contains the relevant concepts. It gives the basic terminology for biorthogonal wavelets of compact support, together with their usage.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16610-1, *Geometrical product specifications (GPS) — Filtration — Part 1: Overview and basic concepts*

ISO 16610-20, *Geometrical product specifications (GPS) — Filtration — Part 20: Linear profile filters: Basic concepts*

ISO 16610-22, *Geometrical product specifications (GPS) — Filtration — Part 22: Linear profile filters: Spline filters*

ISO/IEC Guide 99, *International vocabulary of metrology — Basic and general concepts and associated terms (VIM)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16610-1, ISO 16610-20, ISO 16610-22 and ISO/IEC Guide 99 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
mother wavelet
function of one or more variables which forms the basic building block for wavelet analysis, i.e. an expansion of a signal/profile as a linear combination of wavelets

Note 1 to entry: A mother wavelet, which usually integrates to zero, is localized in space and has a finite bandwidth. Figure 1 provides an example of a real-valued mother wavelet.

3.1.1
biorthogonal wavelet
wavelet where the associated *wavelet transform* (3.3) is invertible but not necessarily orthogonal

Note 1 to entry: The merit of the biorthogonal wavelet is the possibility to construct symmetric wavelet functions, which allows a linear phase filter.

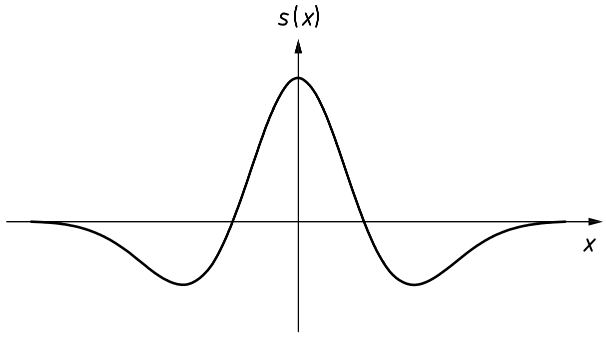

Figure 1 — Example of a real-valued mother wavelet

3.2
wavelet family
$g_{\alpha,b}$
family of functions generated from the *mother wavelet* (3.1) by *dilation* (3.2.1) and *translation* (3.2.2)

Note 1 to entry: If $g(x)$ is the *mother wavelet* (3.1), then the wavelet family $g_{\alpha,b}(x)$ is generated as shown in Formula (1):

$$g_{\alpha,b}(x)=\alpha^{-0,5}\times g\left(\frac{x-b}{\alpha}\right) \tag{1}$$

where

α is the dilation parameter for the wavelet of frequency band $[1/\alpha, 2/\alpha]$;

b is the translation parameter.

3.2.1
dilation
<wavelet> transformation which scales the spatial variable x by a factor α

Note 1 to entry: This transformation takes the function $g(x)$ to $\alpha^{-0,5}g(x/\alpha)$ for an arbitrary positive real number α.

Note 2 to entry: The factor $\alpha^{-0,5}$ keeps the area under the function constant.

3.2.2
translation
transformation which shifts the spatial position of a function by a real number b

Note 1 to entry: This transformation takes the function $g(x)$ to $g(x - b)$ for an arbitrary real number b.

3.3
wavelet transform
unique decomposition of a profile into a linear combination of a *wavelet family* (3.2)

3.4
discrete wavelet transform
DWT
unique decomposition of a profile into a linear combination of a *wavelet family* (3.2) where the *translation* (3.2.2) parameters are integers and the *dilation* (3.2.1) parameters are powers of a fixed positive integer greater than 1

Note 1 to entry: The dilation parameters are usually powers of 2.

3.5
multiresolution analysis
decomposition of a profile by a filter bank into portions of different scales

Note 1 to entry: The portions at different scales are also referred to as resolutions (see ISO 16610-20).

Note 2 to entry: Multiresolution is also called multiscale.

Note 3 to entry: See Figure 2.

Note 4 to entry: Since by definition there is no loss of information, it is possible to reconstruct the original profile from the *multiresolution ladder structure* (3.5.3).

3.5.1
low-pass component
smoothing component
component of the *multiresolution analysis* (3.5) obtained after convolution with a smoothing filter (low-pass) and a *decimation* (3.5.6)

3.5.2
high-pass component
difference component
component of the *multiresolution analysis* (3.5) obtained after convolution with a difference filter (high-pass) and a *decimation* (3.5.6)

Note 1 to entry: The weighting function of the difference filter is defined by the wavelet from a particular family of wavelets, with a particular *dilation* (3.2.1) parameter and no *translation* (3.2.2).

Note 2 to entry: The filter coefficients require the evaluation of an integral over a continuous space unless there exists a complementary function to form the basis expanding the signal/profile.

3.5.3
multiresolution ladder structure
structure consisting of all the orders of the difference components and the highest order smooth component

3.5.4
scaling function
function which defines the weighting function of the smoothing filter used to obtain the smooth component

Note 1 to entry: In order to avoid loss of information on the *multiresolution ladder structure* (3.5.3), the wavelet and scaling function are matched.

Note 2 to entry: The *low-pass component* (3.5.1) is obtained by convolving the input data with the scaling function.

3.5.5
wavelet function
function which defines the weighting function of the difference filter used to obtain the detail component

Note 1 to entry: The *high-pass component* (3.5.2) is obtained by convolving the input data with the wavelet function.

3.5.6
decimation
<wavelet> action which samples every k-th point in a sampled profile, where k is a positive integer

Note 1 to entry: Typically, k is equal to 2.

3.6
lifting scheme
fast *wavelet transform* (3.3) that uses *splitting*, *prediction* and *updating stages* (3.6.1), (3.6.2), (3.6.3)

3.6.1
splitting stage
partition of a profile into "even" and "odd" subsets, in which each sequence contains half as many samples as the original profile

3.6.2
prediction stage
calculation which predicts the odd subset from the even subset and then removes the predicted value from the odd subset value

3.6.3
updating stage
calculation which updates the even subset from the odd subset, in order to preserve as many profile moments as possible

4 General wavelet description

4.1 General

A cubic prediction wavelet claiming to conform with this document shall satisfy the procedure given in Annex A.

A cubic spline wavelet claiming to conform with this document shall satisfy the procedure given in Annex B.

NOTE The relationship to the filtration matrix model is given in Annex C.

4.2 Basic usage of wavelets

Wavelet analysis consists of decomposing a profile into a linear combination of wavelets $g_{a,b}(x)$, all generated from a single mother wavelet[4]. This is similar to Fourier analysis, which decomposes a profile into a linear combination of sinewaves, but unlike Fourier analysis, wavelets are finite in both spatial and frequency domain. Therefore, they can identify the location as well as the scale of a feature in a profile. As a result, they can decompose profiles where the small-scale structure in one portion of the profile is unrelated to the structure in a different portion, such as localized changes (i.e. scratches, defects or other irregularities). Wavelets are also ideally suited for non-stationary profiles. Basically, wavelets decompose a profile into building blocks of constant shape, but of different scales.

4.3 Wavelet transform

The discrete wavelet transform[5] of a profile, $s(x)$, given as height values, $s(x_i)$, at uniformly sampled positions, $x_i = (i-1)\,\Delta x$ (where Δx is the sampling interval, $i = 1, ..., n$ and n being the number of sampling points), with the wavelet function $g((x-b)/a)$, is given by the differences (or details), $d_k(i)$, and the smoothed data, $s_k(i)$, and a subsequent decimation (down-sampling) for each level or rung, k, of decomposition. The smoothed data and differences are obtained by convolving the signal with the scaling function, h, and the wavelet, g, as shown in Formula (2a) and Formula (2b):

$$s_k(i) = \sum_j h_j s_{k-1}(i-j) \quad (2a)$$

$$d_k(i) = \sum_j g_j s_{k-1}(i-j) \quad (2b)$$

where $j = -m, ..., -2, -1, 0, 1, 2, ..., m$; ($m$ is the number of coefficients of the filter on one side from the centre).

The dilation parameter, a, is determined by the level of decomposition, k, and by down-sampling the smoothed data commonly by a factor of two, i.e. $a = 2^{-k}$, respectively. $a = 1/(2^k\,\Delta x)$, such that for each step of the decomposition ladder the number of smoothed data points reduces by a factor of two.

