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Application of statistical and related methods to new technology and product development process — Robust tolerance design (RTD)

Application des méthodes statistiques et des méthodes liées aux nouvelles technologies et de développement de produit — Plans d'expériences robustes

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Foreword

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 8, *Application of statistical and related methodology for new technology and product development*.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The designer of a product typically decides the specifications of the product and passes them on to the manufacturing section for use in manufacturing the product. The specifications include the designed nominal values and tolerances for the parts and/or elements of the product. The optimum nominal values of the design parameters are determined by robust parameter design (RPD), and the optimum tolerances are determined by robust tolerance design (RTD).

RPD, as described in ISO 16336, is applied to the product prior to RTD. In RPD, the major noise factors are used to evaluate robustness as measured by the signal-to-noise ratio, which represents the variability of product output. It is a measure for comparing robustness between levels of control factors. RPD identifies the combination of the values of the design parameters as an optimum RPD condition for minimizing the variability, that is, maximizing the robustness.

RTD, as described in this document, is a method for selecting the degree of errors of the parts or elements of the product from the viewpoint of variability under the optimum RPD condition, that is, the combination of optimum nominal values of the design parameters. If a manufactured product has errors from the designed nominal values, the product output will deviate from the designed value. The error in a design parameter should be smaller than the designed error limit to keep the product output within the designed variability. This is why the design parameters need a tolerance.

The design of a product can be finalized by setting the optimum error limits of the design parameters by using RTD. The expected variance in output of a product manufactured with errored parts or elements can be estimated using RTD. After RPD is used to identify a set of optimum values for the design parameters, RTD is used to check whether the estimated variance is smaller than the target variance under the optimum RPD condition.

RPD can be used to set the optimum nominal values of the design parameters without increasing manufacturing cost while RTD is closely related to the manufacturing cost. Smaller tolerances, meaning higher-grade parts or elements, result in higher costs, while larger tolerances, meaning lower-grade parts or elements, result in lower costs. To finalize the product design, the cost of manufacturing the product is considered. The loss function in the Taguchi methods is used to transform the benefits of an improvement in quality into a monetary amount, the same as a cost.

The cost of the improvement and the benefits of the improvement in quality should be balanced in deciding the tolerances. RPD and RTD together provide a cost-effective way of optimizing product design.

If RPD cannot achieve the product variability smaller than the target variability, the tolerances of the design parameters are reduced to improve the variability, but smaller tolerances result in higher costs.

On the other hand, if RPD can achieve the product variability much smaller than the target variability, the tolerances of the design parameters are increased to reduce manufacturing cost, so larger tolerances result in lower costs.

Products manufactured with optimum nominal values and tolerances of design parameters are robust to noise situations under usage conditions after shipment. Robust products minimize users' quality losses due to defects, failures, and quality problems.

Application of statistical and related methods to new technology and product development process — Robust tolerance design (RTD)

1 Scope

This document specifies guidelines for applying the robust tolerance design (RTD) provided by the Taguchi methods to a product in order to finalize the design of the product.

NOTE 1 RTD is applied to the target product to set the optimum tolerances of the design parameters around the nominal values. RTD identifies the effects of errors in the controllable design parameters on product output and estimates the total variance of the product output if the tolerances are changed. Hence, RTD achieves the target variance of the output from the viewpoints of robustness, performance, and cost.

NOTE 2 The tolerance expresses a maximum allowable error in the value of a design parameter in the manufacturing process. In a perfect world, the parts or elements of every product have the designed nominal values of the design parameters. However, actual manufacturing does not reproduce the exact designed nominal values of the design parameters for all products. The actual products have errors in the values of their parts or elements. These errors are supposed to be within the designed tolerances.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 16336, *Applications of statistical and related methods to new technology and product development process — Robust parameter design (RPD)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 16336 apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1
tolerance
difference between the upper specification limits and lower specification limits

3.2
robust tolerance design
RTD
method of setting optimum tolerances from the viewpoints of robustness, performance, and cost

4 Robust tolerance design

4.1 General

A company's product design section normally gives the specifications of a product, that is, the nominal values and tolerances of the design parameters, to the manufacturing section. The manufacturing section uses the designed specifications in manufacturing the product. When specifications specify the limits of a design parameter as $m \pm \Delta$, the parameter value x in the manufacturing process should satisfy the following restriction:

$$m - \Delta \leq x \leq m + \Delta, \tag{1}$$

where m and Δ denote a nominal value and its permissible difference, respectively. Only the symmetric ($\pm\Delta$) case is discussed in this document. In the symmetric case, the tolerance is 2Δ, and the permissible difference Δ is half the tolerance.

If the absolute error of a design parameter is larger than the specified permissible difference Δ, the variability in the product output cannot meet the designed performance and specifications.

RTD is used by the design section to set the optimum tolerance for each design parameter to achieve the designed performance, which is evaluated based on the total variance of the product output. The permissible difference of a design parameter is the maximum allowable error around the nominal value in the manufacturing process, and it is closely related to the cost of manufacturing.

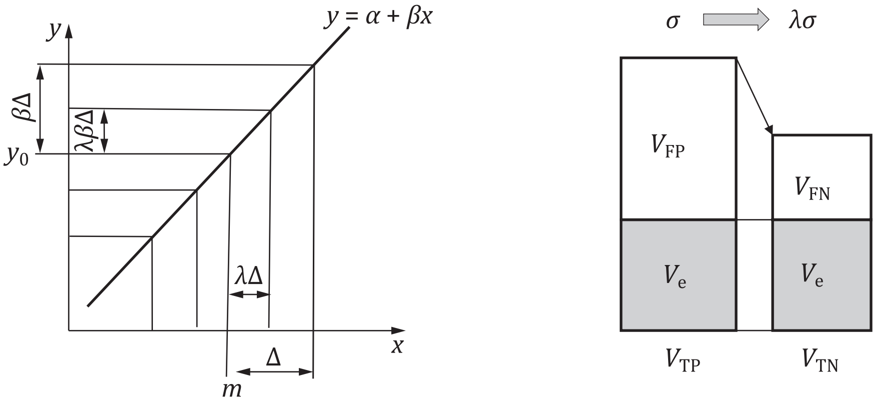
The optimum nominal values of the design parameters can be identified by robust parameter design (RPD) through robustness measure, signal-to-noise ratio[1]. The selection of a robust product by setting the nominal values as the optimum values using RPD prior to RTD is highly recommended. RPD can optimize the target product by choosing the optimum combination of design parameter nominal values from the viewpoint of the variability of the product output without increasing the cost[2].

If RPD cannot achieve a target variability, RTD is used to identify possible tolerances for achieving the target variability even at a higher cost. Smaller tolerances result in smaller variability, but this requires upgrading the parts or elements of the product, which leads to higher manufacturing cost. RTD is used to investigate the balance between product quality and improvement cost.

Even if RPD achieves the target variance, RTD is used, in some cases, to identify larger tolerances than those considered in RPD. Larger tolerances mean larger variability, but if the increased variability satisfies the target variability, the larger tolerances are applicable as they lead to reduced cost of manufacturing the designed product.

The purpose of RTD is to achieve the target variability by setting optimum tolerances from the viewpoints of robustness, performance, and cost. For this purpose, RTD estimates the total variance of the output of the designed product if the tolerance of a design parameter is changed. The total variance can be estimated based on the results of analysis of variance (ANOVA).

Assume that a value x of design parameter F has a linear effect on output y of the product, as shown in Figure 1 a). If the present permissible difference of x in F is $\Delta_\mathrm{P} = \Delta$, the error distribution of F affects output y with a magnitude of $\beta\Delta$. If the permissible difference Δ of F is reduced to new permissible difference $\Delta_\mathrm{N} = \lambda\Delta$ [$\lambda<1$ in Figure 1 a)], the effect of changing Δ in F on the output is reduced to $\lambda\beta\Delta$, and the variance in y due to changing Δ in F is reduced from the present variance V_FP to new variance $V_\mathrm{FN} = \lambda^2 V_\mathrm{FP}$. As a result, the total output variance is reduced from V_TP to V_TN [Figure 1 b)].



a) Linear dependence on *x* in F **b) Change in total variance**

Figure 1 — Effect of changing Δ in design parameter F on total variance

The new total variance V_{TN} can be estimated as

$$V_{TN} = V_{FN} + V_e = \lambda^2 V_{FP} + V_e \, , \tag{2}$$

where $\lambda = {}^{\Delta_N}\!/\!_{\Delta_P}$ is assumed.

If the tolerance of a design parameter is reduced, that is, $\lambda<1$, the magnitude of error of the design parameter becomes smaller, and the total output variance is reduced. A smaller tolerance means that an upgraded part or element is used, so the cost of producing the new design can be higher than that of the present design.

If the tolerance of a design parameter is enlarged, that is, $\lambda>1$, the magnitude of error of the design parameter becomes larger, and the total output variance is enlarged. A larger tolerance means that a down-graded part or element is be used, so the cost of producing the new design can be smaller than that of the present design.

RTD comprises two steps, as follows.

1) RTD experimentation: Collect data on the designed product, and analyse the data to determine the dependence of the product output on the design parameters.

2) Tolerance determination: Estimate the total variance if a tolerance is changed, and compare the effects in quality with the cost of the change to identify the optimum tolerance.

RTD experiments collect the output data of the designed product in which there are errors in the product design parameters, and estimate the total variance and its dependence on the design parameters. The experimental design plan is used to collect the data under the combination of design parameter errors. The ANOVA results show the effects of errors in the design parameters on the product output. The product output has a target variance from the viewpoints of robustness and performance.

In RTD experiments, the design parameters are taken as noise factors. A noise factor is an experimental factor which is taken into experiment for the purpose of estimating its variability. The variance in the linear effect of errors in the design parameters is estimated.

In RPD, on the other hand, the design parameters are taken as control factors. A control factor is an experimental factor which is taken into experiment for the purpose of selecting the optimum level of the factor. Designers can fix the nominal values of design parameters to the optimum RPD values. However, in actual manufacturing, the parts or elements of the product invariably have errors, so the designer cannot specify the error of a design parameter. The designer can set only the permissible difference Δ as an error limit.

Design parameter errors cause variability in product output. If the error of a design parameter has a linear effect on the product output, the output variance can be changed by resetting the tolerance of the design parameter. RTD experimentation is used to determine the contributions of the effects of errors in design parameters to the product output.

In the tolerance determination step of RTD, the change in the output variance due to resetting a tolerance is estimated, and the designer selects optimum tolerance for achieving the target output variance. The optimum tolerance can be determined by balancing the effect in quality due to a tolerance change against the cost of the tolerance change[3].

4.2 RTD experimentation

4.2.1 Data generation

RTD experimentation is used to determine the design parameters' linear effects for the designed product. The relationship between the output by the product and the errors in the design parameters is investigated. The output data can be generated in three ways:

1) by using a theoretical formula,

2) by experimentation with an actual product;

3) by simulation experimentation.

When the theoretical relationship between the product output and the design parameters is known, the output data can be directly calculated for various combination of the design parameter values. RTD offers multi-factor design as an experimental design for generating the output data in various combinations of the level of experimental factors, as shown in case study (1) in Clause 5. ANOVA is used for analysing the dependence of the product output on the factors.

Mathematical analysis can be applied in this case. Mathematical analysis consists of using variance estimates for a system by, for example, propagating an input variance through the system via Taylor series expansions of moment generating functions[4].

If an actual product can be constructed, it can be used for experimentation, and the data output can be collected using the actual experiment. However, in many cases, it is difficult to set the intended levels of the errors of design parameters in an actual product because the noise levels cannot be controlled within the error distribution of the design parameters. Simulation experimentation can be used in such cases. This is why simulation experiments are often used in RTD. A simulation program can provide the product output data, as shown in case study (2) in Clause 6.

4.2.2 Experimental design for data collection

RTD experimentation is used for collecting output data for the designed product under the combinations of design parameter errors. There are many design parameters, and a multi-factor experimental design is used to generate various such combinations. The purpose of RTD experimentation is to determine the main effects of experimental factors. An orthogonal array plan is recommended as a multi-factor experimental plan for collecting the data as it is an efficient way to collect data for an RTD experiment.

An orthogonal array plan can reduce the number of experimental runs compared with a full-factorial plan for the same number of factors and can assign the maximum number of factors in a plan for the same number of experimental runs. The main effects of factors can be estimated under the condition of a balanced combination of the other factors' levels. The choice of the orthogonal array depends on the numbers of factors and their levels[3].

An example orthogonal array (L_{18}) is shown in Table 1. Seven experimental factors with three levels (B-H) and one factor with two levels (A) can be assigned to the columns in the array. Rows represent the experimental run. The number in each cell represents the level of the factor assigned to the column. The experimental run of low No. 1 should be performed under the combination of factor's levels A1B1C1D1E1F1G1H1.

For RTD, the design parameters are assigned to the columns as noise factors. For the purpose of estimating the linear and non-linear effects of a factor, each factor has at least three levels. However, if the proportional property is obvious for a factor, a two-level setting is sufficient. Two-level factor is assigned to the first column. The last column in Table 1 shows the output data y_i calculated for the combination of factors' levels shown in the cells in the same low

Table 1 — Example of orthogonal array L_{18} and output data

Column	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Data output**
No.	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	
1	1	1	1	1	1	1	1	1	y_1
2	1	1	2	2	2	2	2	2	y_2
3	1	1	3	3	3	3	3	3	y_3
4	1	2	1	1	2	2	3	3	y_4
5	1	2	2	2	3	3	1	1	y_5
6	1	2	3	3	1	1	2	2	y_6
7	1	3	1	2	1	3	2	3	y_7
8	1	3	2	3	2	1	3	1	y_8
9	1	3	3	1	3	2	1	2	y_9
10	2	1	1	3	3	2	2	1	y_{10}
11	2	1	2	1	1	3	3	2	y_{11}
12	2	1	3	2	2	1	1	3	y_{12}
13	2	2	1	2	3	1	3	2	y_{13}
14	2	2	2	3	1	2	1	3	y_{14}
15	2	2	3	1	2	3	2	1	y_{15}
16	2	3	1	3	2	3	1	2	y_{16}
17	2	3	2	1	3	1	2	3	y_{17}
18	2	3	3	2	1	2	3	1	y_{18}

Table 2 shows an example of level setting of factors for RTD. The upper and lower permissible differences are assumed to be the same for simplicity. The levels of the factors are set around nominal value *m* with level width *d*. Nominal value *m* is set to an optimum value by RPD from the viewpoint of robustness. Level width *d* is set from the actual standard deviation of the design parameter if it is known.

Table 2 — Example of level settings of factors for RTD

Factor	**1**	**2**	**3**
A	$m_A - d_A$	$m_A + d_A$	—
B	$m_B - d_B$	m_B	$m_B + d_B$
C	$m_C - d_C$	m_C	$m_C + d_C$
D	$m_D - d_D$	m_D	$m_D + d_D$
E	$m_E - d_E$	m_E	$m_E + d_E$
F	$m_F - d_F$	m_F	$m_F + d_F$
G	$m_G - d_G$	m_G	$m_G + d_G$
H	$m_H - d_H$	m_H	$m_H + d_H$

When the actual standard deviation σ_x of the error in the design parameter is not exactly known, the assumption $\sigma_x = \Delta/2$ or $\sigma_x = \Delta/3$ can be applied.

When the actual standard deviation σ_x of the error in the design parameter is known, the level width *d* and the levels of the factors are set as follows.

For a two-level factor, $d=\sigma_x$:

X1: First level $x_1 = m-\sigma_x$, (3)

X2: Second level $x_2 = m+\sigma_x$. (4)

For a three-level factor, $d=\sqrt{\frac{3}{2}}\sigma_x$:

X1: First level $x_1 = m-d = m-\sqrt{\frac{3}{2}}\sigma_x$, (5)

X2: Second level $x_2 = m$, (6)

X3: Third level $x_3 = m+d = m+\sqrt{\frac{3}{2}}\sigma_x$. (7)

Setting the level of the factors in this way makes the estimated variance $\sigma_{y\ell}^2$ of output *y* caused by the linear effect of the error in the factor $\beta^2\sigma_x^2$, where *β* represents the linear coefficient of the relationship $y=\beta x$ between output *y* and input *x*.

If y_{ij} $(i=1,\cdots,n,\ j=1,\cdots,r)$ represents the output from *j*-th run in *r* repeated runs on *i*-th level x_i in *n* level factor, the linear coefficient *β* and the sum of squares of linear effect S_β are calculated as

$$\beta = \frac{\sum_{i=1}^{n}\sum_{j=1}^{r}(x_i-\bar{x})(y_{ij}-\bar{y})}{r\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{8}$$

$$S_\beta = \frac{\left[\sum_{i=1}^{n}\sum_{j=1}^{r}(x_i-\bar{x})(y_{ij}-\bar{y})\right]^2}{r\sum_{i=1}^{n}(x_i-\bar{x})^2} = r\sum_{i=1}^{n}(x_i-\bar{x})^2\cdot\beta^2. \tag{9}$$

For a two-level factor A with levels $x_1=\bar{x}-d$ and $x_2=\bar{x}+d$, the sum of squares of linear effect S_β is calculated as $S_\beta = r\cdot 2d^2\beta^2$. If the linear effect of factor A is significant, S_β approximately represents $2r\sigma_{y\ell}^2$, where 2*r* denotes the number of data items and $\sigma_{y\ell}^2$ denotes the variance of each. If level width *d* is set to σ_x, $S_\beta = 2rd^2\beta^2 = 2r\sigma_x^2\beta^2 \cong 2r\sigma_{y\ell}^2$. Then variance $\sigma_{y\ell}^2$ in output *y* caused by the linear effect of the error in the factor becomes $\sigma_{y\ell}^2 = \beta^2\sigma_x^2$.

For a three-level factor B with levels $x_1=\bar{x}-d, x_2=\bar{x}$, and $x_3=\bar{x}+d$, the sum of squares of linear effect S_β is calculated as $S_\beta = r\cdot 2d^2\beta^2$. If the linear effect of the factor is significant, S_β approximately represents $3r\sigma_{y\ell}^2$, where 3*r* denotes the number of data items. If the level width *d* is set to $\sqrt{\frac{3}{2}}\sigma_x$, $S_\beta = 2rd^2\beta^2 = 2r\cdot\frac{3}{2}\sigma_x^2\cdot\beta^2 = 3r\sigma_x^2\beta^2 \cong 3r\sigma_{y\ell}^2$. Then variance $\sigma_{y\ell}^2$ in output *y* caused by the linear effect of the error in the noise factor becomes $\sigma_{y\ell}^2 = \beta^2\sigma_x^2$.