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Control charts —

Part 4: Cumulative sum charts

Cartes de contrôle —

Partie 4: Cartes de contrôle de l'ajustement de processus

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This second edition of ISO 7870-4 cancels and replaces the first edition (ISO 7870-4: 2011), which has been technical revised.

The main changes compared to the previous edition are as follows:

— Manhattan diagram removed (former 6.7);

— V-mask types in Types of CUSUM decision schemes reduced to one V-mask;

— von Neumann method removed (former Annex A).

A list of all parts in the ISO 7870 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document demonstrates the versatility and usefulness of a very simple, yet powerful, pictorial method of interpreting data arranged in any meaningful sequence. These data can range from overall business figures such as turnover, profit or overheads to detailed operational data such as stock outs and absenteeism to the control of individual process parameters and product characteristics. The data can either be expressed sequentially as individual values on a continuous scale (e.g. 24, 60, 31, 21, 18, 97...), in 'yes'/'no', 'good'/'bad', 'success'/'failure' format, or as summary measures (e.g. mean, range, counts of events).

The method has a rather unusual name, cumulative sum, or CUSUM. This name relates to the process of subtracting a predetermined value, e.g. a target, preferred or reference value from each observation in a sequence and progressively cumulating (i.e. adding) the differences. The graph of the series of cumulative differences is known as a CUSUM chart. Such a simple arithmetical process has a remarkable effect on the visual interpretation of the data.

The CUSUM method is already used unwittingly by golfers throughout the world. By scoring a round as 'plus' 4, or perhaps even 'minus' 2, golfers are using the CUSUM method in a numerical sense. They subtract the 'par' value from their actual score and add (cumulate) the resulting differences. This is the CUSUM method in action. However, it remains largely unknown and hence is a grossly underused tool throughout business, industry, commerce and public service. This is probably due to CUSUM methods generally being presented in statistical language rather than in the language of the workplace.

The intention of this document is, thus, to be readily comprehensible to the extensive range of prospective users and so facilitate widespread communication and understanding of the method. The method offers advantages over the more commonly found Shewhart charts in as much as the CUSUM method detects a change of an important amount up to three times faster. Further, as in golf, when the target changes per hole, a CUSUM plot is unaffected, unlike a standard Shewhart chart where the control lines require constant adjustment.

In addition to Shewhart charts, an EWMA (exponentially weighted moving average) chart can be used. Each plotted point on an EWMA chart incorporates information from all the previous subgroups or observations but gives less weight to process data as they get 'older' according to an exponentially decaying weight. In a similar manner to a CUSUM chart, an EWMA chart can be sensitized to detect any size of shift in a process. This subject is discussed further in 7870-6.

Control charts —

Part 4: **Cumulative sum charts**

1 Scope

This document describes statistical procedures for setting up cumulative sum (CUSUM) schemes for process and quality control using variables (measured) and attribute data. It describes general-purpose methods of decision-making using cumulative sum (CUSUM) techniques for monitoring, control and retrospective analysis.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions, abbreviated terms and symbols

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1 Terms and definitions

3.1.1
target value
T
value for which a departure from an average level is required to be detected

Note 1 to entry: With a charted CUSUM, the deviations from the target value are cumulated.

Note 2 to entry: Using a 'V' mask, the target value is often referred to as the reference value or the nominal control value. If so, it needs be acknowledged that it is not necessarily the most desirable or preferred value, as can appear in other standards. It is simply a convenient target value for constructing a CUSUM chart.

3.1.2
representative out of control value
⟨tabulated CUSUM⟩ value which controls the sensitivity of the procedure

Note 1 to entry: The upper out of control value is $T + f\sigma_e$, for monitoring an upward shift. The lower control value is $T - f\sigma_e$, for monitoring a downward shift.

3.1.3
reference shift
F, *f*
⟨tabulated CUSUM⟩ difference between the *target value* (3.1.1) and the *representative out of control value* (3.1.2)

Note 1 to entry: It is necessary to distinguish between *f*, that relates to a standardized reference shift, and *F*, that relates to an observed reference shift; $F = f\sigma_e$. It plays a crucial role for constructing the tabular form of the CUSUM chart.

3.1.4
decision interval
H, *h*
⟨tabulated CUSUM⟩ cumulative sum of deviations from a *representative out of control value* (3.1.2) required to yield a signal

Note 1 to entry: It is necessary to distinguish between *h*, that relates to a standardized decision interval, and *H*, that relates to an observed decision interval; $H = h\sigma_e$.

3.1.5
average run length
ARL
average number of samples taken up to the point at which a signal occurs

Note 1 to entry: The average run length (ARL) is usually related to a process level, in which case it carries an appropriate subscript as, for example, ARL_0, meaning the average run length when the process is at target level, i.e. zero shift.

3.2 Abbreviated terms

ARL average run length

CS1 CUSUM scheme with a long ARL at zero shift

CS2 CUSUM scheme with a shorter ARL at zero shift

FIR fast initial response

LCL lower control limit

RL run length

SPC statistical process control

UCL upper control limit

3.3 Symbols

a scale coefficient

C CUSUM value

c_4 factor for estimating the within-subgroup standard deviation

δ amount of change to be detected

Δ standardized amount of change to be detected

d lead distance

d_2 factor for estimating the within-subgroup standard deviation from within-subgroup range

F observed reference shift

f standardized reference shift

K reference value, equal to sum of target T and observed reference shift F

H observed decision interval

h standardized decision interval

ARL_{δ} average run length at δ shift

ARL_0 average run length at no shift μ population mean value

n subgroup size

m number of subgroups within a preliminary study

p probability of ‘success’

$\bar{R}$ mean subgroup range

σ process standard deviation

σ_0 within-subgroup standard deviation

$\hat{\sigma}_0$ estimated within-subgroup standard deviation

σ_e standard error

s observed within-subgroup standard deviation

$\bar{s}$ average subgroup standard deviation

$s_{\bar{x}}$ realized standard error of the mean from m subgroups

T target value

τ true change point

$\hat{\tau}$ estimated change point

x individual result

$\bar{x}$ arithmetic mean value (of a subgroup)

$\bar{\bar{x}}$ mean of subgroup means

4 Principal features of cumulative sum (CUSUM) charts

A CUSUM chart is essentially a running total of deviations from some preselected reference value. The mean of any group of consecutive values is represented visually by the current slope of the graph. The principal features of a CUSUM chart are the following.

a) It is sensitive in detecting changes in the mean.

b) Any change in the mean, and the extent of the change, is indicated visually by a change in the slope of the graph:

1) a horizontal graph indicates an ‘on-target’ or at reference value;

2) a downward slope indicates a mean less than the reference or target value: the steeper the slope, the bigger the difference;

3) an upward slope indicates a mean more than the reference or target value: the steeper the slope, the bigger the difference.

c) It can be used retrospectively for investigative purposes, on a running basis for control, and for prediction of performance in the immediate future.

Referring to point b) above, a CUSUM chart has the capacity to clearly indicate points of change; these are clearly indicated by the change in gradient of the CUSUM plot. This has enormous benefit for process management: to be able to quickly and accurately pinpoint the moment when a process altered so that the appropriate corrective action can be taken.

A further very useful feature of a CUSUM system is that it can be handled without plotting, i.e. in tabular form. This is very helpful if the system is to be used to monitor a highly technical process, e.g. plastic film manufacture, where the number of process parameters and product characteristics is large. Data from such a process can be captured automatically, downloaded into CUSUM software to produce an automated CUSUM analysis. A process manager can then be alerted to changes on many characteristics on a simultaneous basis. Annex A contains an example of the method.

5 Basic steps in the construction of CUSUM charts — Graphical representation

The following steps are used to set up a CUSUM chart for individual values.

Step 1 — Choose a reference, target, control or preferred value. The average of past results generally provides good discrimination.

Step 2 — Tabulate the results in a meaningful (e.g. chronological) sequence. Subtract the reference value from each result.

Step 3 — Progressively sum the values obtained in Step 2. These sums are then plotted as a CUSUM chart.

Step 4 — For reasonable discrimination, without undue sensitivity, the following options are recommended:

a) choose a convenient plotting interval for the horizontal axis and make the same interval on the vertical axis equal to 2σ (or $2\sigma_e$ if a CUSUM of means is to be charted), rounding off as appropriate; or

b) where it is required to detect a known change, say δ, choose a vertical scale such that the ratio of the scale unit on the vertical scale divided by the scale unit on the horizontal scale is between δ and 2δ, rounding off as appropriate.

NOTE The scale selection is visually very important since an inappropriate scale gives either the impression of impending disaster due to the volatile nature of the plot, or a view that nothing is changing. The schemes described in a) and b) above can give a scale that shows changes in a reasonable manner, neither too sensitive nor too suppressed.

6 Example of a CUSUM plot — Motor voltages

6.1 Process

Suppose a set of 40 values in chronological sequence is obtained of a characteristic. These happen to be voltages, taken in order of production, on fractional horsepower motors at an early stage of production. But they can be any individual values taken in a meaningful sequence and expressed on a continuous scale. These are now shown:

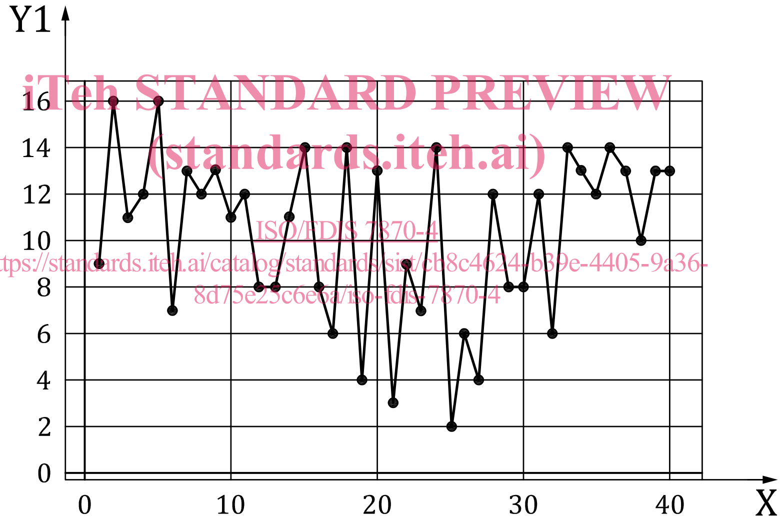
9, 16, 11, 12, 16, 7, 13, 12, 13, 11, 12, 8, 8, 11, 14, 8, 6, 14, 4, 13, 3, 9, 7, 14, 2, 6, 4, 12, 8, 8, 12, 6, 14, 13, 12, 14, 13, 10, 13, 13.

The reference or target voltage value is 10 V.

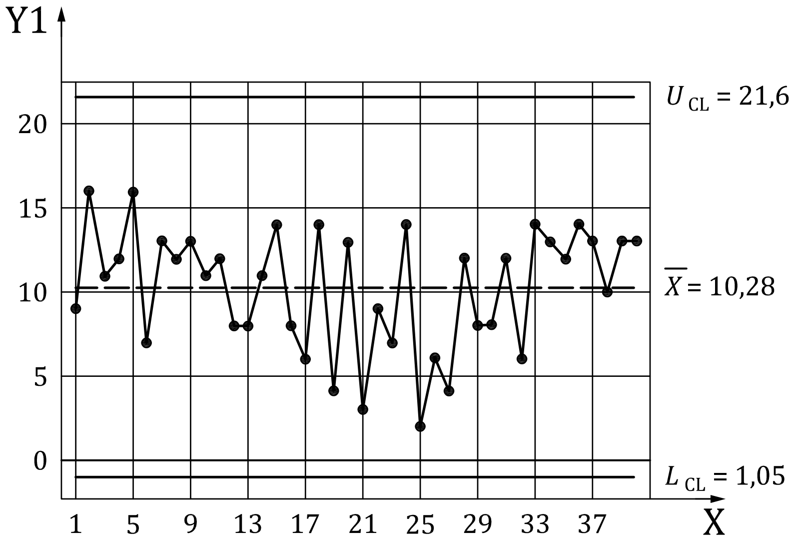
6.2 Simple plot of results

To gain a better understanding of the underlying behaviour of the process, by determining patterns and trends, a standard approach is simply to plot these values in their natural order as shown in Figure 1 a).

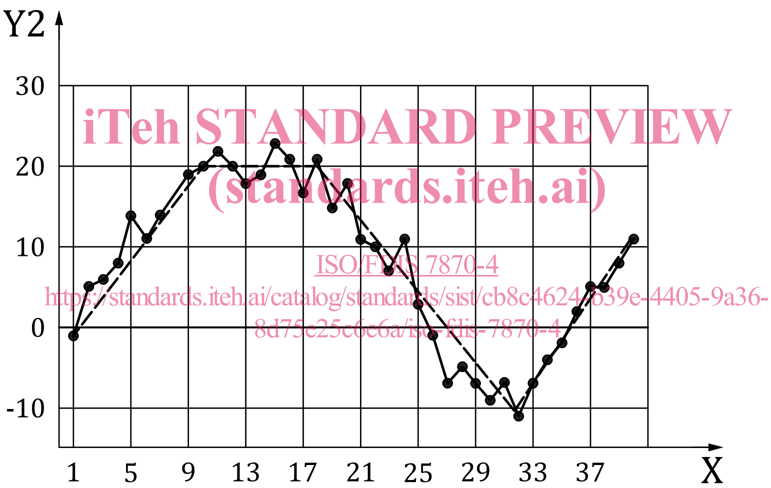
Apart from indicating a general drop away in the middle portion from a high start and with an equally high finish, Figure 1 a) is not very revealing because of the extremely noisy, or spiky, data throughout.



a) Simple plot of motor voltages



b) Standard control chart for individuals



c) CUSUM chart

Key

X motor number

Y1 voltage

Y2 cumulative sum

Figure 1 — Motor voltage example

6.3 Standard control chart for individual results

The next level of sophistication is to establish a standard control chart for individuals as in Figure 1 b).

Figure 1 b) is even less revealing than the previous figure. It is, in fact, quite misleading. The standard statistical process control criterion to test for process stability and control is just “no points lying above the upper control limit (UCL) or below the lower control limit (LCL)”. All points reside within these limits. Hence, one can be led to the conclusion that this is a stable process, one that is ‘in control’ around its overall average value of about 10 V, which is the target value. Further standard analysis would reveal that although the process is stable, it is not capable of meeting the specification requirements.

However, this analysis would not in itself provide any further clues as to why it is incapable of meeting the requirements.

The reason for the inability of the standard control chart for individuals to be of value is that the control limits are based on actual process performance and not on desired or specified requirements. Consequently, if the process naturally exhibits a large variation the control limits are correspondingly wide. What is required is a method that is better at indicating patterns and trends, or even pinpointing points of change, to help determine and remove primary sources of variation.

NOTE By using additional tools, such as an individual and moving range chart, the practitioner can study other process variation issues.

6.4 CUSUM chart construction

The construction of a CUSUM chart using individual values, as in this example, is based on the very simple steps given in Clause 5.

Step 1 — Choose a target value, T. The preferred or reference value is given as 10 V.

Step 2 — Tabulate the results (voltages) in production sequence against motor number as in Table 1, column 2 (and 6). Subtract the reference value of 10 from each result as in Table 1, column 3 (and 7).

Step 3 — Progressively sum the values of Table 1, column 3 (and 7) in column 4 (and 8). Plot column 4 (and 8) against the observation (motor) number as in Figure 1 c), taking note of the scale comments in Steps 4 and 5.

Table 1 — Tabular arrangement for calculating CUSUM values from a sequence of individual values

(1) Motor no.	(2) Voltage	(3) Voltage −10	(4) CUSUM	(5) Motor no.	(6) Voltage	(7) Voltage −10	(8) CUSUM
1	9	−1	−1	21	3	−7	+11
2	16	+6	+5	22	9	−1	+10
3	11	+1	+6	23	7	−3	+7
4	12	+2	+8	24	14	+4	+11
5	16	+6	+14	25	2	−8	+3
6	7	−3	+11	26	6	−4	−1
7	13	+3	+14	27	4	−6	−7
8	12	+2	+16	28	12	+2	−5
9	13	+3	+19	29	8	−2	−7
10	11	+1	+20	30	8	−2	−9
11	12	+2	+22	31	12	+2	−7
12	8	−2	+20	32	6	−4	−11
13	8	−2	+18	33	14	+4	−7
14	11	+1	+19	34	13	+3	-4
15	14	+4	+23	35	12	+2	−2
16	8	−2	+21	36	14	+4	+2
17	6	−4	+17	37	13	+3	+5
18	14	+4	+21	38	10	0	+5
19	4	−6	+15	39	13	+3	+8
20	13	+3	+18	40	13	+3	+11

7 Fundamentals of making CUSUM-based decisions

7.1 Need for decision rules

Decision rules are needed to rationalize the interpretation of a CUSUM chart. When an appropriate decision rule so indicates, some action is taken, depending on the nature of the process. Typical actions are:

a) for in-process control: adjustment of process conditions;

b) in an improvement context: investigation of the underlying cause of the change; and

c) in a forecasting mode: analysis of and, if necessary, adjustment to the forecasting model or its parameters.

7.2 Basis for making decisions

Establishing the base criteria against which decisions are to be made is obviously an essential prerequisite.

To provide an effective basis for detecting a signal, a suitable quantitative measure of 'noise' in the system is required. What represents noise, and what represents a signal, is determined by the monitoring strategy adopted, such as how many observations to take, and how frequently, and how to constitute a sample or a subgroup. Also, the measure used to quantify variation can affect the issue.

It is usual to measure inherent variation by means of a statistical measure termed either of the following.

a) Standard deviation: where individual observations are the basis for plotting CUSUMs.

The individual observations for calculation of the standard deviation are often taken from a homogeneous segment of the process data. This performance then becomes the more onerous criterion from which to judge. Any variation greater than this inherent variation is taken to arise from special causes indicating a shift in the mean of the series or a change in the natural magnitude of the variability, or both.

b) Standard error: where some function of a subgroup of observations, such as mean, median or range, forms the basis for CUSUM plotting.

The concept of subgrouping is that variation within a subgroup is made up of common causes with all special causes of variation occurring between subgroups. The primary role of the CUSUM chart is then to distinguish between common and special cause variation. Hence, the choice of subgroup is of vital importance. For example, making up each subgroup of four consecutively from a high-speed production process each hour, as opposed to one measurement taken every quarter of an hour to make up a subgroup of four every hour, gives very different variabilities on which to base a decision. The standard error would be minuscule in the first instance compared with the second. One CUSUM chart would be set up with consecutive part variation as the basis for decision-making as opposed to 15 min to 15 min variation for the other chart.

However, the prerequisite that stability should exist over a sufficient period to establish reliable quantitative measures, such as standard deviation or standard error, is too restrictive for some potential areas of application of the CUSUM method.

For instance, observations of a continuous process can exhibit small unimportant variations in the average level. It is required that it is against these variations that systematic or sustained changes should be judged. Illustrations are:

a) an industrial process is controlled by a thermostat or other automatic control device;

b) the quality of raw material input can be subject to minor variations without violating specification; and

c) in monitoring a patient's response to treatment, there might be minor metabolic changes connected with meals, hospital or domestic routine, etc., but any effect of treatment should be judged against the overall typical variation.

On the other hand, samples can comprise output or observations from several sources (administrative regions, plants, machines and operators). As such, there can be too much local variation to provide a realistic basis for assessing whether the overall average shifts. Because of this factor, data arising from a combination of sources should be treated with caution, as any local peculiarities within each contributing source might be overlooked. Moreover, variation between the sources might mask any changes occurring over the whole system as time progresses.

One of the important assumptions in CUSUM procedures is that the process standard deviation σ is stable. Therefore, before constructing the CUSUM procedure, any process should be assessed to see if it is in a state of statistical control (by using the R-chart, s-chart or moving range chart) so that a reliable estimate of σ can be obtained.

Serial correlation between observations can also manifest itself — namely, one observation might have some influence over the next. An illustration of negative serial correlation is the use of successive gauge readings to estimate the use of a bulk material, where an overestimate on one occasion tends to produce an underestimate on the next reading. Another example is where overordering in one month is compensated by underordering in the subsequent month. Positive serial correlation is likely in some industrial processes where one batch of material might partially mix with preceding and succeeding batches.

7.3 Measuring the effectiveness of a decision rule

7.3.1 Basic concepts

The ideal performance of a decision rule is for real changes of at least a prespecified magnitude to be detected immediately and for a process with no real changes to be allowed to continue indefinitely without giving rise to false alarms. In real life, this is not attainable. A simple and convenient measure of actual effectiveness of a decision rule is the average run length (ARL).

The ARL is the expected value of the number of samples (usually denoted as run length, RL) taken up to that which gives rise to a decision that a real change is present.

If no real change is present, the ideal value of the ARL is infinity. A practical objective in such a situation is to make the ARL large. Conversely, when a real change is present, the ideal value of the ARL is 1, in which case the change is detected when the next sample is taken. The choice of the ARL is a compromise between these two conflicting requirements. Making an incorrect decision to act when the process has not changed gives rise to 'overcontrol'. This, in effect, increases variability. Not taking appropriate action when the process has changed gives rise to 'under control'. This also, in effect, increases variability and results in increasing the cost of production.

The actual RL itself is subject to statistical variation. Sometimes one can be fortunate in obtaining no false alarms over a long run, or in detecting a change very quickly. Occasionally, an unfortunate run of samples can generate false alarms or mask a real change so that it does not yield a signal. The actual pattern of such variation deserves attention occasionally. Generally, however, the ARL is looked upon as a reasonable measure of effectiveness of a decision rule. The aim is summarized in Table 2.

Table 2 — ARL patterns and process conditions

True process condition	Required CUSUM response	Ideal response
At or near target	Long ARL (few false alarms)	ARL → infinity
Substantial departure from target	Short ARL (rapid detection)	ARL is 1