# INTERNATIONAL STANDARD 

# Information technology - JPEG XS low-latency lightweight image coding system - 

## Part 1:

Core coding system

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ISO/IEC 21122-1:2019

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## Foreword

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## Introduction

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# Information technology — JPEG XS low-latency lightweight image coding system - 

## Part 1: <br> Core coding system

## 1 Scope

This document defines a syntax (and an accompanying decompression process) that is capable to represent continuous-tone grey-scale, or continuous-tone colour digital images without visual loss at moderate compression rates. Typical compression rates are between 2:1 and 6:1 but can also be higher depending on the nature of the image. In particular, the syntax and the decoding process specified in this document allow lightweight encoder and decoder implementations that limit the end-to-end latency to a fraction of the frame size. However, the definition of transmission channel buffer models necessary to ensure such latency is beyond the scope of this document.

This document:

- specifies a decoding process for converting compressed image data to reconstructed image data;
- specifies a codestream syntax containing information forinterpreting the compressed image data;
(standards.ilen.al)
- provides guidance on encoding processes for converting source image data to compressed image data.

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2 Normative references .teh aicatalogstandards/sista0c8ec3e-12b2-4bcb-87a6-
b943331346ec/iso-iec-21122-1-2019
There are no normative references in this document.

## 3 Terms and definitions, abbreviated terms and symbols

### 3.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.
ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at https://www.iso.org/obp
- IEC Electropedia: available at http://www.electropedia.org/


### 3.1.1

## band

input data to a specific wavelet filter type (3.1.49) that contributes to the generation of one of the components (3.1.13) of the image

### 3.1.2

band type
single number collapsing the information on the component (3.1.13), and horizontal and vertical wavelet filter types (3.1.49) that are applied in the filter cascade reconstructing spatial image samples (3.1.42) from inversely quantized wavelet coefficients (3.1.10)

### 3.1.3

bit
binary choice encoded as either 0 or 1

### 3.1.4

bitplane
array of bits (3.1.3) having all the same significance (3.1.31)

### 3.1.5 <br> bitplane count

number of significant bitplanes (3.1.4) of a code group (3.1.9), counting from the LSB up to the most significant, non-empty bitplane

### 3.1.6 <br> bitplane count subpacket

subset of a packet (3.1.34) which decodes to the bitplane counts (3.1.5) of all code groups (3.1.9) within a packet, followed by padding (3.1.35) and optional filler bytes (3.1.24)

Note 1 to entry: See subclause C.5.3.

### 3.1.7

byte
group of 8 bits (3.1.3)

### 3.1.8 codestream

compressed image data representation that includes all necessary data to allow a (full or approximate) reconstruction of the sample (3.1.42) values of a digitatimageelh.ai)
3.1.9 code group ISO/IEC 21122-1:2019
group of quantization indices (37140) in signamagnitudesirepresentation4before6inverse quantization (3.1.25) b943331346ec/iso-iec-21122-1-2019
3.1.10
coefficient
input value to the inverse wavelet transformation resulting from inverse quantization (3.1.25)
3.1.11
column
set of vertically aligned precincts (3.1.36)
3.1.12
compression
process of reducing the number of bits (3.1.3) used to represent source image data

### 3.1.13 <br> component

two-dimensional array of samples (3.1.42) having the same designation such as red, green or blue in the output or display device

### 3.1.14

continuous-tone image
image whose components (3.1.13) have more than one bit (3.1.3) per sample (3.1.42)

### 3.1.15

## data subpacket

subset of a packet (3.1.34) which consists of the quantization index magnitudes (3.1.41), followed by padding (3.1.35) and optional filler bytes (3.1.24)

Note 1 to entry: See subclause C.5.4.

### 3.1.16

## deadzone quantizer

quantizer whose zero bucket has a size different from all other buckets
Note 1 to entry: Based on this, inverse deadzone quantizers can be defined as inverse quantizers whose zero bucket has a size different from all other buckets.

### 3.1.17

## decoder

embodiment of a decoding process (3.1.18)

### 3.1.18

## decoding process

process which takes as its input a codestream (3.1.8) and outputs a continuous-tone image (3.1.14)

### 3.1.19 <br> decomposition level

set of wavelet coefficients (3.1.10) resulting from a particular level of recursive application of a wavelet transform

### 3.1.20

encoder
embodiment of an encoding process (3.1.23)

### 3.1.21 <br> encoding process

process which outputs compressed image data in the form of a codestream (3.1.8)

### 3.1.22 <br> (standards.iteh.ai)

## entropy decoding

lossless (3.1.28) procedure (3.1.38) whish/recovers-the sequence of symbols from the sequence of bits (3.1.3) produced bylanséntropylencoding (3.11233) procedure3e-12b2-4bcb-87a6-
b943331346ec/iso-iec-21 122-1-2019
3.1.23

## entropy encoding

lossless (3.1.28) procedure (3.1.38) which converts a sequence of input symbols into a sequence of bits (3.1.3) such that the average number of bits per symbol approaches the entropy of the input symbols

### 3.1.24

## filler bytes

integer number of bytes (3.1.7) a decoder (3.1.17) will skip over on decoding without interpreting the values of the bytes itself

### 3.1.25

## inverse quantization

inverse procedure (3.1.38) to quantization (3.1.39) by which the decoder (3.1.17) recovers a representation of the coefficients (3.1.10)

### 3.1.26

inverse reversible multi component transformation
inverse RCT
inverse transformation across multiple component (3.1.13) sample (3.1.42) values located at the same sample grid (3.1.43) point that is invertible without loss

Note 1 to entry: See subclauses F. 3 and F.4.

### 3.1.27

LL band
input to a series of wavelet filters where only inverse low-pass filters are applied in horizontal and vertical direction

### 3.1.28

## lossless

being such that, for encoding and decoding procedures (3.1.38), the output of the decoding procedure(s) is identical to the input to the encoding procedure(s)

### 3.1.29

## lossless coding

mode of operation which refers to any one of the coding processes defined in this document in which all of the procedures (3.1.38) are lossless (3.1.28)

### 3.1.30

## sign subpacket

subset of a packet (3.1.34) that consists of the sign information of all non-zero quantization indices (3.1.40) within a packet, followed by padding (3.1.35) and optional filler bytes (3.1.24)

Note 1 to entry: See subclause C.5.5.

### 3.1.31

## significance

attribute of code groups (3.1.9) that applies if, depending on the Run Mode flag in the picture header, either at least one of coefficients (3.1.10) in the code group is non-zero, or the bitplane count (3.1.5) prediction residual of the code group is non-zero

### 3.1.32

## significance group

group of horizontally adjacent'co de groups (3.1.9) Sharing the same significance(3.1.31) information in the significance subpacket (3.1.33)

### 3.1.33

## (standards.iteh.ai)

## significance subpacket

subset of a packet (3.1.34) that identifies which significance groups (3.1.32) within a packet are insignificant, followed by padding (3.1.35) and optionalfiller bytes (3.1.24)

Note 1 to entry: See subclause C.5.2.

### 3.1.34

## packet

segment of the codestream (3.1.8) containing entropy coded information on a single precinct (3.1.36), line and a subset of the bands (3.1.1) within this precinct and line

### 3.1.35

padding
bits (3.1.3) within the codestream (3.1.8) whose only purpose is to align syntax elements to byte (3.1.7) boundaries and that carry no information

### 3.1.36

## precinct

collection of quantization indices (3.1.40) of all bands (3.1.1) contributing to a given spatial region of the image

### 3.1.37

## precision

number of bits (3.1.3) allocated to a particular sample (3.1.42), coefficient (3.1.10), or other binary numerical representation

### 3.1.38 <br> procedure

set of steps which accomplishes one of the tasks which comprise an encoding (3.1.23) or decoding process (3.1.18)

### 3.1.39

## quantization

method of reducing the precision (3.1.37) of the individual coefficients (3.1.10)

### 3.1.40

## quantization index

input to the inverse quantization (3.1.25) process which reconstructs a wavelet coefficient (3.1.10)

### 3.1.41

quantization index magnitude
absolute value of a quantization index (3.1.40)

### 3.1.42

## sample

single element in the two-dimensional image array which comprises a component (3.1.13)

### 3.1.43 <br> sample grid

common coordinate system for all samples (3.1.42) of an image, where the samples at the top left edge of the image have the coordinates $(0,0)$, the first coordinate increases towards the right, the second towards the bottom

### 3.1.44

slice
integral number of precincts (3.1.36) whose wavelet coefficients (3.1.10) can be entropy-decoded independently iTTeh STANDARD PREVIEW
3.1.45
subpacket
(standards.iteh.ai)
substructure of a packet (3.1.34) containing information of one or multiple bands (3.1.1) of one line of a single precinct (3.1.36)
https.//standards.iteh.ai/catalog/standards/sista0c8ec3e-12b2-4bcb-87a6-
3.1.46
b943331346ec/iso-iec-21122-1-2019
truncation position
number of least significant bitplanes (3.1.4) not included in the quantization index (3.1.40) of a wavelet coefficient (3.1.10)

### 3.1.47 <br> uniform quantizer

quantizer whose buckets are all of equal size
Note 1 to entry: Based in this, inverse uniform quantizers can be defined as inverse quantizers whose buckets are all of equal size.

### 3.1.48

upsampling
procedure (3.1.38) by which the spatial resolution of a component (3.1.13) is increased

### 3.1.49

wavelet filter type
single number that uniquely identifies each element of the wavelet filter with regard to the number and type of horizontal and vertical decompositions

Note 1 to entry: Unlike the band type, the wavelet filter type does not include component information.

### 3.2 Abbreviated terms

LSB least significant bit
MSB most significant bit

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### 3.3 Symbols

$\mathrm{B}[\mathrm{c}] \quad$ bit precision of component c
$\beta \quad$ wavelet filter type
b band type

Bw
$\mathrm{B}_{\mathrm{r}} \quad$ number of bits required to encode a bitplane count in raw
Cpih
$c[p, \lambda, b, x]$
$\mathrm{C}_{\mathrm{s}}$
Cw
$D[p, b] \quad$ bitplane count coding mode of band $b$ in precinct $p$
$\mathrm{D}_{\mathrm{r}}[\mathrm{p}, \mathrm{s}] \quad$ raw coding mode override flag for packet s in precinct p
Fs
Fslc
Fq
G[b]
$\mathrm{H}_{\mathrm{b}}[\mathrm{a}]$
$\mathrm{H}_{\mathrm{c}}[\mathrm{i}]$
$\mathrm{H}_{\mathrm{f}}$
$\mathrm{H}_{\mathrm{p}}$
$\mathrm{H}_{\mathrm{sl}}$
$I[p, b, \lambda, s] \quad$ line inclusion flag, set if line $\lambda$ of band $b$ and precinct $p$ is included in packet $s$, reset otherwise
$\mathrm{L}_{0}[\mathrm{p}, \mathrm{b}] \quad$ first line of band b in precinct p
$\mathrm{L}_{1}[\mathrm{p}, \mathrm{b}] \quad$ last line +1 of band b in precinct p
Lcod codestream length in bytes
$\operatorname{Ldat}[\mathrm{p}, \mathrm{s}] \quad$ size of the data subpacket of precinct p and packet s in bytes
$\operatorname{Lcnt}[p, s] \quad$ size of the bitplane count subpacket of precinct $p$ and packet $s$ in bytes
$\operatorname{Lsgn}[p, s] \quad$ size of the sign subpacket of precinct $p$ and packet $s$ in bytes
Lprc[p] length of the entropy coded data in precinct p
Lslc slice length in bytes

| $\mathrm{M}[\mathrm{p}, \lambda, \mathrm{b}, \mathrm{g}]$ | bitplane count of precinct p , line $\lambda$, band b and code group g |
| :---: | :---: |
| $\mathrm{M}_{\text {top }}[\mathrm{p}, \lambda, \mathrm{b}, \mathrm{g}]$ | vertical predictor of the bitplane count of precinct $p$, line $\lambda$, band $b$ and code group $g$ |
| $\mathrm{N}_{\mathrm{c}}$ | number of components in an image |
| $\mathrm{N}_{\mathrm{cg}}[\mathrm{p}, \mathrm{b}]$ | number of code groups in precinct p and band b |
| $N_{\beta}$ | number of bands per component |
| $\mathrm{Ng}_{\mathrm{g}}$ | number of coefficients in a code group |
| $\mathrm{N}_{\mathrm{s}}[\mathrm{p}, \mathrm{b}]$ | number of significance groups per line band b of precinct p |
| $\mathrm{N}_{\mathrm{p}}[\mathrm{t}]$ | number of precincts in slice t |
| $\mathrm{N}_{\mathrm{L}}$ | number of bands in the wavelet decomposition of the image (wavelet filter types times components) |
| $\mathrm{N}_{\mathrm{L}, \mathrm{X}}$ | number of horizontal decomposition levels |
| $\mathrm{N}_{\mathrm{L}, \mathrm{y}}$ | number of vertical decomposition levels |
| $\mathrm{N}_{\mathrm{p}, \mathrm{x}}$ | number of precincts per sampling grid line |
| $\mathrm{N}_{\mathrm{p}, \mathrm{y}}$ | number of precinets per sampling grid column $\mathbf{V} \\| E W$ |
| $\mathrm{N}_{\mathrm{pc}}[\mathrm{p}]$ | number of packets in precinct pls.itelh.ai) |
| $\mathrm{O}[\mathrm{c}, \mathrm{x}, \mathrm{y}]$ | unscaled output of the inverse wavelet transformation at coordinates $x$ and $y$ of the component c <br> ISO/IEC 21122-1:2019 |
| $\Omega[\mathrm{c}, \mathrm{x}, \mathrm{y}]$ | output of the inverse multiple component transformation at position $\mathrm{x}, \mathrm{y}$ for component c |
| P [b] | priority of band b |
| Plev | level a particular codestream complies to |
| Ppih | profile a particular codestream complies to |
| Ppoc | progression order in which bands are transmitted in the codestream |
| Q[p] | quantization parameter of precinct $p$ |
| Qpih | quantization type of the picture |
| Rm | run mode used for significance coding |
| R [p] | refinement of precinct p |
| $\mathrm{R}[\mathrm{c}, \mathrm{x}, \mathrm{y}]$ | reconstructed sample value at position $\mathrm{x}, \mathrm{y}$ for component c |
| $\mathrm{S}_{\text {S }}$ | size of a significance group in code groups |
| $\mathrm{S}_{\mathrm{X}}[\mathrm{i}]$ | sampling factor of component i in horizontal direction |
| $\left.\mathrm{syy}_{\mathrm{y}} \mathrm{i}\right]$ | sampling factor of component i in vertical direction |
| $s[p, \lambda, b, x]$ | sign of the wavelet coefficient in precinct p , line $\lambda$, band b and position x |
| T[p,b] | truncation position of precinct p and band b |


| $\mathrm{T}_{\text {top }}[\mathrm{p}, \mathrm{b}]$ | vertical truncation position predictor of precinct $p$ and band $b$ |
| :---: | :---: |
| $\mathrm{T}[\beta, \mathrm{x}, \mathrm{y}]$ | temporary wavelet coefficient of filter type $\beta$ at location $\mathrm{x}, \mathrm{y}$ |
| $\mathrm{v}[\mathrm{x}, \mathrm{y}]$ | sample value at the sample grid position $\mathrm{x}, \mathrm{y}$ |
| $v[p, \lambda, b, x]$ | quantization index magnitude of the wavelet coefficient in precinct $p$, line $\lambda$, band $b$ and position x |
| $\mathrm{W}_{\mathrm{b}}$ [b] | width of band b in wavelet coefficients |
| $\mathrm{W}_{\mathrm{c}}[\mathrm{i}]$ | width of component $i$ in samples |
| $\mathrm{W}_{\mathrm{f}}$ | width of the image in sampling grid points |
| $\mathrm{W}_{\mathrm{p}}[\mathrm{p}]$ | width of the precinct p in sampling grid points |
| $\mathrm{W}_{\mathrm{pb}}[\mathrm{p}, \mathrm{b}]$ | width of subband $b$ of precinct $p$ in coefficients |
| Wt ${ }_{\text {x }}$ | wavelet filter type for horizontal filtering |
| Wty | wavelet filter type for vertical filtering |
| $\mathrm{X}[\mathrm{y}$ ] | one-dimensional temporal array of wavelet coefficients |
| Yslh | vertical slice order within the picture <br> order within the picture RD PREVIEW |
| $\mathrm{Z}[\mathrm{p}, \lambda, \mathrm{b}, \mathrm{j}]$ | significance flag of precinct p, line $\lambda$, band band significance group $j$ |

## 4 Conventions

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### 4.1 Conformance language b943331346ec/iso-iec-21122-1-2019

The keyword "reserved" indicates a provision that is not specified at this time, shall not be used, and may be specified in the future. The keyword "forbidden" indicates "reserved" and in addition indicates that the provision will never be specified in the future.

### 4.2 Operators

NOTE Many of the operators used in document are similar to those used in the C programming language.

### 4.2.1 Arithmetic operators

$+\quad$ addition

- $\quad$ subtraction (as a binary operator) or negation (as a unary prefix operator)
$\times \quad$ multiplication
/ division without truncation or rounding
$\ll \quad$ left shift: $\mathrm{x} \ll \mathrm{s}$ is defined as $\mathrm{x} \times 2^{\text {s }}$
>> right shift: $\mathrm{x} \gg \mathrm{s}$ is defined as $\left\lfloor\mathrm{x} / 2^{\mathrm{s}}\right\rfloor$
Umod $\quad x$ umod a is the unique value $y$ between 0 and $a-1$ for which $y+N a=x$ with a suitable integer N


### 4.2.2 Logical operators

\| logical OR
\&\& logical AND
! logical NOT

### 4.2.3 Relational operators

| $>$ | greater than |
| :--- | :--- |
| $\geq$ | greater than or equal to |
| $<$ | less than |
| $\leq$ | less than or equal to |
| $==$ | equal to |
| $!=$ | not equal to |

### 4.2.4 Precedence order of operators

NOTE Operators are tisted below in/descending order of precedence. If seyeral operators appear in the same line, they have equal precedence. When several operators of equal precedence appear at the same level in an expression, evaluation proceeds according to the associativity of the operator either from right to left or from left to right.

Operators Type ofloperation2-1:2019 Associativity
()
[]
-
$\times, /$
Umod
+, -
<<, >>
$<,>, \leq, \geq$
\&
httpss//standards.teh.ail catalog/standards/sista0c8ec3e-12b2-4bcb-87a6-
expression'6ec/iso-iec-21122-1-20left to right
indexing of arrays left to right
unary negation
multiplication, division left to right
modulo (remainder) left to right
addition and subtraction left to right
left shift and right shift left to right
relational left to right
bitwise AND left to right

### 4.2.5 Mathematical functions

$\lceil\mathrm{x}\rceil \quad$ ceil of x : returns the smallest integer that is greater than or equal to x
$\lfloor\mathrm{x}\rfloor \quad$ floor of x : returns the largest integer that is less than or equal to x
$|\mathrm{x}| \quad$ absolute value of $\mathrm{x},|\mathrm{x}|$ equals -x for $\mathrm{x}<0$, otherwise x

