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Sampling procedures for inspection by variables —

Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

Règles d'échantillonnage pour les contrôles par mesures —

Partie 1: Spécification pour les plans d'échantillonnage simples indexés d'après un niveau de qualité acceptable (NQA) pour un contrôle lot par lot pour une caractéristique qualité unique et un NQA unique

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ISO copyright office
CP 401 • Ch. de Blandonnet 8
CH-1214 Vernier, Geneva
Phone: +41 22 749 01 11
Email: copyright@iso.org
Website: www.iso.org

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This third edition cancels and replaces the second edition (ISO 3591-1:2013), which has been technically revised.

The main changes are as follows:

— procedures have been introduced to accommodate measurement uncertainty;

— many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in ISO 2859-1.

A list of all parts in the ISO 3951 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL). A more comprehensive and technical treatment of the AQL scheme is given in ISO 3951-2. This document is complementary to ISO 2859-1.

The objectives of the methods laid down in this document are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

a) an automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected; and

b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this document, the acceptance of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This document is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this document is applied to each one separately.

This document is intended for application to a single quality characteristic that is measurable on a continuous scale and is normally distributed. For two or more such quality characteristics, see ISO 3951-2. For information on normality and data transformations, see Clause 12.

It is assumed in the body of this document that measurement error is negligible (see ISO 10576-1:2003). For information on allowing for measurement error, see Annex B.

For double specification limits, this document covers combined control. For other types of control, refer to ISO 3951-2.

CAUTION — The procedures in this document are not suitable for application to lots that have been screened for nonconforming items.

Inspection by variables for nonconforming items, as described in this document, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

— unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;

— a single specification limit, or combined control of double specification limits;

— normal inspection, tightened inspection, or reduced inspection.

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge of both statistics and the product to be inspected. Clause 5 of this document is intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be kept in mind when deciding whether a variables plan is suitable and the choices to be made when selecting an appropriate standard plan.

The basic definitions and notations are provided in Clauses 3 and 4. The basic operational rules are contained in Clauses 5 through 9. Clause 10 informs about the relations between this document and the attributes sampling standard ISO 2859-1. Clauses 11, 12 and 13 provide background on accounting for measurement uncertainty, the normality assumption, and monitoring of inspection results and the underlying process. All tables needed for the sampling procedure can be found in Clause 14 and examples for the s–method and the σ–method for both one and two specification limits can be found in Clause 15.

Nine annexes are provided. Annex A indicates how the sample standard deviation, s, and the presumed known value of the process standard deviation, σ, should be determined. Annex B provides procedures for accommodating measurement uncertainty. Annex C shows five different sampling strategies. Annex D gives the general formula for the operating characteristic of the σ–method and provides tables with values of the operating characteristics of single sampling plans with known σ. Annex E gives the general formula for the operating characteristic of the s–method and provides tables with values of the operating characteristics of single sampling plans with unknown σ. Annex F provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection, as well as for the s–method and σ–method. Annex G provides similar information for the producer's risks. Annex H give details of how Acceptance diagrams for double specification limits are constructed, Annex I shows the use of the underlying software (R package to support implementation of this document).

Sampling procedures for inspection by variables —

Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

1 Scope

This document specifies single sampling plans for lot-by-lot inspection under the following conditions:

a) where the inspection procedure is applied to a continuing series of lots of discrete products, all supplied by one producer using one production process;

b) where only a single quality characteristic, x, of these products is taken into consideration, which is measurable on a continuous scale;

c) where production is under statistical control and the quality characteristic, x, is distributed according to a normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines a lower specification limit, L, an upper specification limit, U, or both. An item is qualified as conforming if its measured quality characteristic, x, satisfies as appropriate one of the following inequalities:

   1) $x \geq L$ (i.e. the lower specification limit is not violated);

   2) $x \leq U$ (i.e. the upper specification limit is not violated);

   3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are cases with a single specification limit, and 3) is a case with double specification limits.

Where double specification limits apply, it is assumed in this document that conformity to both specification limits is equally important to the integrity of the product. In such cases, it is appropriate to apply a single AQL to the combined percentage of a product outside the two specification limits. This is referred to as combined control.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-2, *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1 and ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
inspection by variables
inspection by measuring the magnitude(s) of a characteristic(s) of an item

[SOURCE: ISO 3534-2:2006, 4.1.4]

3.2
sampling inspection
inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2:2006, 4.1.6]

3.3
acceptance sampling inspection
acceptance inspection where the acceptability is determined by means of *sampling inspection* (3.2)

[SOURCE: ISO 3534-2:2006, 4.1.8]

3.4
acceptance sampling inspection by variables
acceptance sampling inspection (3.3) in which the acceptance of the process is determined statistically from measurements from *inspection by variables* (3.1)

[SOURCE: ISO 3534-2:2006, 4.2.11 modified — "on specified quality characteristics of each item in a sample from a lot" has been replaced by "from inspection by variables"]

3.5
process fraction nonconforming
rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

3.6
acceptance quality limit
AQL
(acceptance sampling) worst tolerable *quality level* (3.7)

Note 1 to entry: This concept only applies when an acceptance sampling scheme with rules for switching and for discontinuation, such as ISO 2859-1 and ISO 3951, is used.

Note 2 to entry: See 5.4.

[SOURCE: ISO 3534-2:2006, 4.6.15 modified — Notes 2 to 4 to entry have been deleted and Note 1 to entry has been added.]

3.7
quality level
quality expressed as the fraction nonconforming

[SOURCE: ISO 3534-2:2006, 4.6.16 modified — "or rate of number of nonconformities" has been removed".]

3.8
limiting quality
LQ
quality level (3.7), when a lot is considered in isolation, which, for the purposes of *acceptance sampling inspection* (3.3), is limited to a low probability of acceptance

Note 1 to entry: In this document: 10 %.

[SOURCE: ISO 3534-2:2006, 4.6.13, modified — Note 1 to entry has been added.]

3.9
nonconformity
non-fulfilment of a requirement

[SOURCE: ISO 3534-2:2006, 3.1.11]

3.10
nonconforming unit
unit with one or more nonconformities

[SOURCE: ISO 3534-2:2006, 1.2.15]

3.11
***p**-method acceptance sampling plan**
acceptance sampling (3.3) plan by variables where the the estimated fraction nonconforming $\hat{p}$ is compared to the maximum acceptable value p^*.

Note 1 to entry: The method is applicable to both the *s-method* (3.12) and the *σ-method* (3.13) and gives equivalent results. It has the advantage that it deals directly with the fraction nonconforming.

3.12
***s*–method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the sample standard deviation

Note 1 to entry: See Clause 6.

[SOURCE: ISO 3534-2:2006, 4.3.10 modified — "s method" has been replaced by "s-method" and "acceptance sampling plan" has been added]

3.13
***σ*–method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the presumed value of the process standard deviation

Note 1 to entry: See Clause 7.

[SOURCE: ISO 3534-2:2006, 4.3.9 modified — "sigma method" has been replaced by "σ-method" and "acceptance sampling plan" has been added]

3.14
specification limit
conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2:2006, 3.1.3, modified — "limiting value stated" has been replaced with "conformance boundary specified".]

3.15
lower specification limit
L
specification limit (3.14) that defines the lower conformance boundary

[SOURCE: ISO 3534 2:2006, 3.1.5, modified — "limiting value" has been replaced with "conformance boundary".]

3.16
upper specification limit
U
specification limit (3.14) that defines the upper conformance boundary

[SOURCE: ISO 3534-2:2006, 3.1.4, modified — "limiting value" has been replaced with "conformance boundary".]

3.17
combined control
requirement when both upper and lower limits are specified for the quality characteristic and an *AQL* (3.6) that applies to the combined percent nonconforming beyond the two limits is given

Note 1 to entry: See 5.4.

Note 2 to entry: The use of combined control implies that nonconformity beyond either *specification limit* (3.14) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.18
form *k* acceptance constant
k
constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

Note 1 to entry: See Clause 6 and Clause 7.

[SOURCE: ISO 3534-2:2006, 4.4.4, modified — "acceptability constant" has been replaced with "acceptance constant". The original Note 1 to entry has been deleted, a new Note 1 to entry has been added.]

3.19
form *p acceptance constant**
p*
constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2:2006, 4.4.4, modified — "acceptability constant" has been replaced with "acceptance constant". The original Note 1 to entry has been deleted, a new Note 1 to entry has been added.]

3.20
quality statistic
Q
function of the *specification limit* (3.14), the sample mean and the sample or process standard deviation, used in assessing the acceptance of a lot

Note 1 to entry: For the case of a single specification limit, the lot can be accepted or rejected on the result of comparing *Q* with the *form k acceptance constant* (3.18).

Note 2 to entry: See Clause 6 and Clause 7.

[SOURCE: ISO 3534-2:2006, 4.4.9, modified — In the Note 1 to entry, "may be sentenced" has been replaced with " can be accepted or rejected". Note 2 to entry has been added.]

3.21
lower quality statistic
Q_L
function of the *lower specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single lower specification limit, the lot can be accepted or rejected on the result of comparing Q_L with the *form k acceptance constant* (3.18).

Note 2 to entry: See Clause 4, Clause 6, and Clause 7.

[SOURCE: ISO 3534-2:2006, 4.4.11, modified — Note 2 to entry has been added.]

3.22
upper quality statistic
Q_U
function of the *upper specification limit* (3.16), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single upper specification limit, the lot can be accepted or rejected on the result of comparing Q_U with the *form k acceptance constant* (3.18).

Note 2 to entry: See Clause 4, Clause 6, and Clause 7.

[SOURCE: ISO 3534-2:2006, 4.4.10, modified — Note 2 to entry has been added.]

3.23
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample size code letter and *AQL* (3.6) for which it is possible to satisfy the acceptance criterion for a double specification limit under all inspection severities (i.e. normal, tightened and reduced) when the process variability is known

Note 1 to entry: The MPSD depends on whether the double specification limits are combined, separate or complex, but does not depend on the inspection severity.

Note 2 to entry: See 5.2.

[SOURCE: ISO 3534-2:2006, 4.4.8, modified — Note 2 to entry has been added.]

3.24
switching rule
instruction within an *acceptance sampling* (3.3) scheme for changing from one acceptance sampling plan to another of greater or lesser severity based on demonstrated quality history

Note 1 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of 'severity of sampling'.

Note 2 to entry: See Clause 9.

[SOURCE: ISO 3534-2:2006, 4.3.4, modified — Note 2 to entry has been added.]

3.25
measurement
set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2:2006, 3.2.1, modified — "having the object of determining a value of a quantity" has been replaced with " to determine the value of some quantity". Notes 1 and 2 to entry have been deleted.]

4 Symbols

f_σ factor that relates the maximum process standard deviation to the difference between U and L (see 14.5)

$F_{BETA(\alpha,\beta)}(x)$ the distribution of the standard beta distribution with parameters α and β. In this document $\alpha = \beta = n/2 - 1$ throughout.

$F_{t(\nu,\delta)}(x)$ the distribution function of the non-central t-distribution with ν degrees of freedom and non-centrality parameter δ

K_p the upper p-quantile of the standardized normal distribution i.e. x such that $1 - \Phi(x) = p$, which corresponds to the process fraction nonconforming p

k form k acceptance constant for use with a single quality characteristic and a single specification limit (see 14.1 for the s–method acceptance sampling plan or 14.2 for the σ–method acceptance sampling plan)

L lower specification limit (as a subscript to a variable, it denotes its value at L)

M unknown process mean

N lot size (number of items in a lot)

n sample size (number of items in a sample)

$\hat{p}$ estimate of the process fraction nonconforming

$\hat{p}_L$ estimate of the process fraction nonconforming below the lower specification limit

$\hat{p}_U$ estimate of the process fraction nonconforming above the upper specification limit

p^* form p^* acceptance constant i.e. the maximum acceptable value for the estimate of the process fraction nonconforming

P_a probability of acceptance

$\Phi(x)$ the distribution function of the standardized normal distribution

Q quality statistic

Q_L lower quality statistic

NOTE Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.

Q_U upper quality statistic

NOTE Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.

S sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j=1}^{n}(x_j - \bar{x})^2}{n-1}}$$

(See Annex A.)

σ process standard deviation

NOTE σ^2, the square of the process standard deviation, is known as the process variance.

σ_{max} maximum process standard deviation (MPSD)

σ_{root} weighted root mean square of s

U upper specification limit (as a suffix to a variable, it denotes its value at U)

x_j measured value of the quality characteristic for the j^{th} item of the sample

$\bar{x}$ the arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n}$$

5 Choice of a sampling plan

5.1 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect variables rather than attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time-consuming and costly per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely the quality of the product. Therefore, earlier warning can be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted. For example, two phenomena of variables sampling are difficult to understand:

   1) lots containing 100 % conforming items can be rejected, occasionally even with a high probability (see example 3 in 15.1);

   2) there are cases of larger AQL and smaller lot size where lots containing 100 % nonconforming items are accepted.

d) From a comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes, such as from ISO 2859-1, and the standard plans in this document, the smallest samples are generally required by the σ–method (used when the process standard deviation is presumed to be known). The sample sizes for the s-method (used when the process standard deviation is presumed to be unknown) are larger than for the σ-method but are, in general, substantially smaller than for sampling by attributes.

e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

g) A variables scheme becomes relatively more complicated to operate as the number of measurements to be taken on each item increases.