Designation: C680 – 10

Standard Practice for Estimate of the Heat Gain or Loss and the Surface Temperatures of Insulated Flat, Cylindrical, and Spherical Systems by Use of Computer Programs[1]

This standard is issued under the fixed designation C680; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice provides the algorithms and calculation methodologies for predicting the heat loss or gain and surface temperatures of certain thermal insulation systems that can attain one dimensional, steady- or quasi-steady-state heat transfer conditions in field operations.

1.2 This practice is based on the assumption that the thermal insulation systems can be well defined in rectangular, cylindrical or spherical coordinate systems and that the insulation systems are composed of homogeneous, uniformly dimensioned materials that reduce heat flow between two different temperature conditions.

1.3 Qualified personnel familiar with insulation-systems design and analysis should resolve the applicability of the methodologies to real systems. The range and quality of the physical and thermal property data of the materials comprising the thermal insulation system limit the calculation accuracy. Persons using this practice must have a knowledge of the practical application of heat transfer theory relating to thermal insulation materials and systems.

1.4 The computer program that can be generated from the algorithms and computational methodologies defined in this practice is described in Section 7 of this practice. The computer program is intended for flat slab, pipe and hollow sphere insulation systems.

1.5 The values stated in inch-pound units are to be regarded as standard. The values given in parentheses are mathematical conversions to SI units that are provided for information only and are not considered standard.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

C168 Terminology Relating to Thermal Insulation

C177 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus

C335 Test Method for Steady-State Heat Transfer Properties of Pipe Insulation

C518 Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus

C585 Practice for Inner and Outer Diameters of Thermal Insulation for Nominal Sizes of Pipe and Tubing

C1055 Guide for Heated System Surface Conditions that Produce Contact Burn Injuries

C1057 Practice for Determination of Skin Contact Temperature from Heated Surfaces Using a Mathematical Model and Thermesthesiometer

2.2 *Other Document:*

NBS Circular 564 Tables of Thermodynamic and Transport Properties of Air, U.S. Dept of Commerce

3. Terminology

3.1 *Definitions:*

3.1.1 For definitions of terms used in this practice, refer to Terminology C168.

3.1.2 *thermal insulation system*—for this practice, a thermal insulation system is a system comprised of a single layer or layers of homogeneous, uniformly dimensioned material(s) intended for reduction of heat transfer between two different temperature conditions. Heat transfer in the system is steady-state. Heat flow for a flat system is normal to the flat surface, and heat flow for cylindrical and spherical systems is radial.

3.2 *Symbols:*

[1] This practice is under the jurisdiction of ASTM Committee C16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.30 on Thermal Measurement.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3.2.1 The following symbols are used in the development of the equations for this practice. Other symbols will be introduced and defined in the detailed description of the development.

where:

h = surface transfer conductance, Btu/(h·ft^2·°F) (W/(m^2·K)) h_i at inside surface; h_o at outside surface
k = apparent thermal conductivity, Btu·in./(h·ft^2·°F) (W/(m·K))
k_e = effective thermal conductivity over a prescribed temperature range, Btu·in./(h·ft^2·°F) (W/(m·K))
q = heat flux, Btu/(h·ft^2) (W/m^2)
q_p = time rate of heat flow per unit length of pipe, Btu/(h·ft) (W/m)
R = thermal resistance, °F·h·ft^2/Btu (K·m^2/W)
r = radius, in. (m); $r_{m+1} - r_m$ = thickness
t = local temperature, °F (K)
t_i = inner surface temperature of the insulation, °F (K)
t_1 = inner surface temperature of the system
t_o = temperature of ambient fluid and surroundings, °F (K)
x = distance, in. (m); $x_{m+1} - x_m$ = thickness
ε = effective surface emittance between outside surface and the ambient surroundings, dimensionless
σ = Stefan-Boltzmann constant, 0.1714 × 10^{-8} Btu/(h·ft^2·°R^4) (5.6697 × 10^{-8} W/(m^2·K^4))
T_s = absolute surface temperature, °R (K)
T_o = absolute surroundings (ambient air if assumed the same) temperature, °R (K)
T_m = $(T_s + T_o)/2$
L = characteristic dimension for horizontal and vertical flat surfaces, and vertical cylinders
D = characteristic dimension for horizontal cylinders and spheres
c_p = specific heat of ambient fluid, Btu/(lb·°R) (J/(kg·K))
h_c = average convection conductance, Btu/(h·ft^2·°F) (W/(m^2·K))
k_f = thermal conductivity of ambient fluid, Btu/(h·ft·°F) (W/(m·K))
V = free stream velocity of ambient fluid, ft/h (m/s)
υ = kinematic viscosity of ambient fluid, ft^2/h (m^2/s)
g = acceleration due to gravity, ft/h^2 (m/s^2)
β = volumetric thermal expansion coefficient of ambient fluid, °R^{-1} (K^{-1})
ρ = density of ambient fluid, lb/ft^3 (kg/m^3)
ΔT = absolute value of temperature difference between surface and ambient fluid, °R (K)
Nu = Nusselt number, dimensionless
Ra = Rayleith number, dimensionless
Re = Reynolds number, dimensionless
Pr = Prandtl number, dimensionless

4. Summary of Practice

4.1 The procedures used in this practice are based on standard, steady-state, one dimensional, conduction heat transfer theory as outlined in textbooks and handbooks, Refs (1,2,3,4,5,6). Heat flux solutions are derived for temperature dependent thermal conductivity in a material. Algorithms and computational methodologies for predicting heat loss or gain of single or multi-layer thermal insulation systems are provided by this practice for implementation in a computer program. In addition, interested parties can develop computer programs from the computational procedures for specific applications and for one or more of the three coordinate systems considered in Section 6.

4.1.1 The computer program combines functions of data input, analysis and data output into an easy to use, interactive computer program. By making the program interactive, little training for operators is needed to perform accurate calculations.

4.2 The operation of the computer program follows the procedure listed below:

4.2.1 *Data Input*—The computer requests and the operator inputs information that describes the system and operating environment. The data includes:

4.2.1.1 Analysis identification.

4.2.1.2 Date.

4.2.1.3 Ambient temperature.

4.2.1.4 Surface transfer conductance or ambient wind speed, system surface emittance and system orientation.

4.2.1.5 *System Description*—Material and thickness for each layer (define sequence from inside out).

4.2.2 *Analysis*—Once input data is entered, the program calculates the surface transfer conductances (if not entered directly) and layer thermal resistances. The program then uses this information to calculate the heat transfer and surface temperature. The program continues to repeat the analysis using the previous temperature data to update the estimates of layer thermal resistance until the temperatures at each surface repeat within 0.1°F between the previous and present temperatures at the various surface locations in the system.

4.2.3 *Program Output*—Once convergence of the temperatures is reached, the program prints a table that presents the input data, calculated thermal resistance of the system, heat flux and the inner surface and external surface temperatures.

5. Significance and Use

5.1 Manufacturers of thermal insulation express the performance of their products in charts and tables showing heat gain or loss per unit surface area or unit length of pipe. This data is presented for typical insulation thicknesses, operating temperatures, surface orientations (facing up, down, horizontal, vertical), and in the case of pipes, different pipe sizes. The exterior surface temperature of the insulation is often shown to provide information on personnel protection or surface condensation. However, additional information on effects of wind velocity, jacket emittance, ambient conditions and other influential parameters may also be required to properly select an insulation system. Due to the large number of combinations of size, temperature, humidity, thickness, jacket properties, surface emittance, orientation, and ambient conditions, it is not practical to publish data for each possible case, Refs (7,8).

5.2 Users of thermal insulation faced with the problem of designing large thermal insulation systems encounter substantial engineering cost to obtain the required information. This cost can be substantially reduced by the use of accurate engineering data tables, or available computer analysis tools, or both. The use of this practice by both manufacturers and users of thermal insulation will provide standardized engineering data of sufficient accuracy for predicting thermal insulation system performance. However, it is important to note that the accuracy of results is extremely dependent on the accuracy of the input data. Certain applications may need specific data to produce meaningful results.

5.3 The use of analysis procedures described in this practice can also apply to designed or existing systems. In the rectangular coordinate system, Practice C680 can be applied to heat flows normal to flat, horizontal or vertical surfaces for all types of enclosures, such as boilers, furnaces, refrigerated chambers and building envelopes. In the cylindrical coordinate system, Practice C680 can be applied to radial heat flows for all types of piping circuits. In the spherical coordinate system, Practice C680 can be applied to radial heat flows to or from stored fluids such as liquefied natural gas (LNG).

5.4 Practice C680 is referenced for use with Guide C1055 and Practice C1057 for burn hazard evaluation for heated surfaces. Infrared inspection, in-situ heat flux measurements, or both are often used in conjunction with Practice C680 to evaluate insulation system performance and durability of operating systems. This type of analysis is often made prior to system upgrades or replacements.

5.5 All porous and non-porous solids of natural or man-made origin have temperature dependent thermal conductivities. The change in thermal conductivity with temperature is different for different materials, and for operation at a relatively small temperature difference, an average thermal conductivity may suffice. Thermal insulating materials ($k < 0.85$ {Btu·in}/{h·ft^2·°F}) are porous solids where the heat transfer modes include conduction in series and parallel flow through the matrix of solid and gaseous portions, radiant heat exchange between the surfaces of the pores or interstices, as well as transmission through non-opaque surfaces, and to a lesser extent, convection within and between the gaseous portions. With the existence of radiation and convection modes of heat transfer, the measured value should be called apparent thermal conductivity as described in Terminology C168. The main reason for this is that the premise for pure heat conduction is no longer valid, because the other modes of heat transfer obey different laws. Also, phase change of a gas, liquid, or solid within a solid matrix or phase change by other mechanisms will provide abrupt changes in the temperature dependence of thermal conductivity. For example, the condensation of the gaseous portions of thermal insulation in extremely cold conditions will have an extremely influential effect on the apparent thermal conductivity of the insulation. With all of this considered, the use of a single value of thermal conductivity at an arithmetic mean temperature will provide less accurate predictions, especially when bridging temperature regions where strong temperature dependence occurs.

5.6 The calculation of surface temperature and heat loss or gain of an insulated system is mathematically complex, and because of the iterative nature of the method, computers best handle the calculation. Computers are readily available to most producers and consumers of thermal insulation to permit the use of this practice.

5.7 Computer programs are described in this practice as a guide for calculation of the heat loss or gain and surface temperatures of insulation systems. The range of application of these programs and the reliability of the output is a primary function of the range and quality of the input data. The programs are intended for use with an "interactive" terminal. Under this system, intermediate output guides the user to make programming adjustments to the input parameters as necessary. The computer controls the terminal interactively with program-generated instructions and questions, which prompts user response. This facilitates problem solution and increases the probability of successful computer runs.

5.8 The user of this practice may wish to modify the data input and report sections of the computer programs presented in this practice to fit individual needs. Also, additional calculations may be desired to include other data such as system costs or economic thickness. No conflict exists with such modifications as long as the user verifies the modifications using a series of test cases that cover the range for which the new method is to be used. For each test case, the results for heat flow and surface temperature must be identical (within resolution of the method) to those obtained using the practice described herein.

5.9 This practice has been prepared to provide input and output data that conforms to the system of units commonly used by United States industry. Although modification of the input/output routines could provide an SI equivalent of the heat flow results, no such "metric" equivalent is available for some portions of this practice. To date, there is no accepted system of metric dimensions for pipe and insulation systems for cylindrical shapes. The dimensions used in Europe are the SI equivalents of American sizes (based on Practice C585), and each has a different designation in each country. Therefore, no SI version of the practice has been prepared, because a standard SI equivalent of this practice would be complex. When an international standard for piping and insulation sizing occurs, this practice can be rewritten to meet those needs. In addition, it has been demonstrated that this practice can be used to calculate heat transfer for circumstances other than insulated systems; however, these calculations are beyond the scope of this practice.

6. Method of Calculation

6.1 *Approach:*

6.1.1 The calculation of heat gain or loss and surface temperature requires: (*1*) The thermal insulation is homogeneous as outlined by the definition of thermal conductivity in Terminology C168; (*2*) the system operating temperature is known; (*3*) the insulation thickness is known; (*4*) the surface transfer conductance of the system is known, reasonably estimated or estimated from algorithms defined in this practice based on sufficient information; and, (*5*) the thermal conductivity as a function of temperature for each system layer is known in detail.

6.1.2 The solution is a procedure calling for (*1*) estimation of the system temperature distribution; (*2*) calculation of the thermal resistances throughout the system based on that distribution; (*3*) calculation of heat flux; and (*4*) reestimation of the system temperature distribution. The iterative process continues until a calculated distribution is in reasonable agreement with the previous distribution. This is shown diagrammatically in Fig. 1. The layer thermal resistance is calculated each time with the effective thermal conductivity being obtained by integration of the thermal conductivity curve for the layer being considered. This practice uses the temperature dependence of the thermal conductivity of any insulation or multiple layer combination of insulations to calculate heat flow.

6.2 *Development of Equations*—The development of the mathematical equations is for conduction heat transfer through homogeneous solids having temperature dependent thermal conductivities. To proceed with the development, several precepts or guidelines must be cited:

6.2.1 *Steady-state Heat Transfer*—For all the equations it is assumed that the temperature at any point or position in the solid is invariant with time. Thus, heat is transferred solely by temperature difference from point to point in the solid.

6.2.2 *One-dimensional Heat Transfer*—For all equations it is assumed there is heat flow in only one dimension of the particular coordinate system being considered. Heat transfer in the other dimensions of the particular coordinate system is considered to be zero.

6.2.3 *Conduction Heat Transfer*—The premise here is that the heat flux normal to any surface is directly proportional to the temperature gradient in the direction of heat flow, or

$$q = -k\frac{dt}{dp} \qquad (1)$$

where the thermal conductivity, k, is the proportionality constant, and p is the space variable through which heat is flowing. For steady-state conditions, one-dimensional heat flow, and temperature dependent thermal conductivity, the equation becomes

$$q = -k(t)\frac{dt}{dp} \qquad (2)$$

where at all surfaces normal to the heat flux, the total heat flow through these surfaces is the same and changes in the thermal conductivity must dictate changes in the temperature gradient. This will ensure that the total heat passing through a given surface does not change from that surface to the next.

6.2.4 *Solutions from Temperature Boundary Conditions*—The temperature boundary conditions on a uniformly thick, homogeneous mth layer material are:

$$t = t_m \quad \text{at}\, x = x_m \quad (r = r_m); \qquad (3)$$

$$t = t_{m+1} \quad \text{at}\, x = x_{m+1} \quad (r = r_m)$$

For heat flow in the flat slab, let $p = x$ and integrate Eq 2:

$$q\int_{x_m}^{x_{m+1}} dx = -\int_{t_m}^{t_{m+1}} k(t)dt \qquad (4)$$

$$q = k_{e,m}\frac{t_m - t_{m+1}}{x_{m+1} - x_m}$$

For heat flow in the hollow cylinder, let $p = r$, $q = Q/(2\pi rl)$ and integrate Eq 2:

$$\frac{Q}{2\pi l}\int_{r_m}^{r_{m+1}}\frac{dr}{r} = -\int_{t_m}^{t_{m+1}} k(t)dt \qquad (5)$$

$$Q = k_{e,m}\frac{t_m - t_{m+1}}{\ln(r_{m+1}/r_m)}2\pi l$$

Divide both sides by $2\pi rl$

$$q = k_{e,m}\frac{t_m - t_{m+1}}{r\ln(r_{m+1}/r_m)}$$

For radial heat flow in the hollow sphere, let $p = r$, $q = Q/(4\pi r^2)$ and integrate Eq 2:

$$\frac{Q}{4\pi}\int_{r_m}^{r_{m+1}}\frac{dr}{r^2} = \int_{t_m}^{t_{m+1}} k(t)dt \qquad (6)$$

$$Q = k_{e,m}\frac{t_m - t_{m+1}}{\frac{1}{r_m} - \frac{1}{r_{m+1}}}4\pi$$

Divide both sides by $4\pi r^2$ and multiply both sides by $r_m r_{m+1}/r_m r_{m+1}$

$$q = k_{e,m}\frac{r_m r_{m+1}}{r^2}\frac{t_m - t_{m+1}}{r_{m+1} - r_m}$$

Note that the effective thermal conductivity over the temperature range is:

$$k_{e,m} = \frac{\int_{t_m}^{t_{m+1}} k(t)dt}{t_{m+1} - t_m} \qquad (7)$$

6.3 *Case 1, Flat Slab Systems:*

6.3.1 From Eq 4, the temperature difference across the mth layer material is:

$$t_m - t_{m+1} = qR_m \qquad (8)$$

$$\text{where} \quad R_m = \frac{(x_{m+1} - x_m)}{k_{e,m}}$$

Note that R_m is defined as the thermal resistance of the mth layer of material. Also, for a thermal insulation system of n layers, m = 1,2...n, it is assumed that perfect contact exists between layers. This is essential so that continuity of temperature between layers can be assumed.

6.3.2 Heat is transferred between the inside and outside surfaces of the system and ambient fluids and surrounding surfaces by the relationships:

$$q = h_i(t_i - t_1) \qquad (9)$$

$$q = h_o(t_{n+1} - t_o)$$

where h_i and h_o are the inside and outside surface transfer conductances. Methods for estimating these conductances are found in 6.7. Eq 9 can be rewritten as:

$$t_i - t_1 = qR_i \qquad (10)$$

$$t_{n+1} - t_o = qR_o$$

$$\text{where} \quad R_i = \frac{1}{h_i}, \quad R_o = \frac{1}{h_o}$$

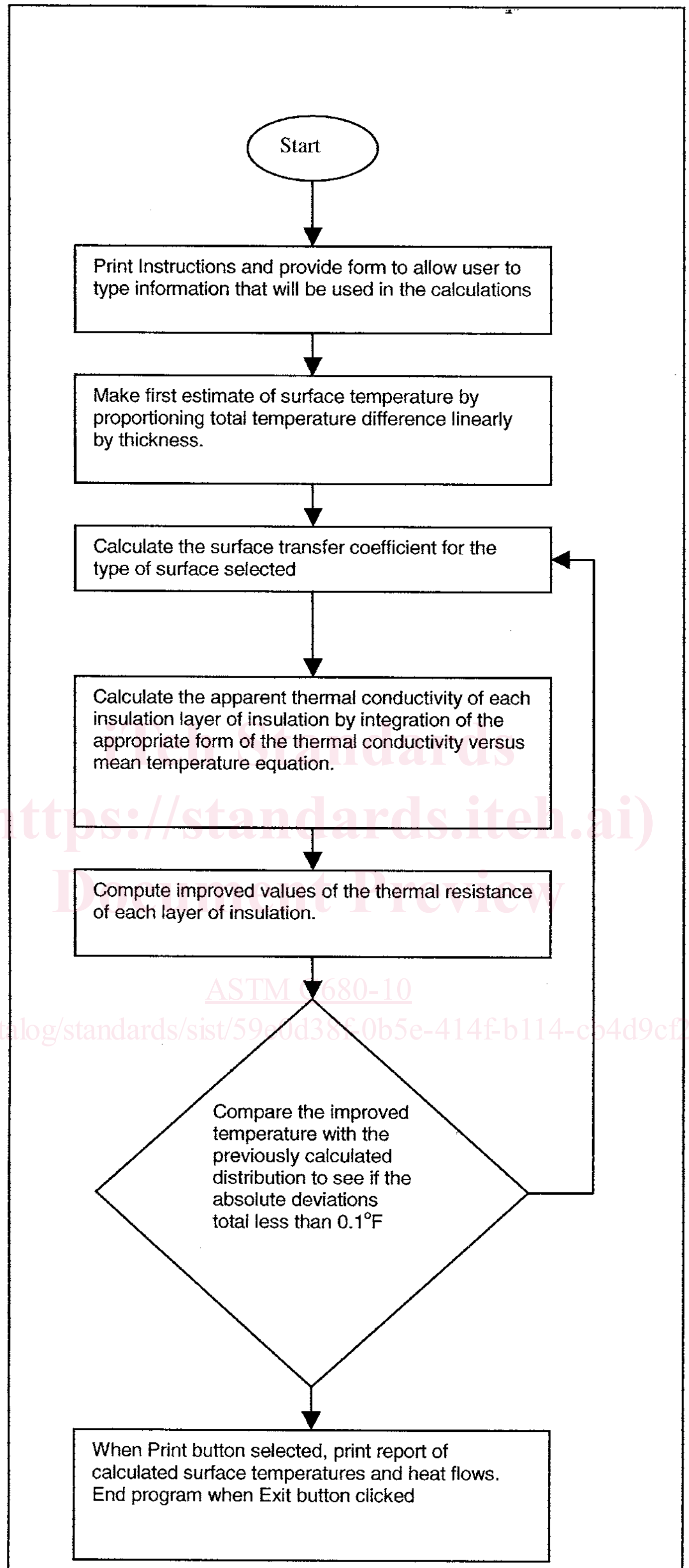

FIG. 1 Flow Chart

For the computer program, the inside surface transfer conductance, h_i, is assumed to be very large such that $R_i = 0$, and $t_1 = t_i$ is the given surface temperature.

6.3.3 Adding Eq 8 and Eq 10 yields the following equation:

$$t_i - t_o = q(R_1 + R_2 + \ldots + R_n + R_i + R_o) \tag{11}$$

From the previous equation a value for q can be calculated from estimated values of the resistances, R. Then, by rewriting Eq 8 to the following:

$$t_{m+1} = t_m - qR_m \tag{12}$$

$$t_1 = t_i - qR_i, \quad \text{for } R_i > 0$$

The temperature at the interface(s) and the outside surface can be calculated starting with $m = 1$. Next, from the calculated temperatures, values of $k_{e,m}$ (Eq 7) and R_m (Eq 8) can be calculated as well as R_o and R_i. Then, by substituting the calculated R-values back into Eq 11, a new value for q can be calculated. Finally, desired (correct) values can be obtained by repeating this calculation methodology until all values agree with previous values.

6.4 *Case 2, Cylindrical (Pipe) Systems:*

6.4.1 From Eq 5, the heat flux through any layer of material is referenced to the outer radius by the relationship:

$$q_n = q_m \frac{r}{r_{n+1}} = k_{e,m} \frac{t_m - t_{m+1}}{r_{n+1}\ln(r_{m+1}/r_m)} \tag{13}$$

and, the temperature difference can be defined by Eq 8, where:

$$R_m = \frac{r_{n+1}\ln(r_{m+1}/r_m)}{k_{e,m}} \tag{14}$$

Utilizing the methodology presented in case 1 (6.3), the heat flux, q_n, and the surface temperature, t_{n+1}, can be found by successive iterations. However, one should note that the definition of R_m found in Eq 14 must be substituted for the one presented in Eq 8.

6.4.2 For radial heat transfer in pipes, it is customary to define the heat flux in terms of the pipe length:

$$q_p = 2\pi r_{n+1} q_n \tag{15}$$

where q_p is the time rate of heat flow per unit length of pipe. If one chooses not to do this, then heat flux based on the interior radius must be reported to avoid the influence of outer-diameter differences.

6.5 *Case 3, Spherical Systems:*

6.5.1 From Eq 6, the flux through any layer of material is referenced to the outer radius by the relationship:

$$q_n = q_m \frac{r^2}{r_{n+1}^2} = k_{e,m} \frac{r_m r_{m+1}(t_m - t_{m+1})}{r_{n+1}^2 (r_{m+1} - r_m)} \tag{16}$$

The temperature difference can be defined by Eq 8, where:

$$R_m = \frac{r_{n+1}^2 (r_{m+1} - r_m)}{k_{e,m} r_m r_{m+1}} \tag{17}$$

Again, utilizing the methodology presented in case 1 (6.3), the heat flux, q_n, and the surface temperature, t_{n+1}, can be found by successive iterations. However, one should note that the definition of R_m found in Eq 17 must be substituted for the one presented in Eq 8.

6.6 *Calculation of Effective Thermal Conductivity:*

6.6.1 In the calculational methodologies of 6.3, 6.4, and 6.5, it is necessary to evaluate $k_{e,m}$ as a function of the two surface temperatures of each layer comprising the thermal insulating system. This is accomplished by use of Eq 7 where *k(t)* is defined as a polynomial function or a piecewise continuous function comprised of individual, integrable functions over specific temperature ranges. It is important to note that temperature can either be in °F (°C) or absolute temperature, because the thermal conductivity versus temperature relationship is regression dependent. It is assumed for the programs in this practice that the user regresses the k versus t functions using °F.

6.6.1.1 When *k(t)* is defined as a polynomial function, such as $k(t) = a + bt + ct^2 + dt^3$, the expression for the effective thermal conductivity is:

$$k_{e,m} = \frac{\int_{m}^{t_{m+1}} (a + bt + ct^2 + dt^3)dt}{(t_{m+1} - t_m)} \tag{18}$$

$$k_{e,m} = \frac{a(t_{m+1} - t_m) + \frac{b}{2}(t_{m+1}^2 - t_m^2) + \frac{c}{3}(t_{m+1}^3 - t_m^3) + \frac{d}{4}(t_{m+1}^4 - t_m^4)}{(t_{m+1} - t_m)}$$

$$k_{e,m} = a + \frac{b}{2}(t_m + t_{m+1}) + \frac{c}{3}(t_m^2 + t_m t_{m+1} + t_{m+1}^2) + \frac{d}{4}(t_m^3 + t_m^2 t_{m+1} + t_m t_{m+1}^2 + t_{m+1}^3)$$

It should be noted here that for the linear case, $c = d = 0$, and for the quadratic case, $d = 0$.

6.6.1.2 When *k(t)* is defined as an exponential function, such as $k(t) = e^{a+bt}$, the expression for the effective thermal conductivity is:

$$k_{e,m} = \frac{\int_{m}^{t_{m+1}} e^{a+bt}dt}{(t_{m+1} - t_m)} \tag{19}$$

$$k_{e,m} = \frac{\frac{1}{b}(e^{a+bt_{m+1}} - e^{a+bt_m})}{(t_{m+1} - t_m)}$$

$$k_{e,m} = \frac{(e^{a+bt_{m+1}} - e^{a+bt_m})}{b(t_{m+1} - t_m)}$$

6.6.1.3 The piece-wise continuous function may be defined as:

$$k(t) = k_1(t) \qquad t_{bl} \le t \le t_l \tag{20}$$

$$= k_2(t) \qquad t_l \le t \le t_u \qquad t_{bl} \le t_m \text{ and } t_{m+1} \le t_{bu}$$

$$= k_3(t) \qquad t_u \le t \le t_{bu}$$

where t_{bl} and t_{bu} are the experimental lower and upper boundaries for the function. Also, each function is integrable, and $k_1(t_l) = k_2(t_l)$ and $k_2(t_u) = k_3(t_u)$. In terms of the effective thermal conductivity, some items must be considered before performing the integration in Eq 8. First, it is necessary to determine if t_{m+1} is greater than or equal to t_m. Next, it is necessary to determine which temperature range t_m and t_{m+1} fit into. Once these two parameters are decided, the effective thermal conductivity can be determined using simple calculus. For example, if $t_{bl} \le t_m \le t_l$ and $t_u \le t_{m+1} \le t_{bu}$ then the effective thermal conductivity would be:

$$k_{e,m} = \frac{\int_{t_m}^{T_l} k_1(t)dt + \int_{T_l}^{T_u} k_2(t) + \int_{T_u}^{t_{m+1}} k_3(t)}{(t_{m+1} - t_m)} \quad (21)$$

It should be noted that other piece-wise functions exist, but for brevity, the previous is the only function presented.

6.6.2 It should also be noted that when the relationship of k with t is more complex and does not lend itself to simple mathematical treatment, a numerical method might be used. It is in these cases that the power of the computer is particularly useful. There are a wide variety of numerical techniques available. The most suitable will depend of the particular situation, and the details of the factors affecting the choice are beyond the scope of this practice.

6.7 *Surface Transfer Conductance:*

6.7.1 The surface transfer conductance, h, as defined in Terminology C168, assumes that the principal surface is at a uniform temperature and that the ambient fluid and other visible surfaces are at a different uniform temperature. The conductance includes the combined effects of radiant, convective, and conductive heat transfer. The conductance is defined by:

$$h = h_r + h_c \quad (22)$$

where h_r is the component due to radiation and h_c is the component due to convection and conduction. In subsequent sections, algorithms for these components will be presented.

6.7.1.1 The algorithms presented in this practice for calculating surface transfer conductances are used in the computer program; however, surface transfer conductances may be estimated from published values or separately calculated from algorithms other than the ones presented in this practice. One special note, care must be exercised at low or high surface temperatures to ensure reasonable values.

6.7.2 *Radiant Heat Transfer Conductance*—The radiation conductance is simply based on radiant heat transfer and is calculated from the Stefan-Boltzmann Law divided by the average difference between the surface temperature and the air temperature. In other words:

$$h_r = \frac{\sigma\varepsilon\,(T_s^4 - T_o^4)}{T_s - T_o} \quad \text{or} \quad (23)$$

$$h_r = \sigma\varepsilon\cdot(T_s^3 + T_s^2 T_o + T_s T_o^2 + T_o^3) \quad \text{or}$$

$$h_r = \sigma\varepsilon\cdot 4T_m^3\left[1 + \left(\frac{T_s - T_o}{T_s + T_o}\right)^2\right]$$

where:

ε = effective surface emittance between outside surface and the ambient surroundings, dimensionless,
σ = Stefan-Boltzman constant, 0.1714 × 10^{-8} Btu/(h·ft^2·°R^4) (5.6697 × 10^{-8} W/(m^2·K^4)),
T_s = absolute surface temperature, °R (K),
T_o = absolute surroundings (ambient air if assumed the same) temperature, °R (K), and
T_m = $(T_s + T_o)/2$

6.7.3 *Convective Heat Transfer Conductance*—Certain conditions need to be identified for proper calculation of this component. The conditions are: (*a*) Surface geometry—plane, cylinder or sphere; (*b*) Surface orientation—from vertical to horizontal including flow dependency; (*c*) Nature of heat transfer in fluid—from free (natural) convection to forced convection with variation in the direction and magnitude of fluid flow; (*d*) Condition of the surface—from smooth to various degrees of roughness (primarily a concern for forced convection).

6.7.3.1 Modern correlation of the surface transfer conductances are presented in terms of dimensionless groups, which are defined for fluids in contact with solid surfaces. These groups are:

$$\text{Nusselt,} \quad \overline{Nu}_L = \frac{\bar{h}_c L}{k_f} \quad \text{or} \quad \overline{Nu}_D = \frac{\bar{h}_c D}{k_f} \quad (24)$$

$$\text{Rayleigh,} \quad Ra_L = \frac{g\cdot\beta\cdot\rho\cdot c_p(\Delta T)L^3}{\nu\cdot k_f} \quad \text{or} \quad Ra_D = \frac{g\cdot\beta\cdot\rho\cdot c_p(\Delta T)D^3}{\nu\cdot k_f} \quad (25)$$

$$\text{Reynolds,} \quad Re_L = \frac{VL}{\nu} \quad \text{or} \quad Re_D = \frac{VD}{\nu} \quad (26)$$

$$\text{Prandtl,} \quad Pr = \frac{\nu\cdot\rho\cdot c_p}{k_f} \quad (27)$$

where:

L = characteristic dimension for horizontal and vertical flat surfaces, and vertical cylinders feet (m), in general, denotes height of vertical surface or length of horizontal surface,
D = characteristic dimension for horizontal cylinders and spheres feet (m), in general, denotes the diameter,
c_p = specific heat of ambient fluid, Btu/(lb·°R) (J/(kg·K)),
$\bar{h}_c$ = average convection conductance, Btu/(h·ft^2·°F) (W/(m^2·K)),
k_f = thermal conductivity of ambient fluid, Btu/(h·ft·°F) (W/(m·K)),
V = free stream velocity of ambient fluid, ft/h (m/s),
ν = kinematic viscosity of ambient fluid, ft^2/h (m^2/s),
g = acceleration due to gravity, ft/h^2 (m/s^2),
β = volumetric thermal expansion coefficient of ambient fluid, °R^{-1} (K^{-1}),
ρ = density of ambient fluid, lb/ft^3 (kg/m^3), and
ΔT = absolute value of temperature difference between surface and ambient fluid, °R (K).

It needs to be noted here that (except for spheres–forced convection) the above fluid properties must be calculated at the film temperature, T_f, which is the average of surface and ambient fluid temperatures. For this practice, it is assumed that the ambient fluid is dry air at atmospheric pressure. The properties of air can be found in references such as Ref (**9**). This reference contains equations for some of the properties and polynomial fits for others, and the equations are summarized in Table A1.1.

6.7.3.2 When a heated surface is exposed to flowing fluid, the convective heat transfer will be a combination of forced and free convection. For this mixed convection condition, Churchill (**10**) recommends the following equation. For each geometric shape and surface orientation the overall average Nusselt number is to be computed from the average Nusselt number for forced convection and the average Nusselt number for natural convection. The film conductance, h, is then computed from Eq 24. The relationship is:

$$\left(\overline{Nu} - \delta\right)^{j} = \left(\overline{Nu}_{f} - \delta\right)^{j} + \left(\overline{Nu}_{n} - \delta\right)^{j} \qquad (28)$$

where the exponent, j, and the constant, δ, are defined based on the geometry and orientation.

6.7.3.3 Once the Nusselt number has been calculated, the surface transfer conductance is calculated from a rearrangement of Eq 24:

$$h_{c} = \overline{Nu}_{L} \cdot k_{f} / L \qquad (29)$$

$$\overline{h}_{c} = \overline{Nu}_{D} \cdot k_{f} / D$$

where L and D are the characteristic dimension of the system. The term k_a is the thermal conductivity of air determined at the film temperature using the equation in Table A1.1.

6.7.4 *Convection Conductances for Flat Surfaces:*

6.7.4.1 From *Heat Transfer* by Churchill and Ozoe as cited in *Fundamentals of Heat and Mass Transfer* by Incropera and Dewitt, the relation for forced convection by laminar flow over an isothermal flat surface is:

$$\overline{Nu}_{f,L} = \frac{0.6774\, Re_{L}^{1/2} Pr^{1/3}}{\left[1 + (0.0468/Pr)^{2/3}\right]^{1/4}} \qquad Re_{L} < 5 \times 10^{5} \qquad (30)$$

For forced convection by turbulent flow over an isothermal flat surface, Incropera and Dewitt suggest the following:

$$\overline{Nu}_{f,L} = \left(0.037\, Re_{L}^{4/5} - 871\right) Pr^{1/3} \qquad 5 \times 10^{5} < Re_{L} < 10^{8} \qquad (31)$$

It should be noted that the upper bound for Re_L is an approximate value, and the user of the above equation must be aware of this.

6.7.4.2 In "Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate" by Churchill and Chu, as cited by Incropera and Dewitt, it is suggested for natural convection on isothermal, vertical flat surfaces that:

$$\overline{Nu}_{n,L} = \left\{0.825 + \frac{0.387\, Ra_{L}^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}}\right\}^{2} \qquad \text{All } Ra_{L} \qquad (32)$$

For slightly better accuracy in the laminar range, it is suggested by the same source (p. 493) that:

$$\overline{Nu}_{n,L} = 0.68 + \frac{0.670\, Ra_{L}^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}} \qquad Ra_{L} < 10^{9} \qquad (33)$$

In the case of both vertical flat and cylindrical surfaces the characteristic dimension, L or D, is the vertical height (ft). To compute the overall Nusselt number (Eq 28), set $j = 3$ and $\delta = 0$. Also, it is important to note that the free convection correlations apply to vertical cylinders in most cases.

6.7.4.3 For natural convection on horizontal flat surfaces, Incropera and Dewitt (p. 498) cite *Heat Transmission* by McAdams, "Natural Convection Mass Transfer Adjacent to Horizontal Plates" by Goldstein, Sparrow and Jones, and "Natural Convection Adjacent to Horizontal Surfaces of Various Platforms" for the following correlations:

$$\text{Heat flow up:} \qquad \overline{Nu}_{n,L} = 0.54\, Ra_{L}^{1/4} \qquad 10^{4} < Ra_{L} < 10^{7} \qquad (34)$$

$$\overline{Nu}_{n,L} = 0.15\, Ra_{L}^{1/3} \qquad 10^{7} < Ra_{L} < 10^{11}$$

$$\text{Heat flow down:} \qquad \overline{Nu}_{n,L} = 0.27\, Ra_{L}^{1/4} \qquad 10^{5} < Ra_{L} < 10^{10}$$

In the case of horizontal flat surfaces, the characteristic dimension, L, is the area of the surface divided by the perimeter of the surface (ft). To compute the overall Nusselt number (Eq 28), set $j = 3.5$ and $\delta = 0$.

6.7.5 *Convection Conductances for Horizontal Cylinders:*

6.7.5.1 For forced convection with fluid flow normal to a circular cylinder, Incropera and Dewitt (p. 370) cite *Heat Transfer* by Churchill and Bernstein for the following correlation:

$$\overline{Nu}_{f,D} = 0.3 + \frac{0.62\, Re_{D}^{1/2}\, Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_{D}}{282\,000}\right)^{5/8}\right]^{4/5} \qquad (35)$$

$$\text{All } Re_{D} \cdot Pr > 0.2$$

In the case of horizontal cylinders, the characteristic dimension, D, is the diameter of the cylinder, (ft). In addition, this correlation should be used for forced convection from vertical pipes.

6.7.5.2 For natural convection on horizontal cylinders, Incropera and Dewitt (p. 502) cite "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder" by Churchill and Chu for the following correlation:

$$\overline{Nu}_{n,D} = \left\{0.60 + \frac{0.387 Ra_{D}^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}}\right\}^{2} \qquad Ra_{D} < 10^{12} \qquad (36)$$

To compute the overall Nusselt number using Eq 28, set $j = 4$ and $\delta = 0.3$.

6.7.6 *Convection Conductances for Spheres:*

6.7.6.1 For forced convection on spheres, Incropera and DeWitt cite S. Whitaker in *AIChE J.* for the following correlation:

$$\overline{Nu}_{f,D} = 2 + \left(0.4\, Re_{D}^{1/2} + 0.06\, Re_{D}^{2/3}\right) Pr^{0.4} \left(\frac{\mu}{\mu_{s}}\right)^{1/4} \qquad (37)$$

$$0.71 < Pr < 380$$

$$3.5 < Re_{D} < 7.6 \times 10^{4}$$

$$1.0 < (\mu/\mu_{s}) < 3.2$$

where μ and μ_s are the free stream and surface viscosities of the ambient fluid respectively. It is extremely important to note that all properties need to be evaluated based on the free stream temperature of the ambient fluid, except for μ_s, which needs to be evaluated based on the surface temperature.

6.7.6.2 For natural convection on spheres, Incropera and DeWitt cite "Free Convection Around Immersed Bodies" by S. W. Churchill in *Heat Exchange Design Handbook* (Schlunder) for the following correlation:

$$\overline{Nu}_{n,D} = 2 + \frac{0.589\, Ra_{D}^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}} \qquad (38)$$

$$0.7 \leq Pr$$

$$Ra_{D} < 10^{11}$$

where all properties are evaluated at the film temperature. To compute the overall Nusselt number for spheres (Eq 28) set j = 4 and δ = 2.

7. Computer Program

7.1 *General:*

7.1.1 The computer program(s) are written in Microsoft® Visual Basic.

7.1.2 The program consists of a main program that utilizes several subroutines. Other subroutines may be added to make the program more applicable to the specific problems of individual users.

7.2 *Functional Description of Program*—The flow chart shown in Fig. 1 is a schematic representations of the operational procedures for each coordinate system covered by the program. The flow chart presents the logic path for entering data, calculating and recalculating system thermal resistances and temperatures, relaxing the successive errors in the temperature to within 0.1° of the temperature, calculating heat loss or gain for the system and printing the parameters and solution in tabular form.

7.3 *Computer Program Variable Descriptions*—The description of all variables used in the programs are given in the listing of the program as comments.

7.4 *Program Operation:*

7.4.1 Log on procedures and any executive program for execution of this program must be followed as needed.

7.4.2 The input for the thermal conductivity versus mean temperature parameters must be obtained as outlined in 6.6. The type code determines the thermal conductivity versus temperature relationship applying to the insulation. The same type code may be used for more than one insulation. As presented, the programs will operate on three functional relationships:

Type	Functional Relationship
Quadratic	$k = a + bt + ct^2$ where a, b, and c are constants
Linear	$k = a_1 + b_1t$; $t < t_L$ $k = a_2 + b_2t$; $t_L < t < t_U$ $k = a_3 + b_3t$; $t > t_U$ where $a1$, $a2$, $a3$, $b1$, $b2$, $b3$ are constants, and t_L and t_U are, respectively, the lower and upper inflection points of an S-shaped curve

Additional or different relationships may be used, but the main program must be modified.

8. Report

8.1 The results of calculations performed in accordance with this practice may be used as design data for specific job conditions, or may be used in general form to represent the performance of a particular product or system. When the results will be used for comparison of performance of similar products, it is recommended that reference be made to the specific constants used in the calculations. These references should include:

8.1.1 Name and other identification of products or components,

8.1.2 Identification of the nominal pipe size or surface insulated, and its geometric orientation,

8.1.3 The surface temperature of the pipe or surface,

8.1.4 The equations and constants selected for the thermal conductivity versus mean temperature relationship,

8.1.5 The ambient temperature and humidity, if applicable,

8.1.6 The surface transfer conductance and condition of surface heat transfer,

8.1.6.1 If obtained from published information, the source and limitations,

8.1.6.2 If calculated or measured, the method and significant parameters such as emittance, fluid velocity, etc.,

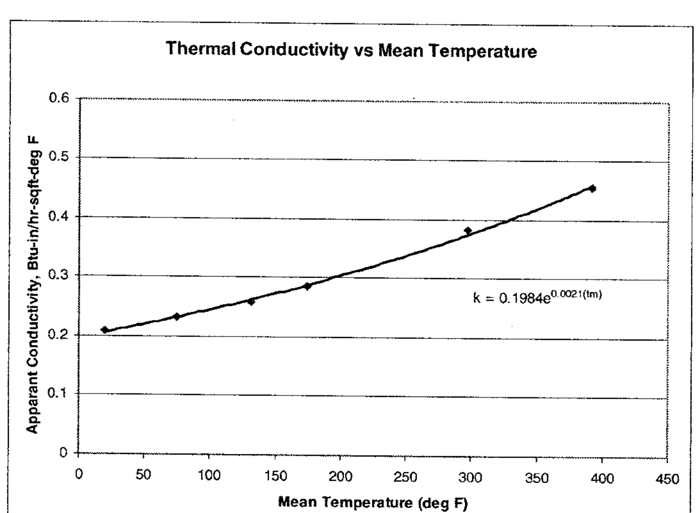

FIG. 2 Thermal Conductivity vs. Mean Temperature

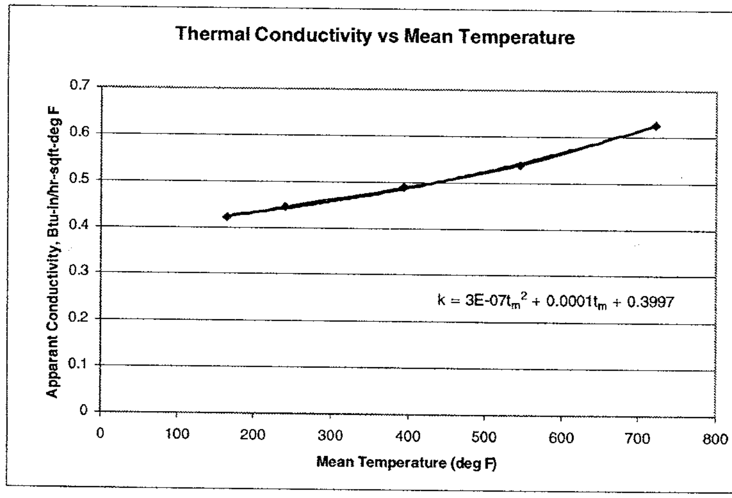

FIG. 3 Mean Temperature vs. Thermal Conductivity

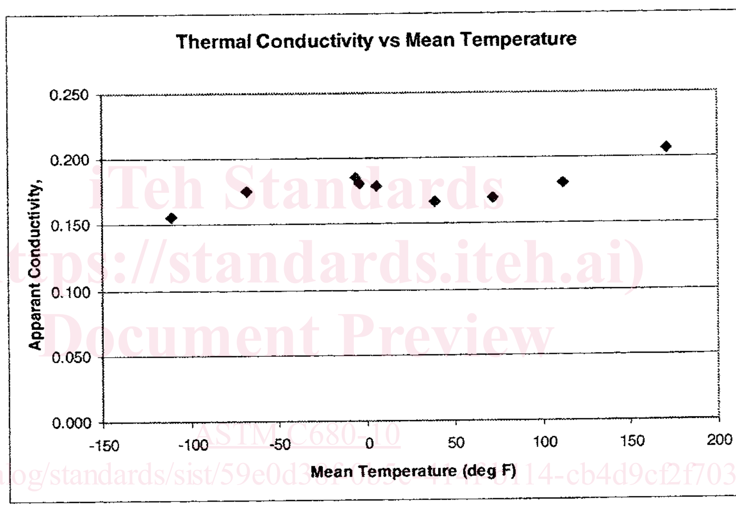

FIG. 4 Thermal Conductivity vs. Mean Temperature

8.1.7 The resulting outer surface temperature, and
8.1.8 The resulting heat loss or gain.

8.2 Either tabular or graphical representation of the calculated results may be used. No recommendation is made for the format in which results are presented.

9. Accuracy and Resolution

9.1 In many typical computers normally used, seven significant digits are resident in the computer for calculations. Adjustments to this level can be made through the use of "Double Precision;" however, for the intended purpose of this practice, standard levels of precision are adequate. The formatting of the output results, however, should be structured to provide a resolution of 0.1 % for the typical expected levels of heat flux and a resolution of 1°F (0.55°C) for surface temperatures.

NOTE 1—The term "double precision" should not be confused with ASTM terminology on Precision and Bias.

9.2 Many factors influence the accuracy of a calculative procedure used for predicting heat flux results. These factors include accuracy of input data and the applicability of the assumptions used in the method for the system under study. The system of mathematical equations used in this analysis has been accepted as applicable for most systems normally insulated with bulk type insulations. Applicability of this practice to systems having irregular shapes, discontinuities and other variations from the one-dimensional heat transfer assumptions should be handled on an individual basis by professional engineers familiar with those systems.

9.3 The computer resolution effect on accuracy is only significant if the level of precision is less than that discussed in 9.1. Computers in use today are accurate in that they will reproduce the calculated results to resolution required if identical input data is used.

9.4 The most significant factor influencing the accuracy of claims is the accuracy of the input thermal conductivity data. The accuracy of applicability of these data is derived from two factors. The first is the accuracy of the test method used to generate the data. Since the test methods used to supply these data are typically Test Methods C177, C335, or C518, the reports should contain some statement of the estimates of error or estimates of uncertainty. The remaining factors influencing the accuracy are the inherent variability of the product and the variability of the installation practices. If the product variability is large, the installation is poor, or both, serious differences might exist between measured performance and predicted performance from this practice.

10. Precision and Bias

10.1 When concern exists with the accuracy of the input test data, the recommended practice to evaluate the impact of possible errors is to repeat the calculation for the range of the uncertainty of the variable. This process yields a range in the desired output variable for a given uncertainty in the input variable. Repeating this procedure for all the input variables would yield a measure of the contribution of each to the overall uncertainty. Several methods exist for the combination of these effects; however, the most commonly used is to take the square root of the sum of the squares of the percentage errors induced by each variable's uncertainty. Eq 39 from *Theories of Engineering Experimentation* by H. Schenck gives the expression in mathematical form:

$$\frac{S}{R} = \left(\sum_{i=1}^{n} \left(\left(\frac{\partial R}{\partial x_i} \right) \Delta x_i \right)^2 \right)^{1/2} \quad (39)$$

where:

S = estimate of the probable error of the procedure,
R = result of the procedure,
x_i = ith variable in procedure,
$\partial R/\partial x_i$ = change in result with respect to change in ith variable,
Δx_i = uncertainty in value of variable, i, and
n = total number of variables in procedure.

10.2 ASTM Subcommittee C16.30, Task Group 5.2, which is responsible for preparing this practice, has prepared Appendix X1. The appendix provides a more complete discussion of the precision and bias expected when using Practice C680 in the analysis of operating systems. While much of that discussion is relevant to this practice, the errors associated with its application to operating systems are beyond the primary Practice C680 scope. Portions of this discussion, however, were used in developing the Precision and Bias statements included in Section 10.

11. Keywords

11.1 computer program; heat flow; heat gain; heat loss; pipe; thermal insulation

ANNEX

(Mandatory Information)

A1. EQUATIONS DERIVED FROM THE NIST CIRCULAR

A1.1 Table A1.1 lists the equations derived from the NBS Circular for the determination of the properties of air as used in this practice.

A1.2 T_k is temperature in degrees Kelvin, T_f is temperature in degrees Farenheit.

TABLE A1.1 Equations and Polynomial Fits for the Properties of Air Between –100ºF and 1300ºF (NBS Circular 564, Department of Commerce [1960])

Property	Equation	Units
Thermal Conductivity, k_a	$\frac{6.325\times10^{-6}\cdot\sqrt{T_k}}{[1+(245.4\cdot10^{=12/T_k})/T_k]}\cdot241.77$	Btu/(hr·ft·°F)
Dynamic Viscosity, μ	$\mu=\frac{145.8\cdot T_k\cdot\sqrt{T_k}}{T_k+110.4}\cdot241.9\times10^{-7}$	lb/(h·ft)
Prandtl Number, *Pr*	$Pr=0.7189-T_f\cdot[1.6349\times10^{-4}-T_f\cdot(1.8106\times10^{-7}-5.6617\times10^{-11}\cdot T_f)]$	
Volumetric Expansion Coefficient, β	$\beta=\frac{1}{1.8\cdot T_k}$	$°R^{-1}$
Density, ρ	$\rho=\frac{22.0493}{T_k}$	lb/ft^3
Kinematic Viscosity, ν	$\nu=\frac{\mu}{\rho}$	ft^2/h
Specific Heat, c_p	$c_p=0.24008-T_f\cdot[1.2477\times10^{-6}-T_f\cdot(4.0489\times10^{-8}-1.6088\times10^{-11}\cdot T_f)]$	Btu/(lb·°R[†])

[†] Editorially corrected June 2007.

APPENDIXES

(Nonmandatory Information)

X1. APPLICATION OF PRACTICE C680 TO FIELD MEASUREMENTS

X1.1 This appendix has been included to provide a more complete discussion of the precision and bias expected when using this practice in the analysis of operating systems. While much of the discussion below is relevant to the practice, the errors associated with its application to operating systems is beyond the immediate scope of this task group. Portions of this discussion, however, were used in developing the Precision and Bias statements included in Section 10.

X1.2 This appendix will consider precision and bias as it relates to the comparison between the calculated results of the Practice C680 analysis and measurements on operating systems. Some of the discussion here may also be found in Section 10; however, items are expanded here to include analysis of operating systems.

X1.3 *Precision*:

X1.3.1 The precision of this practice has not yet been demonstrated as described in Specification E691, but an interlaboratory comparison could be conducted, if necessary, as facilities and schedules permit. Assuming no errors in programming or data entry, and no computer hardware malfunctions, an interlaboratory comparison should yield the theoretical precision presented in X1.3.2.

X1.3.2 The theoretical precision of this practice is a function of the computer equipment used to generate the calculated results. Typically, seven significant digits are resident in the computer for calculations. The use of "Double Precision" can expand the number of digits to sixteen. However, for the intended purpose of this practice, standard levels of precision are adequate. The effect of computer resolution on accuracy is only significant if the level of precision is higher than seven digits. Computers in use today are accurate in that they will reproduce the calculation results to the resolution required if identical input data is used.

X1.3.2.1 The formatting of output results from this has been structured to provide a resolution of 0.1 % for the typically expected levels of heat flux, and within 0.1°F (0.05°C) for surface temperatures.

X1.3.2.2 A systematic precision error is possible due to the choices of the equations and constants for convective and radiative heat transfer used in the program. The interlaboratory comparison of X1.3.3 indicates that this error is usually within the bounds expected in in-situ heat flow calculations.

X1.3.3 *Precision of Surface Convection Equations:*

X1.3.3.1 Many empirically derived equation sets exist for the solution of convective heat transfer from surfaces of various shapes in various environments. If two different equation sets are chosen and a comparison is made using identical input data, the calculated results are never identical, not even when the conditions for application of the equations appear to be identical. For example, if equations designed for vertical surfaces in turbulent cross flow are compared, results from this comparison could be used to help predict the effect of the equation sets on overall calculation precision.

X1.3.3.2 The systematic precision of the surface equation set used in this practice has had at least one through intralaboratory evaluation (**11**). When the surface convective coefficient equation (see 6.6) of this practice was compared to another surface equation set by computer modeling of identical conditions, the resultant surface coefficients for the 240 typical data sets varied, in general, less than 10 %. One extreme case (for flat surfaces) showed variations up to 30 %. Other observers have recorded larger variations (in less rigorous studies) when additional equation sets have been compared.

Unfortunately, there is no standard for comparison since all practical surface coefficient equations are empirically derived. The equations in 6.6 are accepted and will continue to be recommended until evidence suggests otherwise.

X1.3.4 *Precision of Radiation Surface Equation:*

X1.3.4.1 The Stefen-Boltzmann equation for radiant transfer is widely applied. In particular, there remains some concern as to whether the exponents of temperature are exactly 4.0 in all cases. A small error in these exponents cause a larger error in calculated radiant heat transfer. The exactness of the coefficient 4 is well-founded in both physical and quantum physical theory and is therefore used here.

X1.3.4.2 On the other hand, the ability to measure and preserve a known emittance is quite difficult. Furthermore, though the assumptions of an emittance of 1.0 for the surroundings and a "sink" temperature equal to ambient air temperature is often approximately correct in a laboratory environment, operating systems in an industrial environment often diverge widely from these assumptions. The effect of using 0.95 for the emittance of the surroundings rather than the 1.00 assumed in the previous version of this practice was also investigated by the task group **(11)**. Intralaboratory analysis of the effect of assuming a surrounding effective emittance 0.95 versus 1.00 indicates a variation of 5 % in the radiation surface coefficient when the object emittance is 1.00. As the object emittance is reduced to 0.05, the difference in the surface coefficient becomes negligible. These differences would be greater if the surrounding effective emittance is less than 0.95.

X1.3.5 *Precision of Input Data:*

X1.3.5.1 The heat transfer equations used in the computer program of this practice imply possible sources of significant errors in the data collection process, as detailed later in this appendix.

NOTE X1.1—Although data collection is not within the scope of this practice, the results of this practice are highly dependent on accurate input data. For this reason, a discussion of the data collection process is included here.

X1.3.5.2 A rigorous demonstration of the impact of errors associated with the data collection phase of an operating system's analysis using Practice C680 is difficult without a parametric sensitivity study on the method. Since it is beyond the intent of this discussion to conduct a parametric study for all possible cases, X1.3.5.3 – X1.3.5.7 discuss in general terms the potential for such errors. It remains the responsibility of users to conduct their own investigation into the impact of the analysis assumptions particular to their own situations.

X1.3.5.3 *Conductivity Data*—The accuracy and applicability of the thermal conductivity data are derived from several factors. The first is the accuracy of the test method used to generate the data. Since Test Methods C177, C335, and C518 are usually used to supply test data, the results reported for these tests should contain some statement of estimated error or estimated uncertainty. The remaining factors influencing the accuracy are the inherent variability of the product and the variability of insulation installation practice. If the product variability is large or the installation is poor, or both, serious differences might exist between the measured performance and the performance predicted by this method.

X1.3.5.4 *Surface Temperature Data*—There are many techniques for collecting surface temperatures from operating systems. Most of these methods assuredly produce some error in the measurement due to the influence of the measurement on the operating condition of the system. Additionally, the intended use of the data is important to the method of surface temperature data collection. Most users desire data that is representative of some significant area of the surface. Since surface temperatures frequently vary significantly across operating surfaces, single-point temperature measurements usually lead to errors. Sometimes very large errors occur when the data is used to represent some integral area of the surface. Some users have addressed this problem through various means of determining average surface temperature, Such techniques will often greatly improve the accuracy of results used to represent average heat flows. A potential for error still exists, however, when theory is precisely applied. This practice applies only to areas accurately represented by the average point measurements, primarily because the radiation and convection equations are non-linear and do not respond correctly when the data is averaged. The following example is included to illustrate this point:

(1) Assume the system under analysis is a steam pipe. The pipe is jacketed uniformly, but one-half of its length is poorly insulated, while the second half has an excellent insulation under the jacket. The surface temperature of the good half is measured at 550°F. The temperature of the other half is measured at 660°F. The average of the two temperatures is 605°F. The surface emittance is 0.92, and ambient temperature is 70°F. Solving for the surface radiative heat loss rates for each half and for the average yields the following:

(2) The average radiative heat loss rate corresponding to a 605°F temperature is 93.9 Btu/ft^2/h.

(3) The "averaged" radiative heat loss obtained by calculating the heat loss for the individual halves, summing the total and dividing by the area, yields an "averaged" heat loss of 102.7 Btu/ft^2/h. The error in assuming the averaged surface temperature when applied to the radiative heat loss for this case is 8.6 %.

(4) It is obvious from this example that analysis by the methods described in this practice should be performed only on areas which are thermally homogeneous. For areas in which the temperature differences are small, the results obtained using Practice C680 will be within acceptable error bounds. For large systems or systems with significant temperature variations, total area should be subdivided into regions of nearly uniform temperature difference so that analysis may be performed on each subregion.

X1.3.5.5 *Ambient Temperature Variations*—In the standard analysis by the methods described in his practice, the temperature of the radiant surroundings is taken to be equal to the ambient air temperature (for the designer making comparative studies, this is a workable assumption). On the other hand, this assumption can cause significant errors when applied to equipment in an industrial environment, where the surroundings may contain objects at much different temperatures than the surrounding air. Even the natural outdoor environment does not conform well to the assumption of air temperatures when the solar or night sky radiation is considered. When this practice is used in conjunction with in-situ measurements of surface temperatures, as would be the case in an audit survey, extreme care must be observed to record the environmental conditions at the time of the measurements. While the computer program supplied in this practice does not account for these differences, modifications to the program may be made easily to separate the convective ambient temperature from the mean radiative environmental temperature seen by the surface. The key in this application is the evaluation of the magnitude of this mean radiant temperature. The mechanism for this evaluation is beyond the scope of this practice. A discussion of the mean radiant temperature concept is included in the ASHRAE Handbook of Fundamentals (**12**).

X1.3.5.6 *Emittance Data*—Normally, the emittance values used in a Practice C680 analysis account only for the emittance of the subject of the analysis. The subject is assumed to be completely surrounded by an environment which has an assigned emittance of 0.95. Although this assumption may be valid for most cases, the effective emittance used in the calculation can be modified to account for different values of effective emittance. If this assumption is a concern, using the following formula for effective surface emittance will correct for this error:

$$\varepsilon_{eff} = \frac{A_A}{(1-\varepsilon_A)\,\varepsilon_A\,A_A + 1/A_A\,F_{AB} + (1-\varepsilon_B)/\varepsilon_B\,A_B} \quad \text{(X1.1)}$$

where:

ε_{eff} = effective mean emittance for the two surface combination,
ε_A = mean emittance of the surface A,
ε_B = mean emittance of the surrounding region B,
F_{AB} = view factor for the surface A and the surrounding region B,
A_A = area of region A, and
A_B = area of region B.

This equation set is described in most heat transfer texts on heat transfer. See Holman (**1**), p. 305.

X1.3.5.7 *Wind Speed*—Wind speed is defined as wind speed measured in the main airstream near the subject surface. Air blowing across real objects often follows flow directions and velocities much different from the direction and velocity of the main free stream. The equations used in Practice C680 analysis yield "averaged" results for the entire surface in question. Because of this averaging, portions of the surface will have different surface temperatures and heat flux rates from the average. For this reason, the convective surface coefficient calculation cannot be expected to be accurate at each location on the surface unless the wind velocity measurements are made close to the surface and a separate set of equations are applied that calculate the local surface coefficients.

X1.3.6 *Theoretical Estimates of Precision:*

X1.3.6.1 When concern exists regarding the accuracy of the input test data, the recommended practice is to repeat the calculation for the range of the uncertainty of the variable. This process yields a range of the desired output variable for a given input variable uncertainty. Several methods exist for evaluating the combined variable effects. Two of the most common are illustrated as follows:

X1.3.6.2 The most conservative method assumes that the errors propagating from the input variable uncertainties are additive for the function. The effect of each of the individual input parameters is combined using Taylor's Theorem, a special case of a Taylor's series expansion (**13**).

$$\frac{S}{R} = \sum_{i=1}^{n} \left| \frac{\partial R}{\partial x_i} \right| \cdot \Delta x_i \quad \text{(X1.2)}$$

where:

S = estimate of the probable error of the procedure,
R = result of the procedure,
x_i = ith variable of the procedure,
$\partial R/\partial S$ = change in result with respect to a change in the ith variable (also, the first derivative of the function with respect to the ith variable),
x_i = uncertainty in value of variable i, and
n = total number of input variables in the procedure.

X1.3.6.3 For the probable uncertainty of function, R, the most commonly used method is to take the square root of the sum of the squares of the fractional errors. This technique is also known as Pythagorean summation. This relationship is described in Eq 39, Section 10.

X1.3.7 *Bias of Practice C680 Analysis:*

X1.3.7.1 As in the case of the precision, the bias of this standard practice is difficult to define. From the preceding discussion, some bias can result due to the selection of alternative surface coefficient equation sets. If, however, the same equation sets are used for a comparison of two insulation systems to be operated at the same conditions, no bias of results is expected from this method. The bias due to computer differences will be negligible in comparison with other sources of potential error. Likewise, the use of the heat transfer equations in the program implies a source of potential bias errors, unless the user ensures the applicability of the practice to the system.

X1.3.8 *Error Avoidance*—The most significant sources of possible error in this practice are in the misapplication of the empirical formulae for surface transfer coefficients, such as using this practice for cases that do not closely fit the thermal and physical model of the equations. Additional errors evolve from the superficial treatment of the data collection process. Several promising techniques to minimize these sources of error are in stages of development. One attempt to address some of the issues has been documented by Mack (**14**). This technique addresses all of the above issues except the problem of non-standard insulation k values. As the limitations and strengths of in-situ measurements and Practice C680 analysis become better understood, they can be incorporated into additional standards of analysis that should be associated with this practice. Until such methods can be standardized, the best assurance of accurate results from this practice is tat each application of the practice will be managed by a user who is knowledgeable in heat transfer theory, scientific data collection practices, and the mathematics of programs supplied in this practice.