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Standard Practice for
Calculation of Weighting Factors for Tristimulus Integration[1]

This standard is issued under the fixed designation E2022; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice describes the method to be used for calculating tables of weighting factors for tristimulus integration using custom spectral power distributions of illuminants or sources, or custom color-matching functions.

1.2 This practice provides methods for calculating tables of values for use with spectral reflectance or transmittance data, which are corrected for the influences of finite bandpass. In addition, this practice provides methods for calculating weighting factors from spectral data which has not been bandpass corrected. In the latter case, a correction for the influence of bandpass on the resulting tristimulus values is built in to the tristimulus integration through the weighting factors.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to its use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E284 Terminology of Appearance

E308 ~~Practice for Computing the Colors of Objects by Using the CIE System~~ Practice for Computing the Colors of Objects by Using the CIE System

E2729 Practice for Rectification of Spectrophotometric Bandpass Differences

2.2 *CIE Standard:*

CIE Standard S 002 Colorimetric Observers[3]

3. Terminology

3.1 *Definitions*—Appearance terms in this practice are in accordance with Terminology E284.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *illuminant*, *n*—real or ideal radiant flux, specified by its spectral distribution over the wavelengths that, in illuminating objects, can affect their perceived colors.

3.2.2 *source*, *n*—an object that produces light or other radiant flux, or the spectral power distribution of that light.

3.2.2.1 *Discussion*—A source is an emitter of visible radiation. An illuminant is a table of agreed spectral power distribution that may represent a source; thus, Illuminant A is a standard spectral power distribution and Source A is the physical representation of that distribution. Illuminant D65 is a standard illuminant that represents average north sky daylight but has no representative source.

3.2.3 *spectral power distribution, SPD, S(λ), n*—specification of an illuminant by the spectral composition of a radiometric quantity, such as radiance or radiant flux, as a function of wavelength.

4. Summary of Practice

4.1 CIE color-matching functions are standardized at 1-nm wavelength intervals. Tristimulus integration by multiplication of abridged spectral data into sets of weighting factors occurs at larger intervals, typically 10-nm or 20-nm; therefore, intermediate 1-nm interval spectral data are missing, but needed.

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[1] This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from USNC-CIE Publications Office (International Commission on Illumination), C/o Thomas M. Lemons, TLA-Lighting Consultants, Inc., 7 Pond St., Salem, MA 01970, http://www.cie-usnc.org.

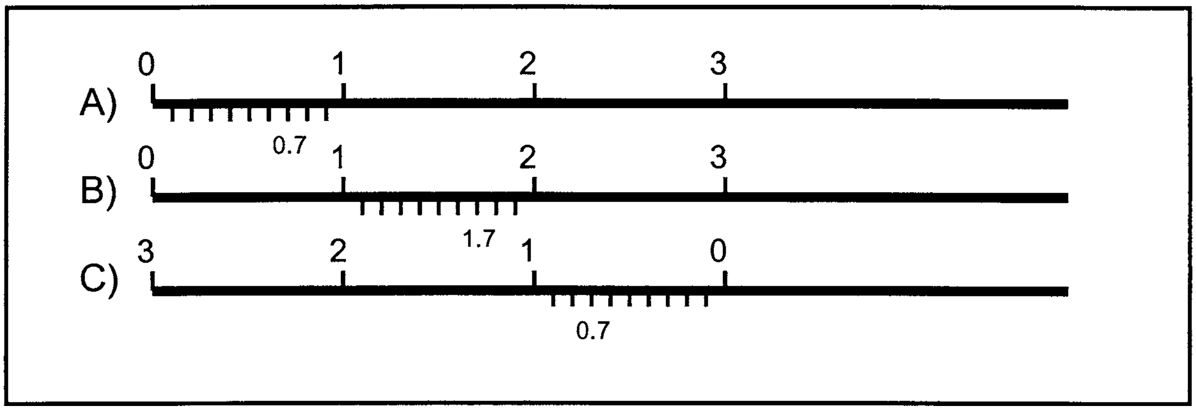

FIG. 1 The Values of *i* in Eq 1 are Plotted Above the Abscissa and the Values of *r* are Plotted Below for A) the First Measurement Interval; B) the Intermediate Measurement Intervals; and, C) the Last Measurement Interval Being Interpolated

4.2 Lagrange interpolating coefficients are calculated for the missing wavelengths. The Lagrange coefficients, when multiplied into the appropriate measured spectral data, interpolate the abridged spectrum to 1-nm interval. The 1-nm interval spectrum is then multiplied into the CIE 1-nm color-matching data, and into the source spectral power distribution. Each separate term of this multiplication is collected into a value associated with a measured spectral wavelength, thus forming weighting factors for tristimulus integration. ~~4.3A correction may be applied to the resulting table of weighting factors to incorporate a correction for the spectral data's bandpass dependence.~~

5. Significance and Use

5.1 This practice is intended to provide a method that will yield uniformity of calculations used in making, matching, or controlling colors of objects. This uniformity is accomplished by providing a method for calculation of weighting factors for tristimulus integration consistent with the methods utilized to obtain the weighting factors for common illuminant-observer combinations contained in Practice E308.

5.2 This practice should be utilized by persons desiring to calculate a set of weighting factors for tristimulus integration who have custom source, or illuminant spectral power distributions, or custom observer response functions.

5.3 This practice assumes that the measurement interval is equal to the spectral bandwidth integral when applying correction for bandwidth.

6. Procedure

6.1 *Calculation of Lagrange Coefficients*—Obtain by calculation, or by table look-up, a set of Lagrange interpolating coefficients for each of the missing wavelengths.[4]

6.1.1 The coefficients should be quadratic (three-point) in the first and last missing interval, and cubic (four-point) in all intervals between the first and the last missing interval.

6.1.2 *Generalized Lagrange Coefficients*—Lagrange coefficients may be calculated for any interval and number of missing wavelengths by Eq 1:

$$L_j(r) = \prod_{i=0}^{n}\prod_{i \neq j} \frac{(r - r_i)}{(r_j - r_i)}, \text{ for } j = 0,1,...n \tag{1}$$

[4] Hildebrand, F. B., *Introduction to Numerical Analysis*, Second Edition, Dover, New York, 1974, Chapter 3.

where:

n = degree of coefficients being calculated,[5]
i and j = indices denoting the location along the abscissa,
π = repetitive multiplication of the terms in the numerator and the denominator, and
indices of the interpolant, r = chosen on the same scale as the values i and j.

6.1.2.1 Fig. 1 assist the user in selecting the values of i, j, and r for these calculations.

6.1.2.2 Eq 1 is general and is applicable to any measurement interval or interpolation interval, regular or irregular.

6.1.3 *10 and 20-nm Lagrange Coefficients*—Where the measured spectral data have a regular or constant interval, the equation reduces to the following:

$$L_0 = \frac{(r-1)(r-2)(r-3)}{-6} \tag{2}$$

$$L_1 = \frac{(r)(r-2)(r-3)}{2} \tag{3}$$

$$L_2 = \frac{(r-1)(r)(r-3)}{-2} \tag{4}$$

$$L_3 = \frac{(r-1)(r-2)(r)}{6} \tag{5}$$

for the cubic case, and to

$$L_0 = \frac{(r-1)(r-2)}{2} \tag{6}$$

$$L_1 = \frac{(r)(r-2)}{-1} \tag{7}$$

$$L_2 = \frac{(r-1)(r)}{2} \tag{8}$$

for the quadratic case. In each of the above equations, as many or as few values of r as required are chosen to generate the necessary coefficients.

6.1.3.1 Eq 2-8 are applicable when the spectral data are abridged at 10-nm or 20-nm intervals, and the interpolated interval is regular with respect to the measurement interval, presumably 1-nm.

6.1.4 Tables 1-4 provide both quadratic and cubic Lagrange coefficients for 10-nm and 20-nm intervals.

TABLE 1 The Lagrange Quadratic Interpolation Coefficients Applicable to the First and Last Missing Interval for Calculation of 10-nm Weighting Factors for Tristimulus Integration

Index of Missing Wavelength	L_0	L_1	L_2
1	0.855	0.190	–0.045
2	0.720	0.360	–0.080
3	0.595	0.510	–0.105
4	0.480	0.640	–0.120
5	0.375	0.750	–0.125
6	0.280	0.840	–0.120
7	0.195	0.910	–0.105
8	0.120	0.960	–0.080
9	0.055	0.990	–0.045

6.2 With the Lagrange coefficients provided, the intermediate missing spectral data may be predicted as follows:

$$P(\lambda) = \sum_{i=0}^{n} L_i m_i \tag{9}$$

[5] Fairman, H. S., "The Calculation of Weight Factors for Tristimulus Integration," *Color Research and Application*, Vol 10 , 1985, pp. 199–203.

TABLE 2 The Lagrange Cubic Interpolation Coefficients Applicable to the Interior Missing Intervals for Calculation of 10-nm Weighting Factors for Tristimulus Integration

Index of Missing Wavelength	L_0	L_1	L_2	L_3
1	–0.0285	0.9405	0.1045	–0.0165
2	–0.0480	0.8640	0.2160	–0.0320
3	–0.0595	0.7735	0.3315	–0.0455
4	–0.0640	0.6720	0.4480	–0.0560
5	–0.0625	0.5625	0.5625	–.0625
6	–0.0560	0.4480	0.6720	–0.0640
7	–0.0455	0.3315	0.7735	–0.0595
8	–0.0320	0.2160	0.8640	–0.0480
9	–0.0165	0.1045	0.9405	–0.0285

TABLE 3 The Lagrange Quadratic Interpolating Coefficients Applicable to the First and Last Missing Interval for Calculation of 20-nm Weighting Factors for Tristimulus Integration.

Index of Missing Wavelength	L_0	L_1	L_2
1	0.92625	0.0975	–0.02375
2	0.85500	0.1900	–0.04500
3	0.78625	0.2775	–0.06375
4	0.72000	0.3600	–0.08000
5	0.65625	0.4375	–0.09375
6	0.59500	0.5100	–0.10500
7	0.53625	0.5775	–0.11375
8	0.48000	0.6400	–0.12000
9	0.42625	0.6975	–0.12375
10	0.37500	0.7500	–0.12500
11	0.32625	0.7975	–0.12375
12	0.28000	0.8400	–0.12000
13	0.23625	0.8775	–0.11375
14	0.19500	0.9100	–0.10500
15	0.15625	0.9375	–0.09375
16	0.12000	0.9600	–0.08000
17	0.08625	0.9775	–0.06375
18	0.05500	0.9900	–0.04500
19	0.02625	0.9975	–0.02375

TABLE 4 The Lagrange Cubic Interpolating Coefficients Applicable to the Interior Missing Intervals for Calculation of 20-nm Weighting Factors for Tristimulus Integration

Index of Missing Wavelength	L_0	L_1	L_2	L_3
1	–0.0154375	0.9725625	0.0511875	–0.0083125
2	–0.028500	0.940500	0.104500	–0.016500
3	–0.0393125	0.9041875	0.1595625	–0.0244375
4	–0.048000	0.864000	0.216000	–0.032000
5	–0.0546875	0.8203125	0.2734375	–0.0390625
6	–0.059500	0.773500	0.331500	–0.045500
7	–0.0625625	0.7239375	0.3898125	–0.0511875
8	–0.064000	0.672000	0.448000	–0.056000
9	–0.0639375	0.6180625	0.5056875	–0.0598125
10	–0.062500	0.562500	0.562500	–0.062500
11	–0.0598125	0.5056875	0.6180625	–0.0639375
12	–0.056000	0.448000	0.672000	–0.064000
13	–0.0511875	0.3898125	0.7239375	–0.0625625
14	–0.045500	0.331500	0.773500	–0.059500
15	–0.0390625	0.2734375	0.8203125	–0.0546875
16	–0.032000	0.216000	0.864000	–0.048000
17	–0.0244375	0.1595625	0.9041875	–0.0393125
18	–0.016500	0.104500	0.940500	–0.028500
19	–0.0083125	0.0511875	0.9725625	–0.0154375

where:
P = the value being interpolated at interval λ,
L = the Lagrange coefficients, and

m = the measured abridged spectral values.

Because the measured spectral values are as yet unknown, it may be best to consider this equation in its expanded form:

$$P(\lambda) = L_0 m_0 + L_1 m_1 + L_2 m_2 + L_3 m_3 \quad (10)$$

6.3 Multiply each $P(\lambda)$ by the 1-nm interval relative spectral power of the source or illuminant being considered.

6.3.1 It may be necessary to interpolate missing values of the source spectral power distribution $S(\lambda)$, if the source has been measured at other than 1-nm intervals.

6.3.2 Doing so results in the following equation:

$$S(\lambda)P(\lambda) = S(\lambda)L_0 m_0 + S(\lambda)L_1 m_1 + S(\lambda)L_2 m_2 + S(\lambda)L_3 m_3 \quad (11)$$

6.4 Multiply the weighted power at each 1-nm wavelength by the appropriate custom color-matching function value for that wavelength. Using the CIE color-matching functions as an example, obtain the CIE 1-nm data from CIE Standard S 002, Colorimetric Observers. Doing so results in the following equation:

$$\bar{x}(\lambda)S(\lambda)P(\lambda) = [\bar{x}(\lambda)S(\lambda)L_0]m_0 + [\bar{x}(\lambda)S(\lambda)L_1]m_1 + [\bar{x}(\lambda)S(\lambda)L_2]m_2 + [\bar{x}(\lambda)S(\lambda)L_3]m_3 \quad (12)$$

where:

$\bar{x}(\lambda)$ = the value of the CIE X color-matching function at wavelength λ, and the calculations are carried out for each of the three CIE color-matching functions, $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$.

6.5 In the four terms on the right-hand side of this equation, the numerical values of the three factors in the brackets are known and should be multiplied into a single coefficient. The fourth factor, m_i, in each of the four additive terms is associated with a different measured wavelength.

6.6 Add all multiplicative coefficients dependent upon each different measured wavelength into a single coefficient applicable to that wavelength. This results in a single set of weighting factors that then will contain one value for each measured wavelength in each of three color-matching functions. The partial contribution to the tristimulus value at wavelength m_0 is:

$$[(\bar{x}(\lambda_0)S(\lambda_0)L_0) + (\bar{x}(\lambda_1)S(\lambda_1)L_0) + \ldots]m_0 = wt_0 m_0 \quad (13)$$

6.7 Normalize the weighting factors by calculating the following normalizing coefficient:

$$k = \frac{100}{\Sigma S(\lambda)\bar{y}(\lambda)} \quad (14)$$

where:

k = the normalizing coefficient,
$S(\lambda)$ = the power in the 1-nm spectrum, and
$y(\lambda)$ = the CIE Y color-matching function.

6.8 Multiply the weighting factors by k to normalize the set to $Y = 100$ for the perfect reflecting diffuser.

~~6.9 *Correction for Bandpass Dependence*—If it is desired to correct the resulting weighting factors for the bandpass dependence of the measured spectral data, apply the following correction to the interior passbands.~~

~~(15) $Wc(i) = -0.083 \cdot WM(i-1) + 1.166 \cdot WM(i) - 0.083 \cdot WM(i+1)$~~

~~where:~~

~~W = the indexed weight,~~
~~c = a corrected weight, and~~
~~m = a weight calculated without bandpass correction.~~

~~The index i varies from the second measured passband to the next to last measured passband. The following correction applies to the first and last measured passband:~~

~~$$W_c(i) = 1.083 \cdot W_M(i) - 0.083 \cdot W_M(i \pm 1) \quad (16)$$~~

~~where the symbols are the same as those of Eq 15 and the index i and ± refer to the first and last measured passbands, respectively.~~

6.9 Beginning in January of 2010, rectification of bandpass differences is no longer accomplished by building the correction factors into a weight set for tristimulus integration. This is because to do so fails to correct the spectrum itself and corrects only the tristimulus values. Bandpass rectification is now under the jurisdiction of Practice E2729.

7. Precision

7.1 The precision of the practice is limited only by the precision of the data provided for the source spectral power distribution. The CIE color-matching functions are precise to six digits by definition. The Lagrange coefficients are precise to seven digits.

8. Keywords

8.1 color-matching functions; illuminant; illuminant-observer weights; source; tristimulus weighting factors

~~APPENDIX~~

APPENDIXES

(Nonmandatory Information)

X1. EXAMPLE OF THE CALCULATIONS

X1.1 Table X1.1 gives the spectral power distribution (SPD) of a typical 3-band fluorescent lamp with a correlated color temperature of about 3000K. The first step is to multiply each value of the SPD by the appropriate CIE color matching function ($\bar{y}$ in this case), wavelength by wavelength, which is shown in Table X1.2 for three spectral regions: near 360 nm, 560 nm, and 830 nm. Table X1.3 shows a typical interpolation of a measured reflectance curve from a 10-nm reported interval to the 1-nm interval that matches the SPD- $\bar{y}$ product in the same three spectral regions. Tables X1.4-X1.6 illustrate how the same measured data, used to interpolate the missing reflectance data in several different intervals, can be combined with the illuminant-color matching function product to form a single weight at a single measurement point. Finally, Table X1.7 shows the resulting weight set for this 3000K source and the 1964 10° color matching functions. Table X1.7 is compatible with Tables 5 in Practice E308~~.~~ ~~The weights in Table X1.7 then can be adjusted by the Stearns[6] bandwidth terms to create a new weight set that is compatible with Tables6 in Practice E308. These bandwidth corrected data are shown in Table X1.8~~.