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Standard Guide for Statistical Analysis of Service Life Data[1]

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1. Scope

1.1 This guide presents briefly some generally accepted methods of statistical analyses which are useful in the interpretation of service life data. It is intended to produce a common terminology as well as developing a common methodology and quantitative expressions relating to service life estimation.

1.2 This guide does not cover detailed derivations, or special cases, but rather covers a range of approaches which have found application in service life data analyses.

1.3 Only those statistical methods that have found wide acceptance in service life data analyses have been considered in this guide.

1.4 The Weibull life distribution model is emphasized in this guide and example calculations of situations commonly encountered in analysis of service life data are covered in detail.

1.5 The choice and use of a particular life distribution model should be based primarily on how well it fits the data and whether it leads to reasonable projections when extrapolating beyond the range of data. Further justification for selecting a model should be based on theoretical considerations.

2. Referenced Documents

2.1 *ASTM Standards:*[2]

G169 Guide for Application of Basic Statistical Methods to Weathering Tests

3. Terminology

3.1 *Definitions:*

3.1.1 *material property*—customarily, service life is considered to be the period of time during which a system meets critical specifications. Correct measurements are essential to producing meaningful and accurate service life estimates.

3.1.1.1 *Discussion*—There exists many ASTM recognized and standardized measurement procedures for determining material properties. As these practices have been developed within committees with appropriate expertise, no further elaboration will be provided.

3.1.2 *beginning of life*—this is usually determined to be the time of manufacture. Exceptions may include time of delivery to the end user or installation into field service.

3.1.3 *end of life*—Occasionally this is simple and obvious such as the breaking of a chain or burning out of a light bulb filament. In other instances, the end of life may not be so catastrophic and free from argument. Examples may include fading, yellowing, cracking, crazing, etc. Such cases need quantitative measurements and agreement between evaluator and user as to the precise definition of failure. It is also possible to model more than one failure mode for the same specimen. (for example, The time to produce a given amount of yellowing may be measured on the same specimen that is also tested for cracking.)

3.1.4 *F(t)*—The probability that a random unit drawn from the population will fail by time *(t)*. Also *F(t)* = the decimal fraction of units in the population that will fail by time *(t)*. The decimal fraction multiplied by 100 is numerically equal to the percent failure by time *(t)*.

3.1.5 *R(t)*—The probability that a random unit drawn from the population will survive at least until time *(t)*. Also R(t) = the fraction of units in the population that will survive at least until time *(t)*

$$R(t) = 1 - F(t) \quad (1)$$

3.1.6 *pdf*—the probability density function (pdf), denoted by f(t), equals the probability of failure between any two points of time t(1) and t(2). Mathematically $f(t)=\frac{dF(t)}{dt}$. For the normal distribution, the pdf is the "bell shape" curve.

3.1.7 *cdf*—the cumulative distribution function (cdf), denoted by F(t), represents the probability of failure (or the population fraction failing) by time = (t). See section 3.1.4.

3.1.8 *weibull distribution*—For the purposes of this guide, the Weibull distribution is represented by the equation:

$$F(t) = 1 - e^{-\left(\frac{t}{c}\right)^{b}} \quad (2)$$

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

where:

$F(t)$	=	defined in paragraph 3.1.4
t	=	units of time used for service life
c	=	scale parameter
b	=	shape parameter

3.1.8.1 The shape parameter (b), section 3.1.6, is so called because this parameter determines the overall shape of the curve. Examples of the effect of this parameter on the distribution curve are shown in Fig. 1, section 5.3.

3.1.8.2 The scale parameter (c), section 3.1.6, is so called because it positions the distribution along the scale of the time axis. It is equal to the time for 63.2 % failure.

NOTE 1—This is arrived at by allowing t to equal c in the above expression. This then reduces to Failure Probability = $1{-}e^{-1}$, which further reduces to equal 1–0.368 or .632.

3.1.9 *complete data*—A complete data set is one where all of the specimens placed on test fail by the end of the allocated test time.

3.1.10 *Incomplete data*—An incomplete data set is one where (*a*) there are some specimens that are still surviving at the expiration of the allowed test time, (*b*) where one or more specimens is removed from the test prior to expiration of the allowed test time. The shape and scale parameters of the above distributions may be estimated even if some of the test specimens did not fail. There are three distinct cases where this might occur.

3.1.10.1 *Time censored*—Specimens that were still surviving when the test was terminated after elapse of a set time are considered to be time censored. This is also referred to as right censored or type I censoring. Graphical solutions can still be used for parameter estimation. At least ten observed failures should be used for estimating parameters (for example slope and intercept).

3.1.10.2 *specimen censored*—Specimens that were still surviving when the test was terminated after a set number of failures are considered to be specimen censored. This is another case of right censored or type I censoring. See 3.1.10.1

3.1.10.3 *Multiply Censored*—Specimens that were removed prior to the end of the test without failing are referred to as left censored or type II censored. Examples would include specimens that were lost, dropped, mishandled, damaged or broken due to stresses not part of the test. Adjustments of failure order can be made for those specimens actually failed.

4. Significance and Use

4.1 Service life test data often show different distribution shapes than many other types of data. This is due to the effects of measurement error (typically normally distributed), combined with those unique effects which skew service life data towards early failure (infant mortality failures) or late failure times (aging or wear-out failures) Applications of the principles in this guide can be helpful in allowing investigators to interpret such data.

NOTE 2—Service life or reliability data analysis packages are becoming more readily available in standard or common computer software packages. This puts data reduction and analyses more readily into the hands of a growing number of investigators.

5. Data Analysis

5.1 In the determinations of service life, a variety of factors act to produce deviations from the expected values. These factors may be of a purely random nature and act to either increase or decrease service life depending on the magnitude of the factor. The purity of a lubricant is an example of one such factor. An oil clean and free of abrasives and corrosive materials would be expected to prolong the service life of a moving part subject to wear. A fouled contaminated oil might prove to be harmful and thereby shorten service life. Purely random variation in an aging factor that can either help or harm

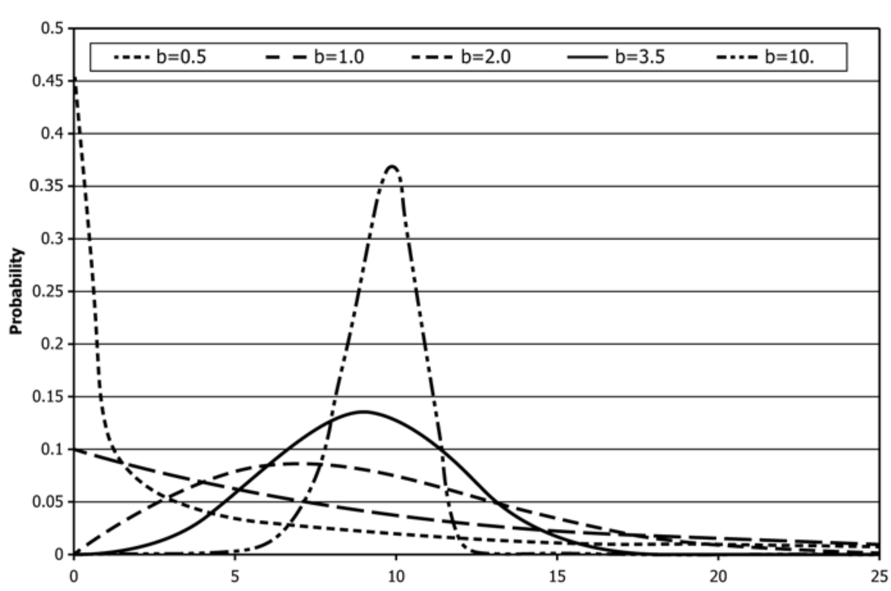

FIG. 1 Effect of the Shape Parameter (b) on the Weibull Probability Density

a service life might lead a normal, or gaussian, distribution. Such distributions are symmetrical about a central tendency, usually the mean.

5.1.1 Some non-random factors act to skew service life distributions. Defects are generally thought of as factors that can only decrease service life. Thin spots in protective coatings, nicks in extruded wires, chemical contamination in thin metallic films are examples of such defects that can cause an overall failure even through the bulk of the material is far from failure. These factors skew the service life distribution towards early failure times.

5.1.2 Factors that skew service life towards the high side also exist. Preventive maintenance, high quality raw materials, reduced impurities, and inhibitors or other additives are such factors. These factors produce life time distributions shifted towards the long term and are those typically found in products having been produced a relatively long period of time.

5.1.3 Establishing a description of the distribution of frequency (or probability) of failure versus time in service is the objective of this guide. Determination of the shape of this distribution as well as its position along the time scale axis are the principle criteria for estimating service life.

5.2 *Normal (Gaussian) Distribution*—The characteristic of the normal, or Gaussian distribution is a symmetrical bell shaped curve centered on the mean of this distribution. The mean represents the time for 50 % failure. This may be defined as either the time when one can expect 50 % of the entire population to fail or the probability of an individual item to fail. The "scale" of the normal curve is the mean value ($\bar{x}$), and the shape of this curve is established by the standard deviation value (σ).

5.2.1 The normal distribution has found widespread use in describing many naturally occurring distributions. Its first known description by Carl Gauss showed its applicability to measurement error. Its applications are widely known and numerous texts produce exhaustive tables and descriptions of this function.

5.2.2 Widespread use should not be confused with justification for its application to service life data. Use of analysis techniques developed for normal distribution on data distributed in a non-normal manner can lead to grossly erroneous conclusions. As described in Section 5, many service life distributions are skewed towards either early life or late life. The confinement to a symmetrical shape is the principal shortcoming of the normal distribution for service life applications. This may lead to situations where even negative lifetimes are predicted.

5.3 *Lognormal Distribution*—This distribution has shown application when the specimen fails due to a multiplicative process that degrades performance over time. Metal fatigue is one example. Degradation is a function of the amount of flexing, cracks, crack angle, number of flexes, etc. Performance eventually degrades to the defined end of life.[3]

5.3.1 There are several convenient features of the lognormal distribution. First, there is essentially no new mathematics to introduce into the analysis of this distribution beyond those of the normal distribution. A simple logarithmic transformation of data converts lognormal distributed data into a normal distribution. All of the tables, graphs, analysis routines etc. may then be used to describe the transformed function. One note of caution is that the shape parameter σ is symmetrical in its logarithmic form and non-symmetrical in its natural form. (for example, $\bar{x} = 1 \pm .2\sigma$ in logarithmic form translates to 10 +5.8 and −3.7 in natural form)

5.3.2 As there is no symmetrical restriction, the shape of this function may be a better fit than the normal distribution for the service life distributions of the material being investigated.

5.4 *Weibull Distribution*—While the Swedish Professor Waloddi Weibull was not the first to use this expression,[4] his paper, A Statistical Distribution of Wide Applicability published in 1951 did much to draw attention to this exponential function. The simplicity of formula given in (1), hides its extreme flexibility to model service life distributions.

5.4.1 The Weibull distribution owes its flexibility to the "shape" parameter. The shape of this distribution is dependent on the value of b. If b is less than 1, the Weibull distribution models failure times having a decreasing failure rate. The times between failures increase with exposure time. If b = 1, then the Weibull models failure times having constant failure rate. If b > 1 it models failure times having an increasing failure rate, if b = 2, then Weibull exactly duplicates the Rayleigh distribution, as b approaches 2.5 it very closely approximates the lognormal distribution, as b approaches 3. the Weibull expression models the normal distribution and as b grows beyond 4, the Weibull expression models distributions skewed towards long failure times. See Fig. 1 for examples of distributions with different shape parameters.

5.4.2 The Weibull distribution is most appropriate when there are many possible sites where failure might occur and the system fails upon the occurrence of the first site failure. An example commonly used for this type of situation is a chain failing when only one link separates. All of the sites, or links, are equally at risk, yet one is all that is required for total failure.

5.5 *Exponential Distribution*—This distribution is a special case of the Weibull. It is useful to simplify calculations involving periods of service life that are subject to random failures. These would include random defects but not include wear-out or burn-in periods.

6. Parameter Estimation

6.1 Weibull data analysis functions are not uncommon but not yet found on all data analysis packages. Fortunately, the expression is simple enough so that parameter estimation may be made easily. What follows is a step-by-step example for estimating the Weibull distribution parameters from experimental data.

6.1.1 The Weibull distribution, (Eq 2) may be rearranged as shown below: (Eq 3)

[3] Mann, N.R. et al, *Methods for Statistical Analysis of Reliability and Life Data*, Wiley, New York 1974 and Gnedenko, B.V. et al, *Mathematical Methods of Reliability Theory*, Academic Press, New York 1969.

[4] Weibull, W., "A statistical distribution of wide applicability ," *J. Appl. Mech.*, 18, 1951, pp 293–297.

$$1 - F(t) = e^{-\left(\frac{t}{c}\right)^b} \quad (3)$$

and, by taking the natural logarithm of both sides twice, this expression becomes

$$ln\left[ln\frac{1}{1 - F(t)}\right] = b\,ln(t) - b\,lnc \quad (4)$$

Eq 4 is in the form of an equation describing a straight line ($y = mx + y_0$) with

$$ln\left[ln\frac{1}{1 - F(t)}\right] \quad (5)$$

corresponding to Y, ln(t) corresponding to x and the slope of the line m equals the Weibull shape parameter b. Time to failure, t, is the independent variable and is defined as the time at which some measurable performance parameter falls below a pre-defined critical value.

6.1.2 The failure probability, F(t), associated with each failure time can be estimated using the median rank estimate approximation shown below:

$$F(t) = \frac{j - 0.3}{n + 0.4} \quad (6)$$

where:

j = the failure order and
n = the total number of specimens on test.

See Tobias and Trindade, section 2.2 and Johnson, section 3.1.3

7. Service Life Estimation

7.1 Select the distribution model that best fits the observed service life data. Often a simple graph will help not only in choosing a model but in detecting outlier data. Further guidance in selecting a distribution model can be obtained from linear regression coefficients of lifetime versus probability. Higher regression coefficients are an indication of a better model fit.

7.1.1 Neither the Weibull distribution nor any other distribution is a universal best choice for every situation or data set. Each data set must be checked and the best fitting model distribution selected for estimation purposes. See section 1.5.

7.2 Determine the shape and scale parameters of the distribution. A minimum of 10 failures is required to properly determine a distribution. The more the better, but there is a point of diminishing returns. A reasonable range of failed specimens is 10 to 50.

7.3 Calculate the probability of failure by a given time t or alternatively, the time to reach a given failure probability. See Example Calculations, section 8, for a step-by-step procedure for this calculation.

8. Example Calculations

8.1 Simple case - complete data set.

8.1.1 Consider a hypothetical case where 20 incandescent lamps are put on test. The lamps are labeled "A" through "T" at the beginning of the test. Each lamp was found to operate satisfactorily at the beginning of the test period. The lamps were all left on and inspected each day to determine if they were still burning. A data sheet was kept and the number of days of operation for each of the 20 lamps was recorded. The results are reported in Table 1.

8.1.2 The failure times were sorted from earliest (78 days) to latest (818 days) and the median rank, F(t), calculated for each lamp. When the median rank has been calculated for each specimen, all of the information will be available that is needed to solve the Weibull expression:

$$1 - F(t) = e^{-\left(\frac{t}{c}\right)^b} \quad (7)$$

8.1.3 Step by step example:

8.1.4 First, calculate F(t) from Eq 6, where j is the failure order and n is the total number of specimens on test. For the first failure j = 1 and n, the number of lamps used in this test, is 20. Therefore

$$F(t) = \frac{j - 0.3}{n + 0.4} \quad (8)$$

$$= \frac{1 - 0.3}{20 + 0.4}$$

$$= 0.034$$

Continuing this operation for all 20 failure times produces Table 2.

8.1.5 Next, substitute the values for F(t) and t into Eq 4. The value for the first failure is shown below.

$$ln\left[ln\frac{1}{1 - 0.034}\right] = 3.355 \quad (9)$$

$$-3.355 = b[ln(78)] - b[ln(c)].$$

8.1.6 Repeating this procedure for the remaining 19 lamps produces a total of 20 such equations. A simple linear regression may now be used to determine the critical parameters b and c.

The resulting regression equation produces the following:

$$Y = 1.62ln(t) - 9.46 \quad (10)$$

8.1.7 The value for the slope, 1.62, is equal to the Weibull shape parameter. The scale parameter, c, can be determined by the expression:

$$c = \exp\left(\frac{-y_0}{b}\right) \quad (11)$$

$$= 344 \text{ days}$$

TABLE 1 Time to Failure (days of operation) for Incandescent Lamps

Lamp ID	Days of Operation	Lamp ID	Days of Operation
A	293	K	189
B	282	L	818
C	535	M	114
D	421	N	550
E	710	O	80
F	166	P	191
G	208	Q	402
H	155	R	210
I	456	S	101
J	203	T	78