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**Lasers and laser-related equipment —  
Test methods for laser beam  
widths, divergence angles and beam  
propagation ratios —**

Part 2:

**General astigmatic beams**

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*Lasers et équipements associés aux lasers — Méthodes d'essai des  
largeurs du faisceau, angles de divergence et facteurs de limite de  
diffraction —*

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Partie 2: Faisceaux astigmatiques généraux



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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 9, *Laser and electro-optical systems*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 123, *Lasers and photonics*, in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

This second edition cancels and replaces the first edition (ISO 11146-2:2005), which has been technically revised. The main changes compared to the previous edition are as follows:

- The terms and definitions were harmonized with the new ISO 11145.
- The "principal axes" were defined more thoroughly and named as  $x'$  and  $y'$ . Quantities related to the principal axes coordinate system refer to this definition and use  $x'$  and  $y'$  in their indices.
- The requirements for the integration range for the determination of the second order moments have been relaxed.

A list of all parts in the ISO 11146 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at [www.iso.org/members.html](http://www.iso.org/members.html).

## Introduction

The propagation properties of laser beams can be characterized by ten independent parameters when applying the method of second order moments (see ISO/TR 11146-3). Most laser beams need few parameters for a complete description due to their higher symmetry. Lasers emit beams which are stigmatic or simple astigmatic due to their resonator design.

ISO 11146-1 describes the measurement methods for stigmatic and simple astigmatic beams while this document deals with the measurement procedures for general astigmatic beams. This document is applicable to beams of unknown type. Beam characterization, based on the method of second order moments as described in ISO 11146-1 and this document, is only valid within the paraxial approximation.

The theoretical description of beam characterization and propagation as well as the classification of laser beams is given in ISO/TR 11146-3, which is a Technical Report. The procedures for background subtraction and offset correction are also given in ISO/TR 11146-3.

In ISO 11146, the second order moments of the power (energy) density distribution function are used for the determination of beam widths. If problems are experienced in the direct measurements of these quantities, other indirect methods of measurement of second order moments may be used as long as comparable results are achievable.

In ISO/TR 11146-3, three alternative methods for beam width measurement and their correlation with the method used in this document are described. These methods are:

- variable aperture method;
- moving knife-edge method;
- moving slit method.

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# Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios —

## Part 2: General astigmatic beams

### 1 Scope

This document specifies methods for measuring beam widths (diameter), divergence angles and beam propagation ratios of laser beams. This document is applicable to general astigmatic beams or unknown types of beams. For stigmatic and simple astigmatic beams, ISO 11146-1 is applicable.

Within this document, the description of laser beams is accomplished by means of the second order moments of the Wigner distribution rather than physical quantities such as beam widths and divergence angles. However, these physical quantities are closely related to the second order moments of the Wigner distribution. In ISO/TR 11146-3, formulae are given to calculate all relevant physical quantities from the measured second order moments.

### 2 Normative references (standards.iteh.ai)

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 11145, *Optics and photonics — Lasers and laser-related equipment — Vocabulary and symbols*

ISO 11146-1, *Lasers and laser-related equipment — Test methods for laser beam widths, divergence angles and beam propagation ratios — Part 1: Stigmatic and simple astigmatic beams*

EN 61040:1992, *Power and energy measuring detectors, instruments and equipment for laser radiation*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 11145, ISO 11146-1, EN 61040 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

**Note** The x-, y- and z-axes in the following definitions refer to the laboratory system (as described in [Clause 4](#)). Here and throughout this document the term “power density distribution  $E(x,y,z)$ ” refers to continuous wave sources. It might be replaced by “energy density distribution  $H(x,y,z)$ ” in case of pulsed sources.

**3.1  
generalized beam diameter**

$d_g$   
measure of the extent of the power density distribution of a beam in a cross-section at an axial location  $z$ , derived from the second order moments by

$$d_g = 2\sqrt{2} \sqrt{\langle x^2 \rangle + \langle y^2 \rangle} \tag{1}$$

Note 1 to entry: This definition is similar to the beam diameter defined in ISO 11145 or ISO 11146-1. But in this context the definition is not restricted to circular power density distributions.

**3.2  
generalized beam waist location**

$z_{0,g}$   
position where the *generalized beam diameter* (3.1) reaches its minimum value along the axis of propagation

**3.3  
generalized Rayleigh length**

$z_{R,g}$   
distance along the beam axis from the generalized beam waist where the generalized beam diameter is a factor of  $\sqrt{2}$  larger than the generalized beam waist diameter

**3.4  
Wigner distribution**

phase space distribution representing a laser beam in a transverse plane at location  $z$

Note 1 to entry: The Wigner distribution is a function of two spatial and two angular coordinates, giving the amount of beam power propagating through the point  $(x, y)$  in the direction  $(\theta_x, \theta_y)$ .

**3.5  
spatial first order moments of the Wigner distribution**

$\langle x \rangle, \langle y \rangle$

subset of the first order moments, which can be directly obtained from measured power density distribution by

$$\langle x \rangle(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) x \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) \, dx \, dy} \tag{2}$$

and

$$\langle y \rangle(z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) y \, dx \, dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) \, dx \, dy} \tag{3}$$

where  $E(x,y,z)$  is the power density distribution at the specific plane  $z = \text{constant}$



### 3.6

#### second order moments of the Wigner distribution

$$\langle x^2 \rangle, \langle y^2 \rangle, \langle xy \rangle, \langle \theta_x^2 \rangle, \langle \theta_y^2 \rangle, \langle \theta_x \theta_y \rangle, \langle x \theta_x \rangle, \langle x \theta_y \rangle, \langle y \theta_x \rangle, \langle y \theta_y \rangle$$

ten second order moments of the *Wigner distribution* (3.4) of the beam at location  $z$

Note 1 to entry: The ten second order moments contain information on the following physical beam properties: beam size and orientation, divergence angles and their orientation, radii of curvature of the phase paraboloid and their orientation and the twist parameter. Details on these relations are given in ISO/TR 11146-3.

Note 2 to entry: In ISO 11146-1, the three spatial second order moments are defined as  $\sigma_x^2$ ,  $\sigma_y^2$  and  $\sigma_{xy}^2$ . In this document and ISO/TR 11146-3, the angular brackets are used to emphasize the coordinates of the moments. This means that  $\sigma_x^2 = \langle x^2 \rangle$ ,  $\sigma_y^2 = \langle y^2 \rangle$  and  $\sigma_{xy}^2 = \langle xy \rangle$ .

Note 3 to entry: Three angular moments  $\langle \theta_x^2 \rangle$ ,  $\langle \theta_y^2 \rangle$  and  $\langle \theta_x \theta_y \rangle$  are independent of  $z$ . The other seven second order moments are, in general, functions of  $z$ .

### 3.7

#### spatial second order moments of the Wigner distribution

$$\langle x^2 \rangle, \langle y^2 \rangle, \langle xy \rangle$$

subset of the second order moments, which can be directly obtained from measured power density distribution by

$$\langle x^2 \rangle (z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) (x - \langle x \rangle)^2 dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy} \quad (4)$$

$$\langle y^2 \rangle (z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) (y - \langle y \rangle)^2 dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy} \quad (5)$$

and

$$\langle xy \rangle (z) = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) (x - \langle x \rangle)(y - \langle y \rangle) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x, y, z) dx dy} \quad (6)$$

### 3.8

#### beam matrix

$P$

symmetric and positive definite 4×4 matrix containing all ten *second order moments of the Wigner distribution* (3.6) and its elements and given by

$$P = \begin{bmatrix} \langle x^2 \rangle & \langle xy \rangle & \langle x \theta_x \rangle & \langle x \theta_y \rangle \\ \langle xy \rangle & \langle y^2 \rangle & \langle y \theta_x \rangle & \langle y \theta_y \rangle \\ \langle x \theta_x \rangle & \langle y \theta_x \rangle & \langle \theta_x^2 \rangle & \langle \theta_x \theta_y \rangle \\ \langle x \theta_y \rangle & \langle y \theta_y \rangle & \langle \theta_x \theta_y \rangle & \langle \theta_y^2 \rangle \end{bmatrix} \quad (7)$$

**3.9 effective beam propagation ratio**

$M_{\text{eff}}^2$   
invariant quantity related to the focusability of a general astigmatic beam, defined as:

$$M_{\text{eff}}^2 = \frac{4\pi}{\lambda} [\det(\mathbf{P})]^{\frac{1}{4}} \tag{8}$$

where  $\det(\mathbf{P})$  is the determinant of matrix  $\mathbf{P}$

Note 1 to entry: The effective beam propagation ratio  $M_{\text{eff}}^2$  is an invariant related to the volume that the beam occupies in the four-dimensional phase space (two lateral spatial and two lateral angular dimensions) and thus a measure for the focusability of the beam.

Note 2 to entry: For simple astigmatic beams, the effective beam propagation ratio is the geometric mean of the beam propagation ratios of the principal axes of the beam:  $M_{\text{eff}}^2 = \sqrt{M_x^2 \times M_y^2}$ . For stigmatic beams  $M_{\text{eff}}^2 = M^2$ .

**3.10 intrinsic astigmatism**

$a$   
degree of how close to a stigmatic beam the general astigmatic beam can be transformed by using lenses and free space propagation

$$a = \frac{8\pi^2}{\lambda^2} \left[ (\langle x^2 \rangle \langle \theta_x^2 \rangle - \langle x\theta_x \rangle^2) + (\langle y^2 \rangle \langle \theta_y^2 \rangle - \langle y\theta_y \rangle^2) + 2(\langle xy \rangle \langle \theta_x \theta_y \rangle - \langle x\theta_y \rangle \langle y\theta_x \rangle) \right] - (M_{\text{eff}}^2)^2 \geq 0 \tag{9}$$

Note 1 to entry: Beams are classified according to their intrinsic astigmatism,  $a$ , which is an invariant quantity. A beam with  $a = 0$  is called intrinsic stigmatic, a beam with  $a > 0$  is called intrinsic astigmatic. For simple astigmatic beams  $a = (1/2)(M_x^2 - M_y^2)^2$ . More details are given in ISO/TR 11146-3.

**3.11 twist parameter**

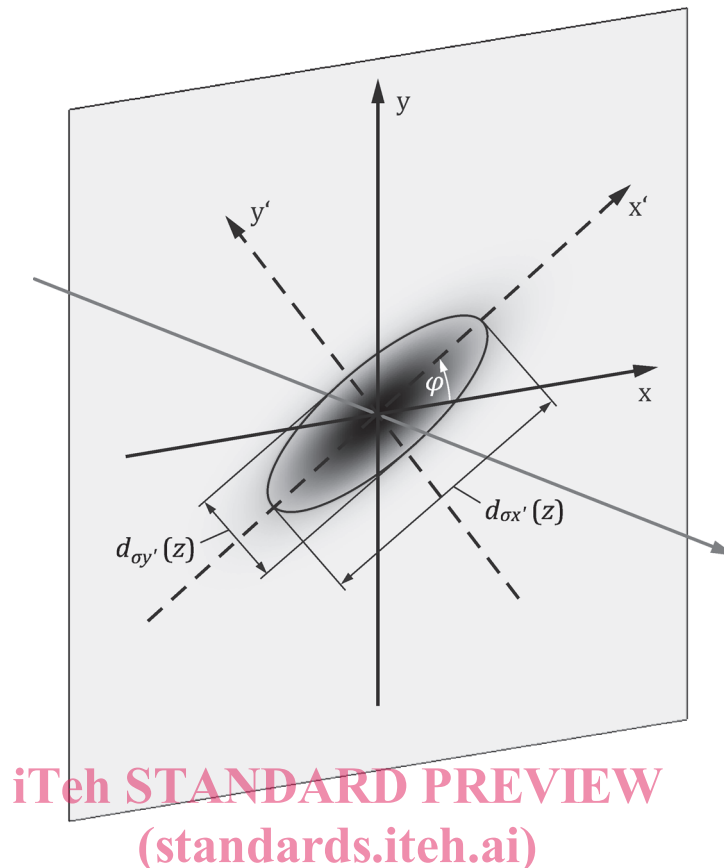
$t$   
parameter related to the rotational properties of the phase front of a beam, and also to the orbital angular momentum carried by the beam

$$t = \langle x\theta_y \rangle - \langle y\theta_x \rangle \tag{10}$$

Note 1 to entry: The twist parameter is invariant under propagation through free space and spherical lenses. It might be altered under propagation through cylindrical lenses.

**3.12 principal axes**

$x', y'$   
<power density distribution> axes of the maximum and minimum beam extent based on the second order moments of the power density distribution in a cross-section of the beam



**Figure 1 — Beam profile with the laboratory and principle axes coordinate systems**

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Note 1 to entry: The axes of maximum and minimum extent are always perpendicular to each other.

Note 2 to entry: Unless otherwise stated, in this document  $x'$  is the *principal axis* which is closer to the  $x$ -axis of the laboratory coordinate system, and  $y'$  is the principal axis which is closer to the  $y$ -axis of the laboratory coordinate system.

Note 3 to entry: If the *principal axes* make the angle  $\pi/4$  with the  $x$ - and  $y$ -axes of the laboratory coordinate system, then the  $x$ -axis is by convention the direction of maximum extent.

Note 4 to entry: See [Figure 1](#).

[SOURCE: ISO 11146-1:2021, 3.3]

### 3.13 azimuthal orientation

$\varphi$

<power density distribution> azimuthal angle between the  $x$ -axis of the laboratory system and the principal axis  $x'$

[SOURCE: ISO 11146-1:2021, 3.4]

### 3.14 beam widths

$d_{\sigma_{x'}}(z), d_{\sigma_{y'}}(z)$

extent of a power density distribution in a cross-section of the beam at an axial location  $z$  along the principal axes  $x'$  and  $y'$ , respectively, based on the second order moments of the power density distribution

Note 1 to entry: This definition differs from that given in ISO 11145:2018, 3.5.2, where the beam widths are defined only in the laboratory system, whereas for the purposes of this document the beam widths are defined in the *principal axes* (3.12) system of the beam.