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Standard Guide for Determining Neutron Energy Spectra from Neutron Sensors for Radiation-Hardness Testing of Electronics¹

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This standard has been approved for use by agencies of the Department of Defense.

1. Scope

- 1.1 This guide covers procedures for determining the energy-differential fluence spectra of neutrons used in radiation-hardness testing of electronic semiconductor devices. The types of neutron sources specifically covered by this guide are fission or degraded energy fission sources used in either a steady-state or pulse mode.
- 1.2 This guide provides guidance and criteria that can be applied during the process of choosing the spectrum adjustment methodology that is best suited to the available data and relevant for the environment being investigated.
- 1.3 This guide is to be used in conjunction with Guide E720 to characterize neutron spectra and is used in conjunction with Practice E722 to characterize damage-related parameters normally associated with radiation-hardness testing of electronic-semiconductor devices.

Note 1—Although Guide E720 only discusses activation foil sensors, any energy-dependent neutron-responding sensor for which a response function is known may be used (1).²

Note 2—For terminology used in this guide, see Terminology E170.

1.4The values stated in SI units are to be regarded as the standard.

1.4 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.5 This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.

2. Referenced Documents

2.1 ASTM Standards:³

E170 Terminology Relating to Radiation Measurements and Dosimetry

E261 Practice for Determining Neutron Fluence, Fluence Rate, and Spectra by Radioactivation Techniques

E262 Test Method for Determining Thermal Neutron Reaction Rates and Thermal Neutron Fluence Rates by Radioactivation Techniques

E263 Test Method for Measuring Fast-Neutron Reaction Rates by Radioactivation of Iron

E264 Test Method for Measuring Fast-Neutron Reaction Rates by Radioactivation of Nickel

E265 Test Method for Measuring Reaction Rates and Fast-Neutron Fluences by Radioactivation of Sulfur-32

E266 Test Method for Measuring Fast-Neutron Reaction Rates by Radioactivation of Aluminum

E393 Test Method for Measuring Reaction Rates by Analysis of Barium-140 From Fission Dosimeters

E523 Test Method for Measuring Fast-Neutron Reaction Rates by Radioactivation of Copper

E526 Test Method for Measuring Fast-Neutron Reaction Rates by Radioactivation of Titanium

E704 Test Method for Measuring Reaction Rates by Radioactivation of Uranium-238

E705 Test Method for Measuring Reaction Rates by Radioactivation of Neptunium-237

E720 Guide for Selection and Use of Neutron Sensors for Determining Neutron Spectra Employed in Radiation-Hardness Testing of Electronics

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² The boldface numbers in parentheses refer to the list of references at the end of this guide.

³ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.



- E722 Practice for Characterizing Neutron Fluence Spectra in Terms of an Equivalent Monoenergetic Neutron Fluence for Radiation-Hardness Testing of Electronics
- E844 Guide for Sensor Set Design and Irradiation for Reactor Surveillance, E 706 (IIC)
- E944 Guide for Application of Neutron Spectrum Adjustment Methods in Reactor Surveillance, E 706 (IIA)
- E1018 Guide for Application of ASTM Evaluated Cross Section Data File, Matrix E706 (IIB)
- E1297 Test Method for Measuring Fast-Neutron Reaction Rates by Radioactivation of Niobium
- E1855 Test Method for Use of 2N2222A Silicon Bipolar Transistors as Neutron Spectrum Sensors and Displacement Damage Monitors

3. Terminology

- 3.1 Definitions: 3.1.1The The following list defines some of the special terms used in this guide:
- 3.1.23.1.1 *effect*—the characteristic which changes in the sensor when it is subjected to the neutron irradiation. The effect may be the reactions in an activation foil.
 - 3.1.3
- 3.1.2 response—the magnitude of the effect. It can be the measured value or that calculated by integrating the response function over the neutron fluence spectrum. The response is an integral parameter. Mathematically, the response, $R = \sum_i R_i$, where R_i is the response in each differential energy region at E_i of width ΔE_i .
 - 3.1.4
- <u>3.1.3</u> response function—the set of values of R_i in each differential energy region divided by the neutron fluence in that differential energy region, that is, the set $f_i = R/(\Phi(E_i)\Delta E_i)$.
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- <u>3.1.4</u> *sensor*—an object or material (sensitive to neutrons) the response of which is used to help define the neutron environment. A sensor may be an activation foil.
 - 3.1.6
- <u>3.1.5</u> spectrum adjustment—the process of changing the shape and magnitude of the neutron energy spectrum so that quantities integrated over the spectrum agree more closely with their measured values. Other physical constraints on the spectrum may be applied.
 - 317
- <u>3.1.6</u> *trial function*—a neutron spectrum which, when integrated over sensor response functions, yields calculated responses that can be compared to the corresponding measured responses.
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- <u>3.1.7</u> *prior spectrum*—an estimate of the neutron spectrum obtained by transport calculation or otherwise and used as input to a least-squares adjustment.
 - 3.2 Abbreviations:
 - 3.2 Abbreviations:
 - 3.2.1 *DUT*—device under test.catalog/standards/sist/882275df-e792-403c-b28e-e2202a350bca/astm-e721-11
 - 3.2.2 ENDF—evaluated nuclear data file.
 - 3.2.3 NNDC—National Nuclear Data Center (at Brookhaven National Laboratory).
 - 3.2.4 RSICC—Radiation Safety Information Computation Center (at Oak Ridge National Laboratory).
 - 3.2.5 TREE—transient radiation effects on electronics.

4. Significance and Use

- 4.1 It is important to know the energy spectrum of the particular neutron source employed in radiation-hardness testing of electronic devices in order to relate radiation effects with device performance degradation.
- 4.2 This guide describes the factors which must be considered when the spectrum adjustment methodology is chosen and implemented. Although the selection of sensors (foils) and the determination of responses (activities) is discussed in Guide E720, the experiment should not be divorced from the analysis. In fact, it is advantageous for the analyst conducting the spectrum determination to be closely involved with the design of the experiment to ensure that the data obtained will provide the most accurate spectrum possible. These data include the following: (1) measured responses such as the activities of foils exposed in the environment and their uncertainties, (2) response functions such as reaction cross sections along with appropriate correlations and uncertainties, (3) the geometry and materials in the test environment, and (4) a trial function or prior spectrum and its uncertainties obtained from a transport calculation or from previous experience.

5. Spectrum Determination With Neutron Sensors

- 5.1 Experiment Design:
- 5.1.1 The primary objective of the spectrum characterization experiment should be the acquisition of a set of response values (activities) from effects (reactions) with well-characterized response functions (cross sections) with responses which adequately define (as a set) the fluence values at energies to which the device to be tested is sensitive. For silicon devices in fission-driven environments the significant neutron energy range is usually from 10 keV to 15 MeV. Lists of suitable reactions along with approximate sensitivity ranges are included in Guide E720. Sensor set design is also discussed in Guide E844. The foil set may

include the use of responses with sensitivities outside the energy ranges needed for the DUT to aid in interpolation to other regions of the spectrum. For example, knowledge of the spectrum below 10 keV helps in the determination of the spectrum above that energy.

- 5.1.2 An example of the difficulty encountered in ensuring response coverage (over the energy range of interest) is the following: If fission foils cannot be used in an experiment because of licensing problems, cost, or radiological handling difficulties (especially with 235 U, 237 Np or 239 Pu), a large gap may be left in the foil set response between 100 keV and 2 MeV—a region important for silicon and gallium arsenide damage (see Figs. A1.1 and A2.3 of Practice E722). In this case two options are available. First, seek other sensors to fill the gap (such as silicon devices sensitive to displacement effects (see Test Method E1855)), 93 Nb(n,n') 93m Nb (see Test Method E1297) or 103 Rh(n,n') 103m Rh. Second, devote the necessary resources to determine a trial function that is close to the real spectrum. In the latter case it may be necessary to carry out transport calculations to generate a prior spectrum which incorporates the use of uncertainty and covariance information.
 - 5.1.3 Other considerations that affect the process of planning an experiment are the following:
- 5.1.3.1 Are the fluence levels low and of long duration so that only long half-life reactions are useful? This circumstance can severely reduce the response coverage of the foil set.
 - 5.1.3.2 Are high gamma-ray backgrounds present which can affect the sensors (or affect the devices to be tested)?
- 5.1.3.3 Can the sensors be placed so as to ensure equal exposure? This may require mounting the sensors on a rotating fixture in steady-state irradiations or performing multiple irradiations with monitor foils to normalize the fluence between runs.
 - 5.1.3.4 Does the DUT perturb the neutron spectrum?
- 5.1.3.5 Can the fluence and spectrum seen in the DUT test later be directly scaled to that determined in the spectrum characterization experiment (by monitors placed with the tested device)?
- 5.1.3.6 Can the spectrum shape and intensity be characterized by integral parameters that permit simple intercomparison of device responses in different environments? Silicon is a semiconductor material whose displacement damage function is well established. This makes spectrum parameterization for damage predictions feasible for silicon.
- 5.1.3.7 What region of the spectrum contributes to the response of the DUT? In other words, is the spectrum well determined in all energy regions that affect device performance?
- 5.1.3.8 How is the counting system set up for the determination of the activities? For example, are there enough counters available to handle up to 25 reactions from a single exposure? (This may require as many as six counters.) Or can the available system only handle a few reactions before the activities have decayed below detectable limits?
- 5.1.4 Once the experimental opportunities and constraints have been addressed and the experiment designed to gather the most useful data, a spectrum adjustment methodology must be chosen.
 - 5.2 Spectrum Adjustment Methodology:
- 5.2.1 After the basic measured responses, response functions, and trial spectrum information have been assembled, apply a suitable spectrum adjustment procedure to reach a "solution" that is as compatible as possible with that information. It must also meet other constraints such as, the fluence spectrum must be positive and defined for all energies. The solution is the energy-dependent spectrum function, $\Phi(E)$, which approximately satisfies the series of Fredholm equations of the first kind represented by Eq. 1 as follows:

$$R_{j} = \int_{0}^{\infty} \sigma_{j}(E) \Phi(E) dE \quad 1 \le j \le n$$
 (1)

where:

 R_i = measured response of sensor j,

 $\sigma_{j}(E)$ = neutron response function at energy E for sensor j,

 $\Phi(E)$ = incident neutron fluence versus energy, and

n = number of sensors which yield n equations.

One important characteristic of this set of equations is that with a finite number of sensors, \underline{jn} , which yield n equations, there is no unique solution. With certain restrictions, however, the range of physically reasonable solutions can be limited to an acceptable degree.

Note 3—Guides E720 and E844 provide general guidance on obtaining a suitable set of responses (activities) when foil monitors are used. Practice E261 and Test Method E262 provide more information on the data analysis that generally is part of an experiment with activation monitors. Specific instructions for some individual monitors can be found in Test Methods E263 (iron), E264 (nickel), E265 (sulfur-32), E266 (aluminum), E393 (barium-140 from fission foils), E523 (copper), E526 (titanium), E704 (uranium-238), E705 (neptunium-237), E1297 (niobium).

5.2.2 Neutron spectra generated from sensor response data may be obtained with either of two types of spectrum adjustment codes. One type is the iterative method, an example of which is SAND II (2). The second is least squares minimization used by codes such as LSL-M2 (3). If used properly and with sufficient, high-quality data, the two methods will usually yield nearly the same values (±10 to 15 %) for the primary integral parameters discussed in E722.

Note 4—Another class of codes often referred to as Maximum Entropy may also prove useful for this type of analysis. These have historically not been used to estimate spectra for radiation-damage purposes.



- 5.2.3 Appendix X1 and Appendix X2 discuss in some detail the implementation and the advantages and disadvantages of the two approaches as represented by SAND II and LSL-M2.
 - 5.3 Iterative Code Characteristics:
- 5.3.1 The "iterative" codes use a trial function supplied by the analyst and integrate it over the response functions of the sensors exposed in the unknown environment to predict a set of calculated responses for comparison with the measured values. The calculated responses are obtained from Eq 1. The code obtains the response functions from a library. See Guide E1018 for the recommendations in the selection of dosimetry-quality cross sections. Available dosimetry-quality cross section libraries include: the International Reactor Dosimetry File (IRDF-2002) cross section library (4), release 6 of the ENDF/B-VI (5, 6) cross section library and the SNLRML package (7) which is available through RSICC. for the recommendations in the selection of dosimetry-quality cross sections.
- 5.3.2 The code compares the measured and calculated responses for each effect and invokes an algorithm designed to alter the trial function so as to reduce the deviations between the measured and calculated responses. The process is repeated with code-altered spectra until the standard deviation drops below a specified value—at which time the code declares that a solution has been obtained and prepares a table of the last spectrum. This should not be the end of the process unless the initial trial was very close to the final result. In each iteration, the SAND II-type code will alter the trial most rapidly where the foil set has the highest response. If the trial is incompatible with the measurements, the spectrum can become distorted in a very unphysical manner
- 5.3.3 For example, if a trial function predicts an incorrect gold activity, it may alter the spectrum by orders of magnitude at the gold high-response resonance at 5 eV while leaving the trial spectrum alone in the immediate vicinity. The analyst must recognize that the trial must be changed in a manner suggested by the previous result. For example, if a peak develops at the gold resonance, this suggests that the trial spectrum values are too low in that whole energy region. A new trial drawn smoothly near the spectrum values where the sensor set has high response may improve the solution. This direct modification becomes an outer iteration on the spectrum adjustment process, as described in Refs (84,95). The outer iteration methodology coupled with good activity data is usually so successful that the form of the initial trial does not overly influence the integral results.
- 5.3.4 For any of the iterative type codes to succeed at producing a spectrum that is both representative of the measured data and likely to be close to the true spectrum of neutrons that caused the activation data, experience has shown that the following are important: (1) the use of sensors with well-established response functions ($\leq 8\%$ for spectrum-averaged cross sections), (2) a sensor set that has good response over all the important regions of the spectrum, and (3) sufficiently accurate measured responses (on the order of $\pm 5\%$). No direct use is made of uncertainty data (variance and covariance information) that exists for each cross section, of uncertainty in the trial spectrum, or in the uncertainties in the measured responses. These uncertainties can vary greatly among sensors or environments. It follows that data with large uncertainties should not be used in the final stages of this methodology because it can cripple the final results.
- Note 5—Response data that exhibits a strong disagreement with other data in the data ensemble can be very useful in the early stages of an analysis. For example, if the activity of a particular reaction is incompatible with the other foils in the spectrum adjustment process, it can indicate one of two important possibilities. First, if it is a reaction whose energy-dependent cross section is well known and has repeatedly demonstrated compatibility in the past, an experimental or transcription error is suggested. Second, if the activity measurement was accurately carried out, and this reaction has repeatedly demonstrated incompatibility in the same direction in other spectra determinations in different environments, an incorrect cross section or energy-specific counting calibration error is indicated (95). A number of specific cross-section problems have been uncovered by analysis of incompatibility data. But in the construction of the neutron spectrum, these "bad" reactions should not be used with a method that does not incorporate uncertainty data.
 - 5.4 Suitability of the Iterative Adjustment Codes:
- 5.4.1 SAND II in its usual form does not have a capability to weight the responses according to uncertainties, it does not provide error or uncertainty analysis, it does not use variance or covariance information, and it provides no direct quantification of the output uncertainties for any calculated quantities. However, it is possible to assign errors in the spectrum in appropriate energy regions using perturbation analysis. (Also computerized perturbation and random draw from response error may be utilized.) The analyst perturbs the trial spectrum upwards and downwards in each energy region and observes to what degree the code brings the two trials into agreement. This is, however, a laborious process and has to be interpreted carefully. In the resonance region where foil responses are spiked, the code will only yield agreement at resonances where there exists high response. The analyst must not only interpolate the spectrum values between high response regions but also the spectrum uncertainties. This step can be rationalized with physical arguments based on the energy-dependence of cross sections but it is difficult to justify mathematically. This situation further supports the arguments for maximizing response coverage. In addition, it is usually the uncertainties of integral parameters that are of primary importance, not the uncertainty of $\Phi(E)$ at individual energy values.
- 5.4.2 Covers are used over many of the foils to restrict the response ranges, as is explained in Guide E720. The SAND II code handles the attenuations in the covers in a simple manner by assuming exponential attenuation through the cover material. There is considerable evidence that for some spectra the calculated exponential attenuation is not accurate because of scattering.
 - 5.5 Least-Squares Code Characteristics:
- 5.5.1 The least-squares codes, represented by LSL-M2 (3) use variance and covariance data for the measured responses, response functions, and prior spectrum. The LSL-M2 code finds a unique solution spectrum which is the most likely solution in the least-squares sense using all the available information. The code, therefore, allows not only the prior spectrum but also the responses and the response functions to be adjusted in a manner constrained by their individual uncertainties and correlations in



- order to find that the most likely solution. In principle this approach provides the best estimate of a spectrum and its uncertainties, but some practical difficulties remain. The least-squares method is described more fully in Guide E944 and in Appendix X2.
- 5.5.2 The input variance and covariance matrix quantities are not always well determined and some may have to be estimated. The analyst must understand that his estimates of these quantities can affect the results.
- 5.5.3 No least-squares code in the form distributed by code libraries conveniently handles the effects of covers over the foils even though the use of covers is strongly recommended. See Section 7.2 of Guide E720 and X2.5.1 of this standard for more information.
- 5.5.4 The code automatically weights the data according to uncertainties. Therefore, data with large uncertainties can be used in the analysis, and will have the appropriately small influence on the results.
- 5.5.5 The trial spectrum shape must correspond fairly well to the final spectrum (within 1 or 2 standard deviations) if the results are to be reliable (106). Experience with this method has shown that the trial spectrum can drive the spectrum determination when its uncertainties are small. See Ref (3).
- 5.5.6 If a transport code prediction of the spectrum is used as the starting point for the analysis, then this methodology can be useful for adjusting spectra at a different location than that in which the foils were exposed. If the transport calculation includes a location where an experiment can be conducted and a similar one where such an experiment would be difficult or impossible (such as inside a test fixture or other structure), then this type of code can be used to adjust both spectra in a predictable manner. In accepting the results for the unmonitored location, it is important that the transport calculation be adjusted minimally.
- 5.5.7 The analyst must be careful the input variances and covariances, including those associated with the prior spectrum, are realistic. It is not sufficient to take statistical scoring errors from a Monte Carlo transport calculation and use these as a measure of the uncertainty in the trial spectrum. All uncertainties, and in particular, uncertainties in the reactor modeling, material densities, and response functions should be represented in the input uncertainty. The value of the chi-squared (χ^2) parameter may be used as a good indication of the consistency of the input data (including the uncertainty data).
- 5.6 Suitability of the Least-Squares Adjustment Codes—The least-squares codes are particularly well suited to situations in which the environment is fairly well characterized physically so that a reasonable trial spectrum can be generated and in which the activity data is limited to a few foils (<10). They work best when detailed transport calculated spectra are available for use as the prior spectra for the analysis. However, it is often difficult to obtain a mathematically defensible covariance matrix for these spectra. In principle, a sensitivity analysis based on the radiation transport code methodology could be used to provide the prior spectrum uncertainty and energy-dependent correlation, but this is not an easy analysis and is seldom attempted.

6. Discussion and Comparison of Methodology Characteristics

- 6.1 The least-squares codes are superior because it should be possible to directly incorporate all that is known about the test environment and about the response functions to arrive at the most likely solution in a least-squares sense. The codes provide mathematically defensible output with uncertainties when covariance data is available for all the input quantities. The iterative codes do not propagate uncertainties nor make use of any variance or covariance information which may exist.
- 6.2 Considerable experience with both approaches has demonstrated that they yield approximately the same integral parameter values when applied to the methodology in E722, provided that adequate and accurate primary experimental information is available. This means the analyst must have access to a set of carefully measured responses, usually activation data. The associated set of responses functions, usually activation cross sections, must cover a broad range of energies. And, the response functions for the measured data must be well established over these energy ranges.
- 6.3 Transient radiation effects testing of electronics (TREE testing) is carried out in a wide variety of different environments that are often customized with complicated filters and shields. For these cases, detailed transport calculations can be time-consuming and expensive. The user may not be aware of the total assemblage of material structure that affects the radiation environment.
- 6.4 The iterative type code performs at its best with accurate response data and well-known response functions because the range of acceptable solutions is then severely restricted, and the acceptance criterion of measured-to-calculated activity values can be set to a low value. Also, incompatible responses, perhaps caused by experimental errors, stand out clearly in the results. The least-squares type code seems much more forgiving because wide variances are assigned to less well-known cross sections and activities, so marginal data can be more easily tolerated. For both methods, a very good trial function or prior spectrum is required when limited or imprecise measured responses are available. In these cases, the solution cannot be allowed to deviate very much from the trial because less use should be made of the measured data.
- 6.5 SAND II should not be used to generate trial functions for LSL-M2, because the SAND II solution spectrum is correlated to the activities, but the LSL method assumes there is no such correlation.
- 6.6 Neither methodology can be used indiscriminately and without careful monitoring by a knowledgeable analyst. The analyst must not only apply physical reasoning but must examine the data to determine if it is of adequate quality. At the very least the analyst must evaluate what is seen in a plot of the solution spectrum. Available versions of the SAND II code provides less subsidiary information than least-squares codes can supply, particularly with regards to uncertainties. More detailed discussions of the SAND II and LSL-M2 methodologies are provided in the appendixes.



7. Precision and Bias

7.1 Precision and bias statements are included in each of the appendixes.

8. Keywords

8.1 neutron sensors; neutron spectra; radiation-hardness testing; spectrum adjustment

APPENDIXES

(Nonmandatory Information)

X1. APPLICATION OF THE SAND II CODE

X1.1 Summary of the Iterative Method, SAND II

X1.1.1 SAND II is discussed here as an example of an iterative adjustment code. Its use in radiation-hardness testing of electronics is discussed in detail in Refs (417, 128). This code employs a mild perturbation method that reduces the formation of spurious structure in the output energy spectrum. The measured responses of the sensor set, along with the response functions and a trial spectrum, are inputs to the code. The output of the code gives the fractional differences between the measured responses and calculated responses that are consistent with the trial spectrum. The code adjusts the trial spectrum to reduce these fractional differences and to obtain better agreement between the measured responses and those calculated from the solution spectrum. Iteration of this process continues until satisfactory agreement is obtained between measured responses and those calculated from the solution energy spectrum. A course of action to take in cases when the solution is unsatisfactory is suggested in X1.2.2 and X1.2.5.

X1.2 Operational Characteristics of the Code

X1.2.1 The measured responses determined for a set of sensors are related to the incident neutron energy-fluence spectrum, $\Phi(E)$, by Eq 1.

X1.2.2 The unknown incident spectrum $\Phi(E)$ is approximated by a trial spectrum. The code calculates the various resultant trial responses, r_{jt} , that are consistent with $\Phi_t(E)$. If the response functions are cross sections, they are obtained from an up-to-date evaluated cross-section library, such as ENDF/B-VI adapted to the SAND II cross-section format for 640 energy groups. A recommended library is provided in Ref (4(9)). It is appropriate here to remind the reader once again of the importance of choosing a set of reactions with well-known and experimentally substantiated cross-section values for use in the spectrum adjustment procedure, because the solution spectrum cannot be well established unless the reaction rates are compatible with a physically reasonable spectrum. See Guide E720. Furthermore, it is very important that the relative responses be accurately established by making certain all sensors are subjected to the same fluence and read with high-statistical and calibration accuracy. The code, when used properly, is quite sensitive to incompatible responses, but when incompatible data are included in the set to be adjusted, the spectrum solution may become severely distorted. While it represents a mathematical solution, it may not be physically meaningful.

X1.2.3 The fractional differences between the measured activities and the trial activities are calculated by the code. They are given as follows:

$$\Delta_{j0} = \frac{R_j - r_{jt}}{r_{jt}} \tag{X1.1}$$

The standard deviation, S_0 of the set of Δ_{j0} values, also is determined. Here the subscript zero indicates the first run of the code and r_{ii} is the calculated value.

X1.2.4 The code operator must choose an input value for the standard deviation S (for example, 5%). If S_0 is less than that value, then $\Phi_i(E)$ is the solution. If S_0 is larger than the chosen input value, then the code adjusts the trial spectrum in the energy regions in which the corresponding values of Δ°_{j0} s are sensitive. On the next iteration, the adjusted trial spectrum, $\Phi_1(E)$, reduces the Δ_{j1} values and consequently, reduces S_1 . This iterative process is repeated to generate the sequence of sets of data:

$$\Phi_1(E), \{\Delta_{11}, \dots \Delta_{n1}\}, S_1$$

.

 $\Phi k (E), \{\Delta 1k, \dots \Delta nk\}, Sk$

This continues until S_k , achieves the preset goal of 5 % (or whatever the operator chose for the standard deviation).

X1.2.5 The procedure of adjusting the trial often leads to a distorted spectrum if the trial is very different from one that is really compatible with the response set. The most direct way to discern any distortion is to examine a plot of the output spectrum. SAND