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Statistical interpretation of data — Techniques of estimation and tests relating to means and variances

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FOREWORD

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Statistical interpretation of data – Techniques of estimation and tests relating to means and variances

SECTION ONE : PRESENTATION OF CALCULATIONS

GENERAL REMARKS

1) This International Standard specifies the techniques required :

a) to estimate the mean or the variance of populations;

b) to examine certain hypotheses concerning the value of those parameters, from samples.

2) The techniques used are valid only if, in each of the populations under consideration, the sample elements are drawn at random and are independent. In the case of a finite population, elements drawn at random may be considered as independent when the population size is sufficiently large or when the sampling fraction is sufficiently small (for instance smaller than 1/10).

3) The distribution of the observed variable is assumed to be normal in each population. However, if the distribution does not deviate very much from the normal, the techniques described remain approximately valid to an extent sufficient for most practical applications, provided the sample size is not too small. For tables A, B, C and D, the sample size should be of the order of 5 to 10 at least; for all the other tables, it should be not less than about 20.[1)]

4) A certain number of techniques exist which permit the verification of the hypothesis of normality. This subject is dealt with briefly in the examples in section two and will also be dealt with in a further document (yet to be prepared). Nevertheless, this hypothesis may be admitted on the basis of information other than that provided by the sample itself. In the case where the hypothesis of normality should be rejected, the obvious method to follow is to resort to non-parametric tests or to use suitable transformations for obtaining normally distributed populations, for example $1/x$, $\log(x + a)$, $\sqrt{x + a}$, but the conclusions reached by applying these procedures described in this International Standard are only directly valid for the transformed variate; caution should be used in the translation to the original variate. For example **exp (mean log x)** is equal to the **geometric mean** of x not the arithmetic mean.

If what is really needed is an estimate of the mean or standard deviation of the variate X itself then, whether the population distribution is normal or not, an unbiassed estimation of the mean m and the population variance σ^2 is produced by the sample mean $\bar{x}$ and characteristic s^2.

5) It is desirable to accompany each statistical operation with all the particulars relevant to the source or to the method of obtaining the observations which may clarify this statistical analysis, and in particular to give the unit or the smallest unit of measurement having practical meaning.

6) It is not permissible to discard any observations or to apply any corrections to apparently doubtful observations without a justification based on experimental, technical or other evident grounds which should be clearly given. In any case the discarded or corrected values and the reason for discarding or correcting them must be mentioned.

7) In problems of estimation, the confidence level $1 - \alpha$ is the probability that the confidence interval covers the true value of the estimated parameter. Its most usual values are 0,95 and 0,99, or $\alpha = 0{,}05$ and $\alpha = 0{,}01$.

8) In problems of testing a hypothesis, the significance level is, in the two-sided cases, the probability of rejecting the null hypothesis (or tested hypothesis) if it is true (error of the first kind); in the one-sided cases, the significance level is the maximum value of this probability (maximum value of the error of the first kind). Values of $\alpha = 0{,}05$ (1 in 20 chance) or 0,01 (1 in 100 chance) are very commonly employed according to the risk which the user is prepared to take. Since a hypothesis may be rejected using $\alpha = 0{,}05$, but not when using 0,01, it is often appropriate to use the phrase : "the hypothesis is rejected at the 5 % level" or, if this is the case, "at the 1 % level". Attention is drawn to the existence of an error of the second kind. This error is committed if the null hypothesis is accepted when it is false. Terms concerning statistical tests are defined in clause 2 of ISO 3534, *Statistics – Vocabulary*[2)].

1) Studies about normal distributions are in progress in TC 69/SC 2.

2) At present at the stage of draft.

9) The calculations can often be greatly reduced by making a change of origin and/or unit on the data. In the case of data classified into groups, reference may be made to the formulae in ISO 2602, *Statistical interpretation of test results — Estimation of the mean — Confidence interval.*

NOTE — A change of origin may be essential to obtain sufficient accuracy when calculating a variance using the stated formulae with a low precision calculator or computer.

10) The methods shown in tables C and C′ deal with the comparison of two means. They assume that the corresponding samples are independent. For the study of certain problems, it may be interesting to pair the observations (for instance in the comparison of two methods or the comparison of two instruments). The statistical treatment of paired observations is the subject of ISO 3301, *Statistical interpretation of data — Comparison of two means in the case of paired observations*, but in annex A an example of treatment of paired observations is given. It uses formally the data of table A′.

11) The symbols and their definitions used in this International Standard are in conformity with ISO 3207, *Statistical interpretation of data — Determination of a statistical tolerance interval.*

TABLES

A — Comparison of a mean with a given value (variance known)

A′ — Comparison of a mean with a given value (variance unknown)

B — Estimation of a mean (variance known)

B′ — Estimation of a mean (variance unknown)

C — Comparison of two means (variances known)

C′ — Comparison of two means (variances unknown, but may be assumed equal)

D — Estimation of the difference of two means (variances known)

D′ — Estimation of the difference of two means (variances unknown, but may be assumed equal)

E — Comparison of a variance or of a standard deviation with a given value

F — Estimation of a variance or of a standard deviation

G — Comparison of two variances or two standard deviations

H — Estimation of the ratio of two variances or of two standard deviations

TABLE A — **Comparison of a mean with a given value** (variance known)

Technical characteristics of the population studied (5) . . . .

Technical characteristics of the sample items (5) . . . .

Discarded observations (6) . . . .

Statistical data	**Calculations**
Sample size : $n =$	$\bar{x} = \frac{\Sigma x}{n} =$
Sum of the observed values : $\Sigma x =$	$[u_{1-\alpha}/\sqrt{n}]\,\sigma =$
Given value : $m_0 =$	$[u_{1-\alpha/2}/\sqrt{n}]\,\sigma =$
Known value of the population variance : $\sigma^2 =$	
Or standard deviation : $\sigma =$	
Significance level chosen (8) : $\alpha =$	

Results

Comparison of the population mean with the given value m_0 :

Two-sided case :

The hypothesis of the equality of the population mean to the given value (null hypothesis) is rejected if :

$$|\bar{x} - m_0| > [u_{1-\alpha/2}/\sqrt{n}]\,\sigma$$

One-sided cases :

a) The hypothesis that the population mean is not smaller than m_0 (null hypothesis) is rejected if :

$$\bar{x} < m_0 - [u_{1-\alpha}/\sqrt{n}]\,\sigma$$

b) The hypothesis that the population mean is not greater than m_0 (null hypothesis) is rejected if :

$$\bar{x} > m_0 + [u_{1-\alpha}/\sqrt{n}]\,\sigma$$

NOTE — The numbers (5), (6) and (8) refer to the corresponding paragraphs of the "General remarks".

Comments

1) The significance level α (see § 8 of the "General remarks") is the probability of rejecting the null hypothesis when this hypothesis is true.

2) U stands for the standardized normal variate : the value u_α is defined by :

$$P\,[U < u_\alpha] = \alpha$$

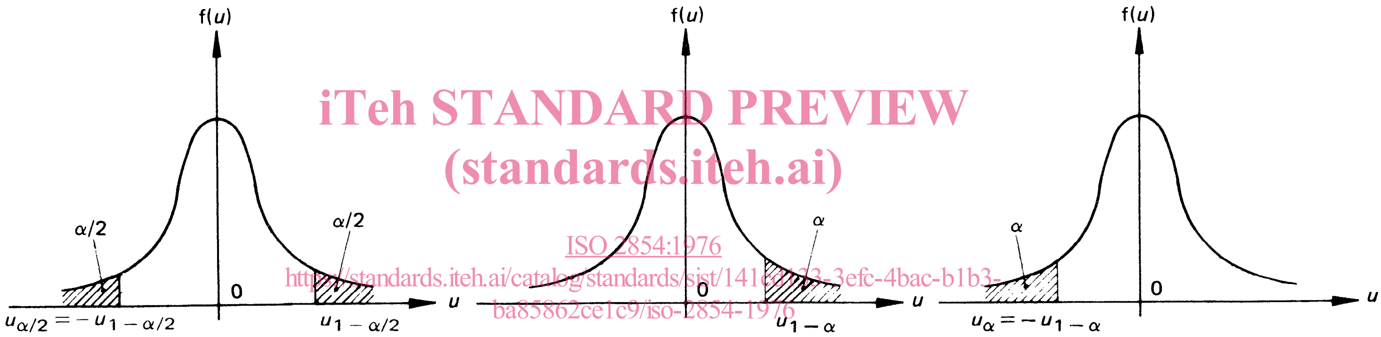
Since the distribution of U is symmetrical around zero, $u_\alpha = -u_{1-\alpha}$.

We therefore have :

$$P\,[U > u_\alpha] = 1 - \alpha$$

$$P\,[-u_{1-\alpha/2} < U < u_{1-\alpha/2}] = 1 - \alpha$$

Probability density of U (standardized normal distribution)



Two-sided case

One-sided cases

3) $\sigma/\sqrt{n}$ is the standard deviation of the mean $\bar{x}$, in a sample of n observations.

4) For convenience in application, values of $u_{1-\alpha}/\sqrt{n}$ and $u_{1-\alpha/2}/\sqrt{n}$ are given in table 1 of annex B for $\alpha = 0{,}05$ and $\alpha = 0{,}01$.

EXAMPLE : see section two, "Explanatory notes and examples".

TABLE A′ – **Comparison of a mean with a given value** (variance unknown)

Technical characteristics of the population studied (5) . . . .

Technical characteristics of the sample items (5) . . . .

Discarded observations (6) . . . .

Statistical data	**Calculations**
Sample size : $n =$	$\bar{x} = \frac{\Sigma x}{n} =$
Sum of the observed values : $\Sigma x =$	$\frac{\Sigma (x - \bar{x})^2}{n - 1} = \frac{\Sigma x^2 - (\Sigma x)^2/n}{n - 1}$
Sum of the squares of the observed values : $\Sigma x^2 =$	$\sigma^* = s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n - 1}} =$
Given value : $m_0 =$	$[t_{1-\alpha}(\nu)/\sqrt{n}\,]\,s =$
Degrees of freedom : $\nu = n - 1$	
Significance level chosen (8) : $\alpha =$	$[t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\,s =$

Results

Comparison of the population mean with the given value m_0 :

Two-sided case :

The hypothesis of the equality of the population mean to the given value (null hypothesis) is rejected if :

$$|\bar{x} - m_0| > [t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\,s$$

One-sided cases :

a) The hypothesis that the population mean is not smaller than m_0 (null hypothesis) is rejected if :

$$\bar{x} < m_0 - [t_{1-\alpha}(\nu)/\sqrt{n}\,]\,s$$

b) The hypothesis that the population mean is not greater than m_0 (null hypothesis) is rejected if :

$$\bar{x} > m_0 + [t_{1-\alpha}(\nu)/\sqrt{n}\,]\,s$$

NOTE – The numbers (5), (6) and (8) refer to the corresponding paragraphs of the "General remarks".

Comments

1) The significance level α (see § 8 of the "General remarks") is the probability of rejecting the null hypothesis when this hypothesis is true.

2) $t(\nu)$ stands for Student's variate with $\nu = n - 1$ degrees of freedom : the value $t_\alpha(\nu)$ is defined by

$$P\,[t(\nu) < t_\alpha(\nu)] = \alpha$$

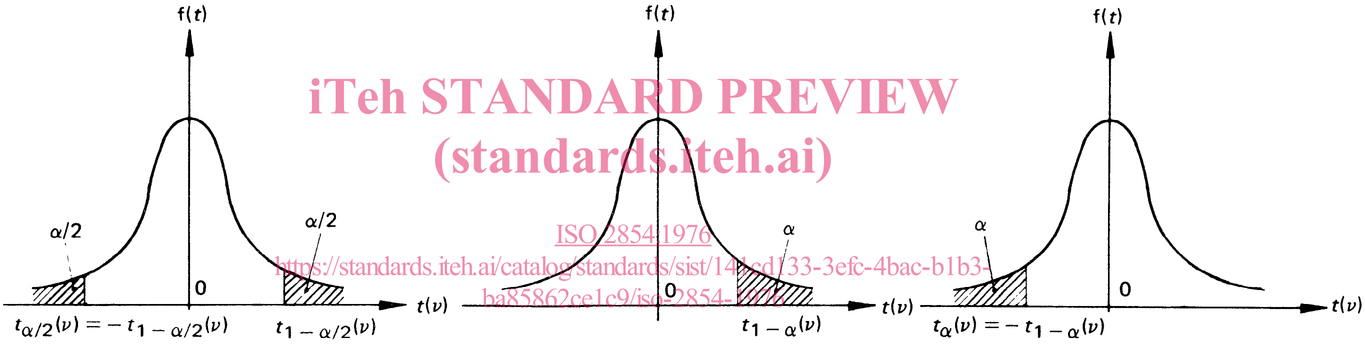
Since the distribution of $t(\nu)$ is symmetrical around zero, $t_\alpha(\nu) = - t_{1-\alpha}(\nu)$.

We therefore have :

$$P\,[t(\nu) > t_\alpha(\nu)] = 1 - \alpha$$

$$P\,[- t_{1-\alpha/2}(\nu) < t(\nu) < t_{1-\alpha/2}(\nu)] = 1 - \alpha$$

Probability density of Student's $t(\nu)$ with $\nu = n - 1$ degrees of freedom



Two-sided case

One-sided cases

3) $\sigma^*/\sqrt{n}$ is the estimated standard deviation of the mean $\bar{x}$, in a sample of n observations.

4) For convenience in application, values of $t_{1-\alpha/2}(\nu)/\sqrt{n}$ and $t_{1-\alpha}(\nu)/\sqrt{n}$ are given in table IIb of annex B for $\alpha = 0{,}05$ and $\alpha = 0{,}01$

EXAMPLE : see section two, "Explanatory notes and examples".

TABLE B — **Estimation of a mean** (variance known)

Technical characteristics of the population studied (5) . . . .

Technical characteristics of the sample items (5) . . . .

Discarded observations (6) . . . .

Statistical data	**Calculations**
Sample size : $n =$	$\bar{x} = \frac{\Sigma x}{n} =$
Sum of the observed values : $\Sigma x =$	$[u_{1-\alpha}/\sqrt{n}]\ \sigma =$
Known value of the population variance : $\sigma^2 =$	
Or standard deviation : $\sigma =$	$[u_{1-\alpha/2}/\sqrt{n}]\ \sigma =$
Confidence level chosen (7) : $1 - \alpha =$	

Results

Estimation of the population mean m :

$$m^* = \bar{x} =$$

Two-sided confidence interval :

$$\bar{x} - [u_{1-\alpha/2}/\sqrt{n}]\ \sigma < m < \bar{x} + [u_{1-\alpha/2}/\sqrt{n}]\ \sigma$$

One-sided confidence intervals :

$$m < \bar{x} + [u_{1-\alpha}/\sqrt{n}]\ \sigma$$

or

$$m > \bar{x} - [u_{1-\alpha}/\sqrt{n}]\ \sigma$$

NOTE — The numbers (5), (6) and (7) refer to the corresponding paragraphs of the "General remarks".

Comments

1) The confidence level $1 - \alpha$ (see § 7 of the "General remarks") is the probability that the confidence interval covers the true value of the mean.

2) U stands for the standardized normal variate : the value u_α is defined by :

$$P\,[U < u_\alpha] = \alpha$$

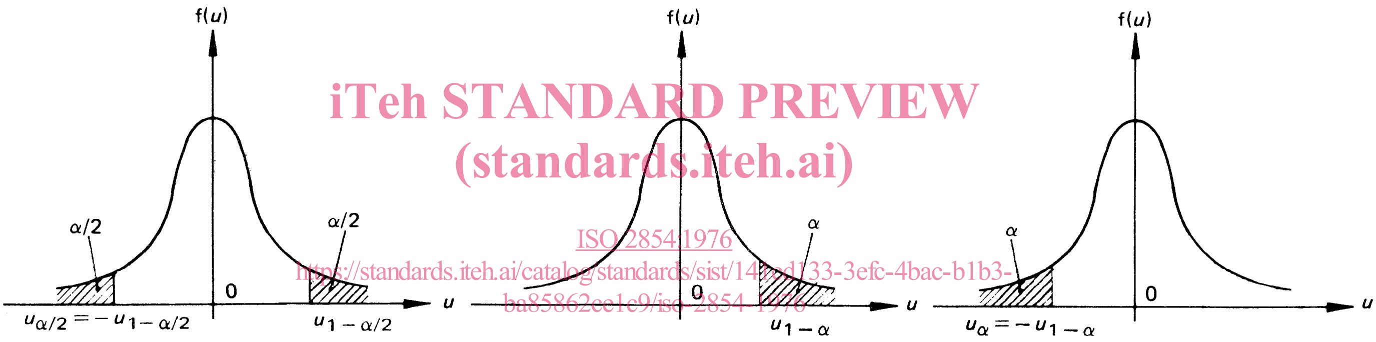
Since the distribution of U is symmetrical around zero, $u_\alpha = - u_{1-\alpha}$

We therefore have :

$$P\,[U > u_\alpha] = 1 - \alpha$$

$$P\,[- u_{1-\alpha/2} < U < u_{1-\alpha/2}] = 1 - \alpha$$

Probability density of U (standardized normal distribution)



Two-sided case

One-sided cases

3) $\sigma/\sqrt{n}$ is the standard deviation of the mean $\bar{x}$, in a sample of n observations.

4) For convenience in application, values of $u_{1-\alpha/2}/\sqrt{n}$ and $u_{1-\alpha}/\sqrt{n}$ are given in table I of annex B for $\alpha = 0{,}05$ and $\alpha = 0{,}01$.

EXAMPLE : see section two, "Explanatory notes and examples".

TABLE B′ — **Estimation of a mean** (variance unknown)

Technical characteristics of the population studied (5) . . . .

Technical characteristics of the sample items (5) . . . .

Discarded observations (6) . . . .

Statistical data	**Calculations**
Sample size : $n =$	$\bar{x} = \frac{\Sigma x}{n} =$
Sum of the observed values : $\Sigma x =$	$\frac{\Sigma (x - \bar{x})^2}{n-1} = \frac{\Sigma x^2 - (\Sigma x)^2/n}{n-1} =$
Sum of the squares of the observed values : $\Sigma x^2 =$	$\sigma^* = s = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}} =$
Degrees of freedom : $\nu = n - 1 =$	$[t_{1-\alpha}(\nu)/\sqrt{n}\,]\, s =$
Confidence level chosen (7) : $1 - \alpha =$	$[t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\, s =$

Results

Estimation of the population mean m :

$$m^* = \bar{x} =$$

Two-sided confidence interval :

$$\bar{x} - [t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\, s < m < \bar{x} + [t_{1-\alpha/2}(\nu)/\sqrt{n}\,]\, s$$

One-sided confidence intervals :

$$m < \bar{x} + [t_{1-\alpha}(\nu)/\sqrt{n}\,]\, s$$

or

$$m > \bar{x} - [t_{1-\alpha}(\nu)/\sqrt{n}\,]\, s$$

NOTE — The numbers (5), (6) and (7) refer to the corresponding paragraphs of the "General remarks".

Comments

1) The confidence level $1 - \alpha$ (see § 7 of the "General remarks") is the probability that the confidence interval covers the true value of the mean.

2) $t(\nu)$ stands for Student's variate with ν degrees of freedom; the value $t_\alpha(\nu)$ is defined by

$$P[t(\nu) < t_\alpha(\nu)] = \alpha$$

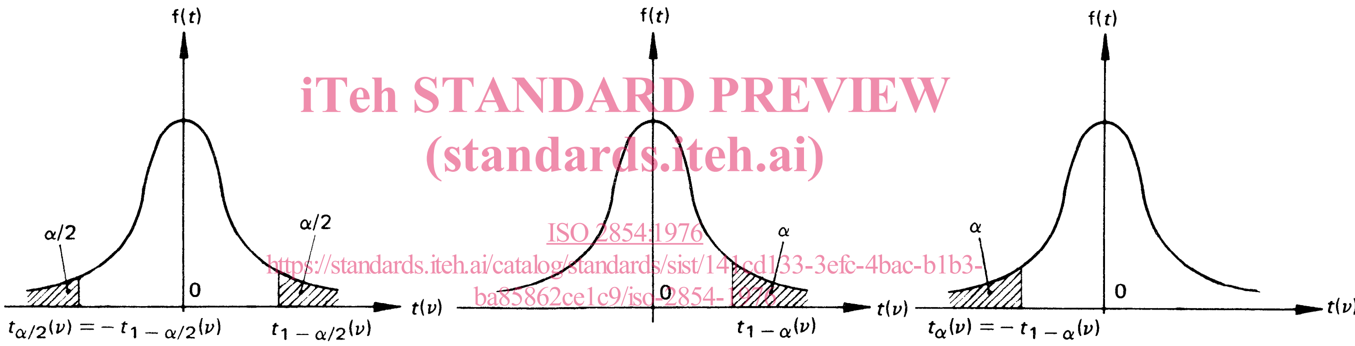
Since the distribution of $t(\nu)$ is symmetrical around zero, $t_\alpha(\nu) = -t_{1-\alpha}(\nu)$.

We therefore have :

$$P[t(\nu) > t_\alpha(\nu)] = 1 - \alpha$$

$$P[-t_{1-\alpha/2}(\nu) < t(\nu) < t_{1-\alpha/2}(\nu)] = 1 - \alpha$$

Probability density of Student's $t(\nu)$ with $\nu = n - 1$ degrees of freedom



Two-sided case — One-sided cases

3) $\sigma^*/\sqrt{n}$ is the estimated standard deviation of the mean $\bar{x}$, in a sample of n observations.

4) For convenience in application, values of $t_{1-\alpha/2}(\nu)/\sqrt{n}$ and $t_{1-\alpha}(\nu)/\sqrt{n}$ are given in table IIb of annex B for $\alpha = 0{,}05$ and $\alpha = 0{,}01$.

EXAMPLE : see section two, "Explanatory notes and examples".