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Standard Guide for
Selecting Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Buildings and Building Systems[1]

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1. Scope

1.1 This guide covers techniques for treating uncertainty in input values to an economic analysis of a building investment project. It also recommends techniques for evaluating the risk that a project will have a less favorable economic outcome than what is desired or expected.[2]

1.2 The techniques include breakeven analysis, sensitivity analysis, risk-adjusted discounting, the mean-variance criterion and coefficient of variation, decision analysis, simulation, and simulation. stochastic dominance.

1.3 The techniques can be used with economic methods that measure economic performance, such as life-cycle cost analysis, net benefits, the benefit-to-cost ratio, internal rate of return, and payback.

2. Referenced Documents

2.1 *ASTM Standards:*[3]
E631 Terminology of Building Constructions
E833 Terminology of Building Economics
E917 Practice for Measuring Life-Cycle Costs of Buildings and Building Systems
E964 Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios for Buildings and Building Systems
E1057 Practice for Measuring Internal Rate of Return and Adjusted Internal Rate of Return for Investments in Buildings and Building Systems
E1074 Practice for Measuring Net Benefits and Net Savings for Investments in Buildings and Building Systems
E1121 Practice for Measuring Payback for Investments in Buildings and Building Systems
E1185 Guide for Selecting Economic Methods for Evaluating Investments in Buildings and Building Systems Guide for Selecting Economic Methods for Evaluating Investments in Buildings and Building Systems
E1946 Practice for Measuring Cost Risk of Buildings and Building Systems

2.2 *Adjuncts:*
Discount FactorTables,Adjunct to Practices
Discount Factor Tables Adjunct to Practices E917, E964, E1057, E1074, and E1121[4]

3. Terminology

3.1 *Definitions*—For definitions of terms used in this guide, refer to Terminologies E631 and E833.

4. Summary of Guide

4.1 This guide identifies related ASTM standards and adjuncts. It describes circumstances when measuring uncertainty and risk may be helpful in economic evaluations of building investments. This guide defines uncertainty, risk exposure, and risk attitude. It presents nonprobabilistic and probabilistic techniques for measuring uncertainty and risk exposure. This guide describes briefly each technique, gives the formula for calculating a measure where appropriate, illustrates the techniques with a case example, and summarizes its advantages and disadvantages.

4.2 Since there is no best technique for measuring uncertainty and risk in every economic evaluation, this guide concludes with a discussion of how to select the appropriate technique for a particular problem.

4.3 This guide describes in detail how risk exposure can be measured by probability functions and distribution functions (see Annex A1). It also describes how risk attitude can be incorporated using utility theory and other approaches (see Annex A2).

5. Significance and Use

5.1 Investments in long-lived projects such as buildings are characterized by uncertainties regarding project life, operation and maintenance costs, revenues, and other factors that affect project economics. Since future values of these variable factors are generally not known, it is difficult to make reliable economic evaluations.

5.2 The traditional approach to project investment analysis has been to apply economic methods of project evaluation to best-guess estimates of project input variables as if they were certain estimates and then to present results in single-value, deterministic terms. When projects are evaluated without regard to uncertainty of inputs to the analysis, decision makers may have insufficient information to measure and evaluate the risk of investing in a project having a different outcome from what is expected.

5.3 Risk analysis is the body of theory and practice that has evolved to help decision makers assess their risk exposures and risk attitudes so that the investment that is the best bet for them can be selected.

NOTE 1—The decision maker is the individual or group of individuals responsible for the investment decision. For example, the decision maker may be the chief executive officer or the board of directors.

5.4 Uncertainty and risk are defined as follows. Uncertainty (or certainty) refers to a state of knowledge about the variable inputs to an economic analysis. If the decision maker is unsure of input values, there is uncertainty. If the decision maker is sure, there is certainty. Risk refers either to risk exposure or risk attitude.

5.4.1 Risk exposure is the probability of investing in a project that will have a less favorable economic outcome than what is desired (the target) or is expected.

5.4.2 Risk attitude, also called risk preference, is the willingness of a decision maker to take a chance or gamble on an investment of uncertain outcome. The implications of decision makers having different risk attitudes is that a given investment of known risk exposure might be economically acceptable to an investor who is not particularly risk averse, but totally unacceptable to another investor who is very risk averse.

NOTE 2—For completeness, this guide covers both risk averse and risk taking attitudes. Most investors, however, are likely to be risk averse. The principles described herein apply both to the typical case where investors have different degrees of risk aversion and to the atypical case where some investors are risk taking while others are risk averse.

5.5 No single technique can be labeled the best technique in every situation for treating uncertainty, risk, or both. What is best depends on the following: availability of data, availability of resources (time, money, expertise), computational aids (for example, computer services), user understanding, ability to measure risk exposure and risk attitude, risk attitude of decision makers, level of risk exposure of the project, and size of the investment relative to the institution's portfolio.

6. Procedures

6.1 The recommended steps for carrying out an evaluation of uncertainty or risk are as follows:

6.1.1 Determine appropriate economic measure(s) for evaluating the investment (see Guide E1185).

6.1.2 Identify objectives, alternatives, and constraints (see Practices E917, E964, E1057, E1074, and E1121).

6.1.3 Decide whether an uncertainty and risk evaluation is needed, and, if so, choose the appropriate technique (see Sections 5, 7, 8, and 10).

6.1.4 Compile data and establish assumptions for the evaluation.

6.1.5 Determine risk attitude of the decision maker (see Section 7 and Annex A2).

6.1.6 Compute measures of worth[5] and associated risk (see Sections 7 and 8).

6.1.7 Analyze results and make a decision (see Section 9).

6.1.8 Document the evaluation (see Section 11).

7. Techniques: Advantages and Disadvantages

7.1 This guide considers in detail three nonprobabilistic techniques (breakeven analysis, sensitivity analysis, and risk-adjusted discounting) and ~~three~~four probabilistic techniques (mean-variance criterion and coefficient of variation, decision analysis,

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[5] The NIST Building Life-Cycle Cost (BLCC) Computer Program helps users calculate measures of worth for buildings and building components that are consistent with ASTM standards. The program is downloadable from http://www.eere.energy.gov/femp/information/download_blcc.html.

simulation, and ~~simulation)~~stochastic dominance) for treating uncertainty and risk. This guide also summarizes several additional techniques that are used less frequently.

7.2 *Breakeven Analysis*:

7.2.1 When an uncertain variable is critical to the economic success of a project, decision makers frequently want to know the minimum or maximum value that variable can reach and still have a breakeven project; that is, a project where benefits (savings) equal costs. For example, the breakeven value of an input *cost* variable is the maximum amount one can afford to pay for the input and still break even compared to benefits earned. A breakeven value of an input *benefit* variable is the minimum amount the project can produce in that benefit category and still cover the projected costs of the project.

Note 3—Benefits and costs are treated throughout this guide on a discounted cash-flow basis, taking into account taxes where appropriate. (See Practice E917 for an explanation of discounted cash flows considering taxes.)

7.2.2 To perform a breakeven analysis, an equation is constructed wherein the benefits are set equal to the costs for a given investment project, the values of all inputs except the breakeven variable are specified, and the breakeven variable is solved algebraically.

7.2.3 Suppose a decision maker is deciding whether or not to invest in a piece of energy conserving equipment for a government-owned building. The deviation of the formula for computing breakeven investment costs for the equipment is as follows:

$$S = C \tag{1}$$
$$C = I + \text{O\&M} + R$$
$$S = I + \text{O\&M} + R$$
$$I = S - \text{O\&M} - R$$

where:
S = savings (benefits) in reduced energy costs from using the equipment,
C = all costs associated with the equipment,
I = initial investment costs of the equipment,
O&M = operation and maintenance costs of the equipment, and
R = replacement costs required to keep the equipment functional over the study period, and where all cost and benefit cash flows are discounted to present values.

7.2.4 By rearranging terms, the breakeven investment unknown is isolated on the left side of the equation. Substitution of known values for the terms on the right side allows the analyst to solve for the breakeven value. For example, if S = \$20 000, O&M = \$2500, and R = \$1000,
then

$$I = \$20\,000 - \$2500 - \$1000 \tag{2}$$

or

$$I = \$16\,500 \tag{3}$$

7.2.5 This means that \$16 500, the breakeven value, is the maximum amount that can be paid for the energy-conserving equipment and still recover all costs through energy savings.

7.2.6 An advantage of breakeven analysis is that it can be computed quickly and easily with limited information. It also simplifies project evaluation in that it gives just one value to decision makers to use as a benchmark for comparison against the predicted performance of that uncertain variable. Breakeven analysis helps decision makers assess the likelihood of achieving the breakeven value and thereby contributes implicitly to the analysis of project risk.

7.2.7 A disadvantage is that it provides no probabilistic picture of input variable uncertainty or of project risk exposure. Furthermore, it includes no explicit treatment of risk attitude.

7.3 *Sensitivity Analysis*:

7.3.1 Sensitivity analysis measures the impact on project outcomes of changing a key input value about which there is uncertainty. For example, choose a pessimistic, expected, and optimistic value for an uncertain variable. Then do an economic analysis for each of the three values to see how the outcome changes as they change, with other things held the same.

7.3.2 Sensitivity analysis also applies to different combinations of input values. That is, alter several variables at once and then compute a measure of worth. For example, one scenario might include a combination of all pessimistic values, another all expected values, and a third all optimistic values; or a combination might include optimistic values for some variables in conjunction with pessimistic or expected values for others. Examining different combinations is required if the uncertain variables are interrelated.

7.3.3 The following illustration of sensitivity analysis treats an accept/reject decision. Consider a decision on whether or not to install a programmable time clock to control heating, ventilating, and air conditioning (HVAC) equipment in a building. The time clock reduces electricity consumption by turning off that part of the HVAC equipment that is not needed during hours when the building is unoccupied. Using the benefit-to-cost ratio (BCR) as the economic method, the time clock is acceptable on economic

grounds if its BCR is greater than 1.0. The energy reduction benefits from the time clock, however, are uncertain. They are a function of three factors: the initial price of energy, the rate of change in energy prices over the life cycle of the time clock, and the number of kilowatt hours saved. Assume that the initial price of energy and the number of kilowatt-hours saved are relatively certain, and that the sensitivity of the BCR is being tested with respect to the following three values of energy price change: a low rate of energy price escalation (slowly increasing benefits from energy savings); a moderate rate of escalation (moderately increasing benefits); and a high rate of escalation (rapidly increasing benefits). These three assumed values of energy price change might correspond to our projections of pessimistic, expected, and optimistic values. Three BCR estimates result from repeating the BCR computation for each of the three energy price escalation rates. For example, BCRs of 0.8, 2.0, and 4.0 might result. Whereas a deterministic approach might have generated a BCR estimate of 2.0, now it is apparent that the BCR *could be* significantly less than 2.0, and even less than 1.0. Thus accepting the time clock could lead to an inefficient outcome.

7.3.4 There are several advantages of sensitivity analysis. First, it shows how significant a single input variable is in determining project outcomes. Second, it recognizes the uncertainty associated with the input. Third, it gives information about the range of output variability. And fourth, it does all of these when there is little information, resources, or time to use more sophisticated techniques.

7.3.5 Disadvantages of sensitivity analysis in evaluating risk are that it gives no explicit probabilistic measure of risk exposure and it includes no explicit treatment of risk attitude. The findings of sensitivity analysis are ambiguous. How likely is a pessimistic or expected or optimistic value, for example, and how likely is the corresponding outcome value? Sensitivity analysis can in fact be misleading if all pessimistic assumptions or all optimistic assumptions are combined in calculating economic measures. Such combinations of inputs are unlikely in the real world.

7.3.6 Sensitivity results can be presented in text, tables, or graphs. One type of graph that is useful in showing the sensitivity of project worth to a critical variable is illustrated in Fig. 1. Net benefits (NB) for Projects A and B decrease as the discount rate increases. The slopes of the functions show that NB is more sensitive to discount rate changes for Project A than for Project B, assuming other variables remain unchanged. These functions also help in making comparisons as to which project is more cost effective. At a discount rate below 7 %, for example, Project A has the greater NB. At a rate above 7 %, Project B yields the greater NB. And at 7 %, the two projects provide identical NB.

7.3.7 Note that the functions indicate the potential values of NB *if* different values of the discount rate occur. If decision makers have some idea as to the likelihood of specific discount rates, the graph will help them evaluate the NB implications for these two projects. The sensitivity graph in this sense contributes to an implicit description of risk exposure. Yet the graph fails to provide a quantitative measure of the probability of any given NB occurring.

7.3.8 Another special graph for sensitivity analysis that presents a snapshot of potential impacts of uncertain input variables on project outcomes is the spider diagram. The one illustrated in Fig. 2 shows for a prospective commercial building investment the sensitivity of the adjusted internal rate of return (AIRR) to three variables: operation, maintenance, and replacement costs (OM&R); project life (PL); and the reinvestment rate (RR). Each variable is represented by a labeled function that shows what AIRR values would result from different values of the uncertain variable. For example, the downward-sloping OM&R function indicates that the AIRR is inversely proportional to OM&R costs. By design, the OM&R function (as well as the other two functions) passes through the horizontal axis at the best-guess estimate of the AIRR (15 % in this case), based on the best-guess estimates of the three uncertain variables. Since each of the variables is measured by different units (money, years, and percent), the vertical axis is denominated in positive and negative percent changes from the best-guess values fixed at the horizontal axis. The AIRR value corresponding to any given percent variation indicated by a point on the function is found by extending a line perpendicular to the horizontal axis and reading directly the AIRR value. Thus a 30 % increase in the best-guess reinvestment rate would yield a 25 % AIRR, assuming other values remain unchanged.

7.3.9 The contribution of the spider diagram is its picture of the relative importance of the different uncertain variables. It shows immediately that the lesser the slope of a function, the more sensitive is the AIRR to that variable. For example, any given percent
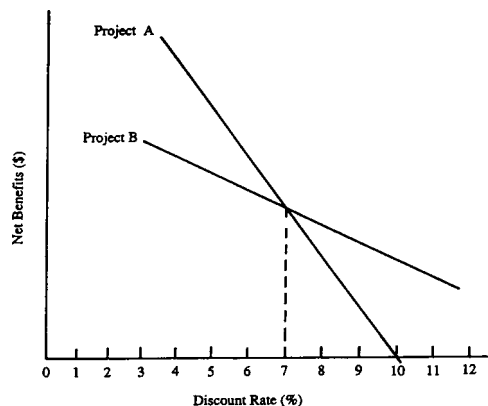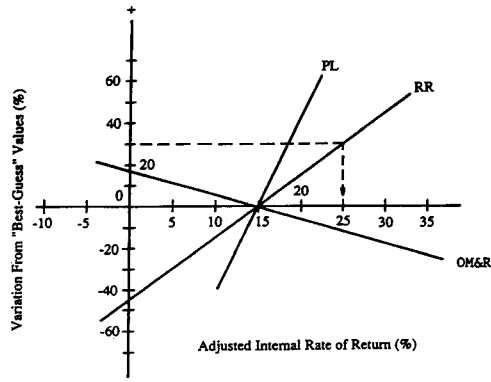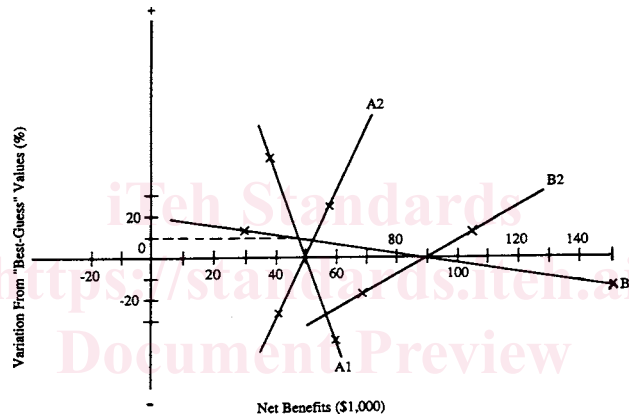

FIG. 1 Sensitivity of Net Benefits of Projects A and B to Discount
Rate

NOTE 1—PL = project life,
RR = reinvestment rate, and
OM&R = operation, maintenance, and replacement costs.
**FIG. 2 Spider Diagram Showing Sensitivity of the Adjusted
Internal Rate of Return to Variations in Uncertain Variables**

FIG. 3 Spider Diagrams for Competing Projects

change in OM&R will have a greater impact on the AIRR than will an equal percent change in RR or PL.

7.3.10 Spider diagrams can be helpful when comparing competing projects as long as the decision maker keeps in mind that extreme values of the measure of worth reflect variations in one variable only. For example, look at the spider diagram for Projects A and B in Fig. 3. The NB of Project A is a function of variables A1 and A2, and the NB of Project B is a function of variables B1 and B2. The horizontal axis suggests that Project B has a higher present value net benefits ($90 000) than Project A ($50 000). That is, if only best-guess values were used in a single-value, deterministic approach, Project B would be the preferred project. However, if we assign, say a 50 % confidence interval about the uncertain variables A1, A2, B1, and B2, as shown by X's on the functions, there appears the possibility that Project A could yield a higher NB than Project B. That is, within that confidence interval, if the extreme B1 value to the left were to occur, Project B would yield a lesser NB than would Project A for A1 or A2 extreme values to the left. Furthermore, if A1 and B1 were the same input variable, we would know that Project A would be preferred at values of A1 and B1 above 10 % over the best-guess value, and Project B would be preferred at values of A1 and B1 below 10 %.

7.3.11 Once again, however, sensitivity analysis gives no indication of the probability of any given value of NB. Furthermore, because only one variable is allowed to change at a time, and NB is a function of more than one variable, sensitivity analysis gives an incomplete description of the possible outcomes.

7.4 *Risk-Adjusted Discounting*:

7.4.1 One technique used by the business community to account for risk is the risk-adjusted discount rate (RADR). The objective of using the RADR technique is to raise the likelihood that the investor will earn a return over time sufficient to compensate for extra risk associated with specific projects.

7.4.2 Projects with anticipated high variability in distributions of project worth have their net benefits or returns discounted at higher rates than projects with low variability. Thus in computing net benefits or the benefit-to-cost ratio, the discount rate is higher for benefit streams of risky projects than for those with certain outcomes. Or when applying rate-of-return methods, the minimum acceptable rate of return (MARR) is raised above the risk-free rate to compensate for the higher variability of returns in risky projects.

7.4.3 Calculate the RADR as follows:

$$RADR = RF + AR1 + AR2 \qquad (4)$$

where:
RF = risk-free rate,
AR1 = adjustment for normal risk encountered in the firm's operations, and
AR2 = adjustment for extra risk above or below normal risk.
All terms are expressed as percents.

7.4.4 The risk-free rate (RF) component accounts for the time value of money. It is what might be earned, for example, on government treasury bills, the closest thing to a riskless investment available to most investors. The adjustment for normal risk (AR1) is the risk premium that a firm might impose to cover the average riskiness of its normal operations. The sum of RF and AR1 should equal the MARR the firm requires on typical investments. The AR2 component adjusts for projects with more or less risk than what is normally associated with the firm. The adjustment can be positive or negative.

7.4.5 For discounting *benefit* streams, AR2 is an increasing function of (*1*) the perceived variability in project outcomes (risk exposure) and (*2*) the degree to which the decision maker is risk averse (risk attitude). For *cost* streams, AR2 is a decreasing function of those same risk factors.

7.4.6 For computing the RADR, each benefit and cost stream should be discounted with a unique RADR that includes AR1 and AR2 values that describe that stream's uncertainty. For benefit or savings streams, AR1 and AR2 are adjusted upwards as perceived risk increases; that is, as future benefits become more uncertain, the RADR technique requires raising the discount rate to make the project look less desirable. For cost streams, AR1 and AR2 are adjusted downwards as perceived risk increases; that is, as future costs become more uncertain, the correct application of the RADR technique requires lowering the discount rate to make the project look less cost effective. It follows then that the appropriate adjustment for risk when using life cycle cost (LCC) analysis is a decrease in the discount rate for each cost stream to make project costs appear higher. Otherwise LCC analysis will be biased in favor of projects with a greater risk of higher-than-anticipated costs.

7.4.7 Let us look once again at the BCR of the time clock for an illustration of the RADR when making an accept/reject decision. If no unusual risk is associated with the time clock, the discount rate is equal to the sum of RF and AR1 as shown under Eq 4. Let us suppose that the BCR for the time clock is 1.1 in this case. Thus it appears economically sound.

7.4.8 Now let us assume instead that the economic performance of the time clock is more risky than average. This might arise, for example, from the impact of uncertain kilowatt-hour reductions or uncertain future energy prices. Furthermore, let us assume that the decision maker is risk averse. Using the RADR technique, we raise the discount rate for evaluating energy cost savings by some positive value of AR2. If the resulting BCR falls below 1.0, the project no longer appears economically acceptable.

7.4.9 Advantages of the RADR technique are that it is relatively simple to understand; it is easy to compute; and it accounts to some extent for uncertainty of inputs, risk exposure, and risk attitude.

7.4.10 A major limitation in using the RADR is the lack of any accepted procedure for establishing the RADR value. It is typically estimated based on the decision maker's best judgment. One common approach is to simply lump projects into risk categories, each of which has an assigned RADR. There is little fine tuning. Furthermore, there is no distinction between adjustments for handling risk exposure and risk attitude.

7.4.11 A common mistake in application is to use a constant AR2 over the entire study period. This distorts risk adjustment when there are periods for which no special adjustment is necessary above or below what is considered normal risk. A constant AR2 also distorts risk adjustment because it implies in effect that returns become exponentially more uncertain over time, which is often not the case. Thus a discount rate that includes a constant AR2 severely reduces the weight of net benefits accrued in later years, regardless of the certainty of their occurrence. This biases selection towards projects with early payoffs. To avoid this common mistake in application and its resulting bias, use a variable AR2.

7.5 *Mean-Variance Criterion and Coefficient of Variation*:

7.5.1 Comparing mean values and standard deviations of measures of project worth can help decision makers evaluate returns and risk exposure of one project versus another and determine stochastic dominance. If two projects competing for limiting funds are compared on the basis of BCRs, for example, the mean-variance criterion dictates that the one with the higher mean (that is, expected value) and lower standard deviation be chosen. This presumes that decision makers prefer higher BCRs to lower BCRs and less risk to more risk.

7.5.2 If one project has a higher mean and higher standard deviation of the measure of project worth, then the choice is not clear with the mean-variance criterion. In this case, the coefficient of variation can be computed to determine the relative risk of the two projects. The coefficient of variation is found by dividing the standard deviation by the mean as follows:

$$CV = \sigma/\mu, \qquad (5)$$

where:
CV = coefficient of variation,
σ = standard deviation, and
μ = mean or expected value.

7.5.3 The project with the lower coefficient of variation has the lesser risk per unit of return or project worth. It will be preferred

by risk-averse decision makers. Risk-taking decision makers, on the other hand, will prefer the project with the higher coefficient.

7.5.4 An advantage of the coefficient of variation is that it provides an explicit measure of relative risk exposure. Another is that risk attitude is considered when the decision maker evaluates the coefficients of variation to choose among alternative projects. The major limitation is in acquiring the σ and μ values for the measure of project worth.

7.6 *Decision Analysis*:

7.6.1 Decision analysis is one of the few techniques for making economic decisions in an uncertain environment that treats formally both risk exposure and risk attitude. It provides a methodology that allows a decision maker to include alternative outcomes, risk attitudes, and subjective impressions about uncertain events in an evaluation of investments.

7.6.2 Decision analysis typically uses decision trees to represent all possible outcomes, costs, and probabilities associated with a given decision problem. A decision tree is a decision-flow diagram that serves as a road map to clarify possible alternatives and outcomes of sequential decisions. A decision tree is used in this section to illustrate how it helps bring order to complex decisions about risky investments.

7.6.3 In general, the decision analysis approach has three steps. The first is to structure the problem. This includes defining variables, describing with models their relationships, assigning values to possible outcomes, and measuring the importance of variables through sensitivity analysis. The second step is to assign subjective probabilities to important variables and possible outcomes, and to find the best bet alternative. This includes describing uncertainty with subjective probability distributions, describing risk attitude with a utility function (see Annex A2), and finding the alternative that is expected to yield the greatest economic return (or utility if the decision maker is not risk neutral). The third step, which is not always taken, is to determine whether obtaining additional information is worth the cost. If it is, then the information is collected, and steps 1 and 2 are repeated.

NOTE 4—Subjective probability distributions are developed by the decision analyst asking the decision maker or an expert(s) designated by the decision maker a series of probing questions designed to reveal the best judgments available on the likelihood of uncertain events.

7.6.4 *Decision Analysis of Energy Conservation Investment*:

7.6.4.1 This illustration examines an energy investment problem facing a state energy office. The office has been directed to make a choice regarding an energy conservation project from among six possibilities for retrofitting two public buildings. The purpose of the conservation project is to demonstrate to private companies that energy conservation is profitable. The objective of the decision analysis exercise is to choose the retrofit package that yields the maximum expected net benefits (NB), that is, shows the greatest profit potential. If none of the packages yields a positive NB, the choice will be not to invest at all.

7.6.4.2 Two types of retrofit costs are considered. The first is a fixed retrofit investment cost that is incurred for energy conservation work in each building regardless of which retrofit package is chosen. The second is the cost of implementing the individual retrofits in each package. The present value fixed investment (F1 and F2) costs and retrofit package (R1 through R6) costs are shown in Table 1. All costs are assumed to be *certain*.

7.6.4.3 The predicted benefit outcomes (dollar energy savings in present value terms) are *uncertain* for the different retrofit packages. Table 2 shows estimates of these possible benefit outcomes with their respective probabilities of occurrence. Since the state is assumed to be risk neutral and act so as to maximize the expected monetary value of its investments, there is no need to consider risk attitude and the corresponding utility measures of outcomes. Furthermore, since the state pays no taxes, they are not included in the analysis.

7.6.4.4 The decision tree in Fig. 4 clarifies the possible alternatives and outcomes listed in Table 1 and Table 2. The following explanation describes the potential paths of the decision tree starting from the left side.

NOTE 5—The procedure for finding the package that yields maximum net benefits requires starting from the right side of the tree, as will be shown later. It is easier to explain the tree structure, however by starting from the left.

7.6.4.5 The basic alternative of not investing is indicated by the top line segment coming out of the box on the left side of Fig. 4. The fixed investment of $500 000 in Building I is shown by the next line, and the investment of $800 000 in Building II is shown by the bottom line. Each box in a decision tree represents a decision juncture or node, and the line segments represent alternative branches on the decision tree. The state energy office will select that branch sequence that they expect will maximize the present value of net benefits from conservation.

7.6.4.6 Associated with each building is another decision node, requiring a decision regarding a specific set of retrofit choices, R1 through R3 or R4 through R6. The known costs of each retrofit package are shown under each alternative branch.
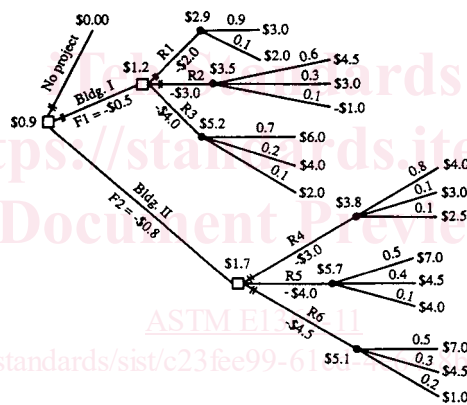
7.6.4.7 The benefit outcomes (dollar energy savings) are uncertain for the different retrofit packages. Thus at the end of each retrofit package branch is a chance node or juncture followed by alternative outcomes. Retrofit package R1, for example, is followed by a chance node, indicated by a circle, with two potential outcomes. The probability of each alternative outcome is indicated on top of its line and the value of each alternative outcome at the tip of its line.

TABLE 1 Fixed Investment and Retrofit Package Cost for Buildings I and II (Cost in Millions of Dollars)

Building I				Building II			
F1	R1	R2	R3	F2	R4	R5	R6
0.5	2.0	3.0	4.0	0.8	3.0	4.0	4.5

TABLE 2 Possible Benefit Outcomes and Their Estimated Probabilities of Occurrence for the Six Retrofit Packages

Retrofit Packages	Possible Benefit Outcomes, millions of dollars	Estimated Probabilities
R1	3.0	0.9
	2.0	0.1
R2	4.5	0.6
	3.0	0.3
	−1.0	0.1
R3	6.0	0.7
	4.0	0.2
	2.0	0.1
R4	4.0	0.8
	3.0	0.1
	2.5	0.1
R5	7.0	0.5
	4.5	0.4
	4.0	0.1
R6	7.0	0.5
	4.5	0.3
	1.0	0.2

NOTE 1—□ = Ðdecision N node,
O = Cchance N node, and
R = Rretrofit P package.

FIG. 4 Decision Tree for Conservation Investment (Dollar Values are in Millions)

7.6.4.8 One way to establish the probability and outcome values is for the analyst to discuss with engineers, architects, building managers, equipment manufacturers, and other knowledgeable people the implications of alternative retrofit packages in Buildings I and II. The outcome values at the branch tips will be based on anticipated potential impacts of changes in uncertain input variables, including energy prices, length of system life, performance of the conservation retrofits, and the quantity of energy saved.

7.6.4.9 Let us trace out one set of decisions with its possible outcomes. If the state energy office chooses the R1 package of retrofits in Building I for a total cost of $2 500 000, there is a 90 % probability that the outcome (payoff) will be $3 000 000 and a 10 % chance it will be only $2 000 000.

7.6.4.10 Let us also examine how some of the outcome and probability values might have been derived. The 90 % probability associated with R1 for a $3 000 000 payoff might be due to the R1 conservation package being a well-tested one with predictable results. On the other hand, R2 might contain conservation options that are new and untried, thereby explaining the spread of possible outcomes and the lower probabilities. And since there is no record of performance, and there is some chance of the conservation options not working, a 10 % probability of a loss of $1 000 000 is included in R2, as shown on the bottom outcome branch.

7.6.4.11 The outcome values at the tips of the outcome branches, the probabilities on the outcome branches, and the retrofit and fixed building costs on the alternative branches are estimated (see Table 1 and Table 2). The values shown at each decision node and chance node, on the other hand, must be calculated. The following steps describe the calculation process for the node values

and how to determine the retrofit choice that maximizes expected net benefits. Note that the calculation process starts from the right side of the tree and works backwards to the left side.

7.6.4.12 Starting from the right-hand side of the tree, average out for each chance node its expected value; that is, calculate the weighted average for each probability fan by summing the products of the possible outcomes weighted by their respective probabilities. The expected value of the probability fan of R1, for example, is computed as follows:

$$0.9(\$3\,000\,000) + 0.1(\$2\,000\,000) = \$2\,900\,000 \tag{6}$$

Write the expected value atop each chance node, as shown in Fig. 4.

7.6.4.13 Next, fold back to the next preceding stage. That is, at each square decision node, compare the alternative branches with respect to their costs and expected benefits. Choose the one with the highest expected net benefits and write it atop the decision node box. For example, if you fold back the decision node values on the Building II path sequence, expected NB values (before subtracting the \$800 000 Building II fixed cost) for retrofit packages R4 through R6 are as follows:

$$R4_{NB} = \$3\,800\,000 - \$3\,000\,000 = \$800\,000 \tag{7}$$
$$R5_{NB} = \$5\,700\,000 - \$4\,000\,000 = \$1\,700\,000 \tag{8}$$
$$R6_{NB} = \$5\,100\,000 - \$4\,500\,000 = \$600\,000 \tag{9}$$

7.6.4.14 The preferred (that is, maximum expected NB) alternative branch is R5. Write its value, \$1 700 000, atop the decision node box. Truncate the other two paths by parallel slash marks to indicate that they are less economical choices.

7.6.4.15 The final step is to fold back one more time. Retrofit package R5 in Building II is the most efficient choice because its expected value of net benefits is \$900 000 (that is, \$1 700 000 – \$800 000) compared to \$700 000 (that is, \$1 200 000 – \$500 000) for retrofit package R3 in Building I and zero dollars for having no project. Enter the maximum expected value at the initial decision node box at the far left of the decision tree. Use parallel slash marks to truncate the no project and Building I alternatives. The decision tree, once all values are written in, shows explicitly the economically efficient path sequence (Building II/R5 in this case) and the expected value of net benefits (\$900 000) for that path sequence.

7.6.4.16 Note that risk attitude was not addressed explicitly with utility values in this example because the state is assumed to be risk neutral. If the decision maker were risk averse or a risk taker, however, the projected earnings and costs associated with each decision branch could be converted to utility values. A utility function (see Fig. A2.5) is used to find the utility value corresponding to these benefits and costs. The averaging out to find expected values (now expected utility values) and the rolling back process are the same as described earlier. Once the alternative that maximizes utility is identified, the certain equivalent dollar value corresponding to that alternative's utility is found on the utility function. The certain equivalent value shows what the risky investment is worth, taking into consideration the decision maker's risk attitude.

7.7 *Simulation*:

7.7.1 Simulation is a well-documented technique used to determine risk exposure from an investment decision. To perform a simulation, probability functions of significant input variables must be estimated. The simulation process for building a probability density function (pdf) and cumulative distribution function (cdf) of the measure of project worth is as follows: draw a value for each input variable randomly from its probability function, substitute the set of input values for that round of draws into the formula for computing the measure of economic worth, and repeat the process over and over until a pdf and cdf can be constructed for the measure of worth.

7.7.2 For example, in analyzing the time clock, the initial energy price, the rate of energy price escalation, and the kilowatt hour savings are uncertain input variables. If each of these inputs could be described by a probability distribution, a simulation could be used to arrive at a probability distribution of the time clock's BCR (or some other measure of worth). Specifically, a random combination of each of the three variables would be selected and combined with constant inputs to compute a BCR. By repeating this random sampling over and over, typically 500 to 1000 times, and computing the BCR for each combination, a pdf and cdf can be generated for evaluating the cost effectiveness of the time clock.

7.7.3 *Construction Contingency Simulation Example*:

7.7.3.1 Contingency analysis is routinely used by cost engineers in estimating the costs of construction projects. A contingency is a cost element included in project cost estimation to cover costs that have some likelihood of occurrence, but whose amounts cannot be predicated with certainty. By adding the contingency to the line-item estimate of project cost, the cost engineer hopes to project the most likely final cost. Typical uses of contingencies are to cover possible increases in material or labor costs beyond normal escalation, unanticipated developments in applying a first-time technology, changes in project scope due to omission or error, or unforeseen work disruptions from operating in a volatile foreign country.

7.7.3.2 Contingencies are often estimated simply as a percent of the base estimate of project cost. Historical data on the differences between actual and estimated costs for similar projects can be used to determine an average percent of underestimation (or overestimation). The percent can apply to the overall project or to specific elements of the project that are estimated separately. This simple approach is typical in estimating costs of small projects. There is no distinction, however, between accounting for risk exposure and risk attitude in the contingency estimate.

7.7.3.3 For large construction projects with many uncertain variables, a sophisticated risk-analysis technique based on simulations is sometimes employed in estimating contingencies. It provides decision makers with the probabilities of cost overruns (that is, risk exposure) associated with every possible contingency markup in the relevant range. The following example adapted

from S. H. Zaheer[6] illustrates how to use simulation to measure risk exposure when making a cost estimate for a specific construction project. Note that the intent here is to show how useful simulation can be in describing risk of an investment and not to describe every step the computer program takes to do the simulation.

7.7.3.4 Construction cost is being estimated for Project X. It is expected to cost $140 million exclusive of contingencies. Of the $140 million, $60 million are spent dollars or firm commitments. Being relatively certain, they require no consideration for contingency. The other $80 million are uncertain and make up the base on which the contingency is calculated.

7.7.3.5 The process for carrying out a contingency/risk analysis is as follows. Generate subjective probability distributions for every activity that is deemed particularly uncertain. The distributions describe the percent of estimated costs of these activities, where the midpoint is 100 % of the estimated value. Enter these data, along with the estimated dollar costs of both certain and uncertain activities into a computer simulation package. It generates a probability distribution of the contingency percent of total project cost and a graph that plots probability of cost overrun against contingency percent and amount.

7.7.3.6 Fig. 5 shows how the probability of a cost overrun (that is, risk exposure) varies with the contingency adjustment for this construction project. To use the contingency/risk analysis to select a single cost estimate, the decision maker considers risk exposure and risk attitude. Risk exposure, as indicated by the rising probability of cost overrun, increases as the percent contingency markup goes down. Risk exposure decreases as the percent contingency markup increases. Risk attitude enters when the decision maker chooses a contingency amount, thereby establishing a probability of overrun that will be acceptable.

7.7.3.7 The risk neutral decision maker will choose the most likely cost estimate of $144 million, which includes the $140 million without contingency plus a contingency of $4 million (0.05·$80 million of uncertain costs). That is, the vertical axis on the right side of the graph shows the most likely contingency percent (where the probability of overrun is 50 %) to be about 5 %. This assumes an underlying probability distribution that is symmetric.

7.7.3.8 A more risk-averse decision maker might opt for a lower risk of overrun by choosing a larger contingency. For example, if an overrun probability of only 20 % were acceptable, the contingency would be $7.2 million (0.09·$80 million), and the total cost estimate would be $147.2 million ($140 million plus $7.2 million contingency).

7.7.3.9 On the other hand, a risk taker might choose a relatively low contingency. For example, if an overrun probability of 70 % were acceptable, the contingency would be $1.6 million (0.02·$80 million), and the total cost estimate would be $141.6 million ($140 million plus $1.6 million contingency). Note that Fig. 5 is really a cdf, although it differs in two respects from what is typically used in risk analysis. First, Fig. 5 has the axes reversed. The item whose distribution is being measured (contingency percent and amount) is on the vertical axis instead of the horizontal axis, and the cumulative distribution measure (probability of overrun) is on the horizontal axis. Second, the cdf in Fig. 5 measures cumulative probabilities of a value being greater than instead of less than. Thus, for example, there would be about a 35 % probability of a cost overrun with a contingency of 7 %. That is, there is a 35 % probability that the necessary contingency to avoid an overrun would be >7 %.

7.7.4 There are several advantages of the simulation technique applied to risk analysis. First, simulation works with any distribution of input variables, so it is not limited to certain classes of well-behaved distributions.

7.7.5 Second, it can handle interdependencies between inputs; that is, where one input is related to another, the two can be tied together.

7.7.6 Third, simulation can be used in generating pdfs and cdfs for any of the economic measures described in 1.3.

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[6] Zaheer, S. H., "Contingency and Capital Cost Estimates," *Cost Engineers' Notebook*, American Association of Cost Engineers, Morgantown, ~~WV (USA);~~WV, March 1983, p. 13.
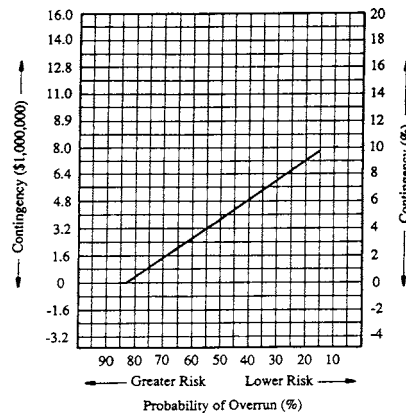
NOTE 1—Adapted from Zaheer, S. H., "Contingency and Capital Cost Estimates," *Cost Engineers' Notebook*, American Association of Cost Engineers, Morgantown, WV ~~(USA)~~, March 1983, p. 13.
FIG. 5 Contingency/Risk Graph for Construction Project X