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Sampling procedures for inspection by variables —

Part 6: Specification for single sampling plans for isolated lot inspection indexed by limiting quality (LQ)

Règles d'échantillonnage pour les contrôles par mesures —

Partie 6: Spécification pour les plans d'échantillonnage simples pour les contrôles de lots isolés, indexés d'après la qualité limite (QL)

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC *69, Applications of statistical methods,* Subcommittee SC 5, *Acceptance sampling*.

A list of all parts in the ISO 3951 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the limiting quality (LQ) for the inspection of lots where switching rules as used in ISO 3951-1 are not applicable. These switching rules provide protection to the consumer (by the prospect of switching to tightened inspection and discontinuation) and also provide an incentive to the supplier to improve the quality level. However, there are various cases where the switching rules of ISO 3951-1 are not applicable, such as isolated lots or a short series of lots.

This document is designed for the inspection of a single quality characteristic that is measurable on a continuous scale and is normally distributed, under conditions where ISO 3951-1 is not applicable, and is complementary to the attributes standard ISO 2859-2. The operating characteristic curves (OC curves) of the variables plans in this document are similar but not identical to those of the corresponding attributes plans in ISO 2859-2. The OC curves have been matched by minimizing the difference of the OC curves on condition of getting a comprehensible sample size structure (see Clause 9).

In this document, the acceptance of a lot is implicitly determined from an estimate of the fraction of nonconforming items in the process, based on a random sample of items from the lot. The objectives of the methods laid down in this document are to ensure that lots of limiting quality have a probability of acceptance about 10 % and that the probability of accepting lots with good quality is as high as practicable.

It is assumed in the main body of this document that measurement error is negligible. For information on accommodating measurement error, see Annex B, which was derived from References [24], [29] and [30].

CAUTION — The procedures in this document are not suitable for application to lots that have been screened for nonconforming items.

Inspection by variables for nonconforming items, as described in this document, includes several possible modes, the combination of which leads to a presentation that can appear quite complex to the user:

— unknown standard deviation, or known since the start of inspection;

— a single specification limit, or combined control of double specification limits.

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge of both statistics and the product to be inspected. Clause 5 is intended to help those responsible for specifying sampling plans in making this choice. It suggests the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

The basic definitions and notations are provided by Clauses 3 and 4. The basic operational rules are contained in Clauses 5 through 8. Clause 9 informs about the relations between this document and the attributes sampling standard ISO 2859-2. Clauses 10 and 11 provide background on accounting for measurement uncertainty and the underlying normality assumption. All tables needed for the sampling procedure can be found in Clause 12, and examples for the s–method and the σ–method for both single and double specification limits can be found in Clause 13.

Nine annexes are provided. Annex A indicates how the sample standard deviation, s, and the presumed known value of the process standard deviation, σ, should be determined. Annex B provides procedures for accommodating measurement uncertainty. Annex C shows five different sampling strategies. Annex D gives the general formula for the operating characteristics of the s–method. Annex E gives the general formula for the operating characteristics of the σ–method. Annex F gives the theory underlying the calculation of consumer’s risks. Annex G gives the theory underlying the calculation of producer’s risk quality. Annex H gives details of how acceptance diagrams for double specification limits are constructed. Annex I gives a description of the use of the underlying software, R package ISO 3951, to support implementation of this document.

Sampling procedures for inspection by variables —

Part 6: Specification for single sampling plans for isolated lot inspection indexed by limiting quality (LQ)

1 Scope

This document specifies an acceptance sampling system of single sampling plans for inspection by variables, primarily designed for use under the following conditions:

a) where the inspection procedure is applied to an isolated lot of discrete products all supplied by one producer using one production process;

b) where only a single quality characteristic, x, of this process is taken into consideration, which is measurable on a continuous scale;

c) where the quality characteristic, x, is distributed according to a normal distribution or a close approximation to a normal distribution;

d) where the quality characteristic can be measured without error or with moderate measurement error;

e) where a contract or standard defines a lower specification limit, L, an upper specification limit, U, or both; an item is qualified as conforming if and only if its measured quality characteristic, x, satisfies the appropriate one of the following inequalities:

   1) $x \geq L$ (i.e. the lower specification limit is not violated);

   2) $x \leq U$ (i.e. the upper specification limit is not violated);

   3) $x \geq L$ and $x \leq U$ (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are cases with a single specification limit, whereas inequality 3) is a case with double specification limits.

Where double specification limits apply, it is assumed in this document that conformance to both specification limits is equally important to the integrity of the product. In such cases, it is appropriate to apply a single LQ to the combined fraction of a product outside the two specification limits. This is referred to as combined control.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 2859-2, *Sampling procedures for inspection by attributes — Part 2: Sampling plans indexed by limiting quality (LQ) for isolated lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 2859-2, ISO 3534-1, and ISO 3534-2 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
inspection by variables
inspection by measuring the magnitude of a characteristic of an item

[SOURCE: ISO 3534-2:2006, 4.1.4, modified — "the magnitude(s) of the characteristic(s)" replaced with "the magnitude of a characteristic".]

3.2
sampling inspection
inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2:2006, 4.1.6]

3.3
acceptance sampling inspection
acceptance sampling
sampling inspection (3.2) to determine whether or not to accept a lot or other amount of product, material, or service

[SOURCE: ISO 3534-2:2006, 4.1.8, modified — "acceptance sampling" added as second preferred term; original definition, "acceptance inspection where the acceptability is determined by means of sampling inspection" replaced with the current one.]

3.4
acceptance sampling inspection by variables
acceptance sampling inspection (3.3) in which the acceptance of a lot is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

[SOURCE: ISO 3534-2:2006, 4.2.11, modified — “the process” replaced by “a lot”, and “on specified quality characteristics of each item in a sample from a lot” has been replaced by “from inspection by variables”]

3.5
process fraction nonconforming
rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

3.6
quality level
quality expressed as the fraction nonconforming

3.7
consumer's risk
CR
probability of acceptance when the *quality level* (3.6) has a value stated by the acceptance sampling plan as unsatisfactory

Note 1 to entry: For the purposes of this document, the consumer's risk is approximately 10 %.

[SOURCE: ISO 3534-2:2006, 4.6.2, modified — Deleted symbol β; original Note 1 to entry replaced with the current one.]

3.8
consumer's risk quality
CRQ
quality level (3.6) of a lot or process which, in the acceptance sampling plan, corresponds to a specified *consumer's risk* (3.7)

Note 1 to entry: For the purposes of this document, the consumer's risk quality is the *limiting quality* (3.9).

[SOURCE: ISO 3534-2:2006, 4.6.9, modified — Deleted symbol Q_{CR}; original Note 1 to entry replaced with the current one.]

3.9
limiting quality
LQ
quality level (3.6), when a lot is considered in isolation, which, for the purposes of *acceptance sampling inspection* (3.3), is limited to a low probability of acceptance

[SOURCE: ISO 3534-2:2006, 4.6.13]

3.10
producer's risk
PR
probability of non-acceptance when the *quality level* (3.6) has a value stated by the plan as acceptable

Note 1 to entry: For the purposes of this document, the producer's risk is 5 %.

[SOURCE: ISO 3534-2:2006, 4.6.4, modified — Deleted symbol α; original Notes 1 and 2 to entry replaced with the current one.]

3.11
producer's risk quality
PRQ
quality level (3.6) of a lot or process which, in the acceptance sampling plan, corresponds to a specified *producer's risk* (3.10)

[SOURCE: ISO 3534-2:2006, 4.6.10, modified — Deleted symbol Q_{PR}; deleted Notes 1 and 2 to entry.]

3.12
nonconformity
non-fulfilment of a requirement

[SOURCE: ISO 3534-2:2006, 3.1.11]

3.13
***s*–method acceptance sampling plan**
***s*–method**
acceptance sampling (3.3) plan by variables using the sample standard deviation.

Note 1 to entry: See Clause 6.

[SOURCE: ISO 3534-2:2006, 4.3.10, modified – "*s* method" has been replaced by "*s*-method", "acceptance sampling plan" has been added; "*s*-method" left as a second preferred term; in the definition, "acceptance sampling inspection by variables" replaced with "acceptance sampling plan by variables"; added Note 1 to entry.]

3.14
σ–method acceptance sampling plan
σ–method
acceptance sampling (3.3) plan by variables using the presumed value of the process standard deviation

Note 1 to entry: See Clause 7.

[SOURCE: ISO 3534-2:2006, 4.3.9, modified — "sigma method" has been replaced with "σ-method"; "acceptance sampling plan" has been added with "σ-method" left as a second preferred term; in the definition, "acceptance sampling inspection by variables" replaced with "acceptance sampling plan by variables"; added Note 1 to entry.]

3.15
specification limit
conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2:2006, 3.1.3, modified — "limiting value stated" replaced with " conformance boundary specified".]

3.16
lower specification limit
L
specification limit (3.15) that defines the lower conformance boundary

[SOURCE: ISO 3534-2:2006, 3.1.5, modified — " limiting value" replaced with " conformance boundary".]

3.17
upper specification limit
U
specification limit (3.15) that defines the upper conformance boundary

[SOURCE: ISO 3534-2:2006, 3.1.4, modified — " limiting value" replaced with " conformance boundary".]

3.18
combined control
requirement when both upper and lower limits are specified for the quality characteristic and an *LQ* (3.9) that applies to the combined fraction nonconforming beyond the two limits is given

Note 1 to entry: The use of combined control implies that *nonconformity* (3.12) beyond either *specification limit* (3.15) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.19
form *k* acceptance constant
k
constant depending on the specified value of the *limiting quality* (3.9) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

Note 1 to entry: See Clauses 6 and 7.

[SOURCE: ISO 3534-2:2006, 4.4.4, modified – "acceptability constant" has been replaced with " form k acceptance constant"; "value of the acceptance quality limit" replaced with "value of the limiting quality"; added Note 1 to entry.]

3.20
form p^* acceptance constant
p^*
constant depending on the specified value of the *limiting quality* (3.9) and the sample size, used in the criteria for accepting the lot in an *acceptance* (3.3) plan by variables

Note 1 to entry: See Clause 8.

[SOURCE: ISO 3534-2:2006, 4.4.4, modified — “acceptability constant” has been replaced with “form p* acceptance constant”; “value of the acceptance quality limit" replaced with "value of the limiting quality"; added Note 1 to entry.]

3.21
lower quality statistic
Q_L
function of the *lower specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single lower specification limit , the lot is sentenced on the result of comparing Q_L with the *form k acceptance constant* (3.19) *k*.

Note 2 to entry: See Clauses 6 and 7.

[SOURCE: ISO 3534-2:2006, 4.4.11, modified — In the Note 1 to entry, "acceptability constant" has been replaced with "form k acceptance constant"; Note 2 to entry added.]

3.22
upper quality statistic
Q_U
function of the *upper specification limit* (3.17), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single upper specification limit , the lot is sentenced on the result of comparing Q_U with the *form k acceptance constant* (3.19) *k*.

Note 2 to entry: See Clauses 6 and 7.

[SOURCE: ISO 3534-2:2006, 4.4.10, modified — In the Note 1 to entry, "acceptability constant" has been replaced with "form k acceptance constant"; Note 2 to entry added.]

3.23
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample size and *LQ* (3.9) for which it is possible to satisfy the acceptance criterion for double *specification limits* (3.15) with a combined *LQ* (3.9) when the process variability is known

[SOURCE: ISO 3534-2:2006, 4.4.8, modified — Added symbol σ_{max}; "or a given sample size code letter and AQL" replaced with "for a given sample size and LQ "; "for a double specification limit under all inspection severities (i.e. normal, tightened and reduced) when the process variability is known" replaced with "for double specification limits with a combined *LQ* when the process variability is known"; Note 1 to entry deleted.]

3.24
measurement
set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2:2006, 3.2.1, modified – "having the object of determining a value of a quantity" replaced with "to determine the value of some quantity".]

4 Symbols

f_σ factor that relates the maximum process standard deviation to the difference between U and L (see Table 3)

$F_{BETA(\alpha,\beta)}(x)$ the distribution of the standard beta distribution with parameters α and β. In this document $\alpha = \beta = n/2 - 1$ throughout.

$F_{t(\nu,\delta)}(x)$ the distribution function of the non-central *t*-distribution with ν degrees of freedom and non-centrality parameter δ

K_p the upper *p*-quantile of the standardized normal distribution, i.e. *x* such that $1 - \Phi(x) = p$, which corresponds to the process fraction nonconforming *p*

k form *k* acceptance constant for use with a single quality characteristic and a single specification limit (see Table 2 for the *s*–method or Table 4 for the *σ*–method)

L lower specification limit (as a subscript to a variable, it denotes its value at *L*)

n sample size (number of items in a sample)

P_a probability of acceptance

p lot quality in fraction nonconforming

$\hat{p}$ estimate of the process fraction nonconforming

$\hat{p}_L$ estimate of the process fraction nonconforming below the lower specification limit

$\hat{p}_U$ estimate of the process fraction nonconforming above the upper specification limit

p^* form *p** acceptance constant, i.e. the maximum acceptable value for the estimate of the process fraction nonconforming (see Table 5)

$\Phi(x)$ the distribution function of the standardized normal distribution.

Q_L lower quality statistic

NOTE Q_L is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.

Q_U upper quality statistic

NOTE Q_U is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.

s sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j=1}^{n} (x_j - \bar{x})^2}{n-1}}$$

(see Annex A)

σ standard deviation of a process that is under statistical control

NOTE 1 σ^2, the square of the process standard deviation, is known as the process variance.

σ_{max} maximum process standard deviation (MPSD) (see Table 3)

U upper specification limit (as a subscript to a variable, it denotes its value at U)

x_j measured value of the quality characteristic for the j^{th} item of the sample

$\bar{x}$ Sample arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n}$$

(see Annex A)

5 Choice of a sampling plan

5.1 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time consuming and costly per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely how good the product is.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items and vice versa (see 13.2 Example 2 a and Example 2 b).

d) From a comparison of the size of the samples required for the same LQ from standard plans for inspection by attributes, such as from ISO 2959, and the standard plans in this document, the smallest samples are generally required by the σ–method (used when the process standard deviation is presumed to be known). The sample sizes for the s–method (used when the process standard deviation is presumed to be unknown) are larger than for the σ-method but are, in general, substantially smaller than for sampling by attributes.

e) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

f) For two or more quality characteristics, ISO 3951 series does not contain specifications for sampling plans indexed by LQ.

5.2 General

The following procedures shall be followed in advance of the inspection by variables.

a) Specify the limiting quality (LQ) in accordance with 5.4.

b) Determine the lot size (N).

c) Determine the quality characteristic x and an upper limit U and/or a lower limit L for x.

d) For a quality characteristic with double specification limits, check that nonconformities beyond each limit are of equal importance.

e) Check whether the *s*–method (Clause 6) should be used or whether the standard deviation is stable and known, in which case the σ–method (Clause 7) should be used (see 5.3);

f) for the σ–method and a quality characteristic with double specification limits, a process capability study in the following sense should be done:

   1) enter Table 3 with the LQ to determine the value of the factor f_σ;

   2) calculate the maximum allowable value of the process standard deviation using the formula $\sigma_{\max} = (U - L) f_\sigma$;

   3) If σ exceeds $\sigma_{\max}$, the process is not capable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.

With the specified lot size and the limiting quality as indexing values, the sample size n and the acceptance constant k are given in Table 2 (*s*-method) or Table 4 (σ-method).

Although the primary index is the limiting quality, the producer/supplier needs guidance on the quality level required if lots are to have a high probability of acceptance.

5.3 Choice between the *s*-method and the σ-method

If it is desired to apply inspection by variables as proposed in this document, the decision shall be made whether to use the *s*–method or the σ–method. The σ–method is the more economical in terms of sample size, but before this method can be applied, it is necessary to have a reliable value of σ, usually obtained from previous process analyses.

In case no reliable assumptions on the value of σ can be made, it is necessary to use the *s*–method.

5.4 Choice of the limiting quality (LQ)

The purpose of this document is to guard against unsatisfactory quality. The determination of unsatisfactory quality is generally a decision that should be made by quality management. The choice of the LQ is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service. In this document, the LQ is the parameter used to protect against unsatisfactory quality. The sampling plans in this document have a probability of accepting the lot at the LQ of about 10 %. In this document, the sampling tables are indexed by a set of specified LQ values.

If the user's chosen LQ value is not among those specified in Table 1, then an applicable LQ value shall be the specified LQ corresponding to the range containing the user's chosen LQ, which is the closest specified LQ below the user's chosen LQ (see Example).

Table 1 — Specified LQ values

Limiting quality (LQ) in percent nonconforming					
range	$0{,}05 \leq LQ < 0{,}08$	$0{,}08 \leq LQ < 0{,}125$	$0{,}125 \leq LQ < 0{,}2$	$0{,}2 \leq LQ < 0{,}315$	$0{,}315 \leq LQ < 0{,}5$
specified	0,05	0,08	0,125	0,2	0,315
range	$0{,}5 \leq LQ < 0{,}8$	$0{,}8 \leq LQ < 1{,}25$	$1{,}25 \leq LQ < 2$	$2 \leq LQ < 3{,}15$	$3{,}15 \leq LQ < 5$
specified	0,5	0,8	1,25	2	3,15
range	$5 \leq LQ < 8$	$8 \leq LQ < 12{,}5$	$12{,}5 \leq LQ < 20$	$20 \leq LQ < 31{,}5$	$31{,}5 \leq LQ$
specified	5	8	12,5	20	31,5