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Control charts —

Part 2:
Shewhart control charts

Cartes de contrôle —

Partie 2: Cartes de contrôle de Shewhart

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This second edition cancels and replaces the first edition (ISO 7870-2:2013), which has been technically revised.

The main changes are as follows:

— various clauses have been modified for better understanding;

— some examples for control charts have been modified;

— new examples for control charts have been included.

A list of all parts in the ISO 7870 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

A traditional approach to manufacturing has been to depend on production to make the product and on quality control to inspect the final product and screen out items not meeting specifications. This strategy of detection is often wasteful and uneconomical because it involves after-the-event inspection when the wasteful production has already occurred. Instead, it is much more effective to institute a strategy of prevention to avoid waste by not producing unusable output in the first place. This can be accomplished by gathering process information and analysing it so that timely action can be taken on the process itself.

Dr. Walter Shewhart in 1924 developed the control chart method for controlling the quality during production. Control chart theory recognizes two kinds of variability. The first kind is random variability (also known as natural/inherent/uncontrollable variation) arising due to causes known as chance/common/random causes. This is due to the wide variety of causes that are consistently present and not readily identifiable, each of which constitutes a very small component of the total variability but none of them contributes any significant amount. Nevertheless, the sum of the contributions of all of these unidentifiable random causes is measurable and is assumed to be inherent to the process. The elimination or correction of common causes may well require a decision to allocate resources to fundamentally change the process and system.

The second kind of variability represents a real change in the process. Such a change can be attributed to some identifiable causes that are not an inherent part of the process and which can, at least theoretically, be eliminated. These identifiable causes are referred to as “assignable causes” (also known as special/unnatural/systematic/controllable causes) of variation. They may be attributable to such matters as the lack of uniformity in material, a broken tool, workmanship or procedures, the irregular performance of equipment, or environmental changes.

A process is said to be in a state of statistical control, or simply “in control”, if the process variability results only from random causes. Once this level of variation is determined, any deviation from this level is assumed to be the result of assignable causes that should be identified and eliminated.

The major statistical tool used to do this is the control chart, which is a method of presenting and comparing information based on a sequence of observations representing the current state of a process against limits established after consideration of inherent process variability. The control chart method helps first to evaluate whether a process has attained, or continues in, a state of statistical control. When the process is deemed to be stable and predictable, then further analysis regarding the ability of the process to satisfy the requirements of the customer may be conducted. The control chart also can be used to provide a continuous record of a quality characteristic of the process output while process activity is ongoing. Control charts aid in the detection of unnatural patterns of variation in data resulting from repetitive processes and provide criteria for detecting a lack of statistical control. The use of a control chart and its careful analysis leads to a better understanding of the process and will often result in the identification of ways to make valuable improvements.

Control charts —

Part 2: Shewhart control charts

1 Scope

This document establishes a guide to the use and understanding of Shewhart control chart approach to the methods for statistical control of a process.

This document is limited to the treatment of statistical process control methods using only Shewhart system of charts. Some supplementary material that is consistent with Shewhart approach, such as the use of warning limits, analysis of trend patterns and process capability is briefly introduced. However, there are several other types of control charts which can be used in different situations.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

3.1 General presence

For the purposes of this document, the terms and definitions given in ISO 3534-2 apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.2 Symbols

NOTE The ISO/IEC Directives make it necessary to depart from common SPC usage in respect to the differentiation between abbreviated terms and symbols. In ISO standards an abbreviated term and its symbol can differ in appearance in two ways: by font and by layout. To distinguish between abbreviated terms and symbols, abbreviated terms are given in Cambria upright and symbols in Cambria or Greek italics, as applicable. Whereas abbreviated terms can contain multiple letters, symbols consist only of a single letter. For example, the conventional abbreviation of upper control limit, UCL, is valid but its symbol in equations becomes U_{CL}. The reason for this is to avoid misinterpretation of compound letters as an indication of multiplication.

3.2.1 For the purposes of this document, the following symbols apply

n Subgroup size; the number of sample observations per subgroup

k Number of subgroups

L Lower specification limit

L_{CL} Lower control limit

L_{CLi} Lower control limit for i^{th} subgroup

CL Centre line

U_{CL} Upper control limit

U_{CLi} Upper control limit for i^{th} subgroup

X Measured quality characteristic (individual values are expressed as (X_1, X_2, X_3,...). Sometimes the symbol Y is used instead of X

$\bar{X}$ (X bar) Subgroup average

$\bar{\bar{X}}$ (X double bar) Average of the subgroup averages

μ True process mean

μ_0 A given or prespecified value of μ

σ True process standard deviation

σ_0 A given or prespecified value of σ

$\tilde{X}$ Median of a subgroup

$\bar{\tilde{X}}$ Average of the subgroup medians

R Subgroup range

$\bar{R}$ Average of subgroup ranges

R_m Subgroup moving range

$\bar{R}_m$ Average moving range

s Subgroup sample standard deviation

$\bar{s}$ Average of subgroup sample standard deviations

p Proportion of nonconforming items in a subgroup

$\bar{p}$ Average proportion of nonconforming items for all subgroups

np Number of nonconforming items in a subgroup

p_0 A given value of p

np_0 A given value of np (for a given p_0)

c Number of nonconformities in a subgroup

c_0 A given value of c

$\bar{c}$ Average number of nonconformities for all subgroups

u Number of nonconformities per unit in a subgroup

$\bar{u}$ Average number of nonconformities per unit

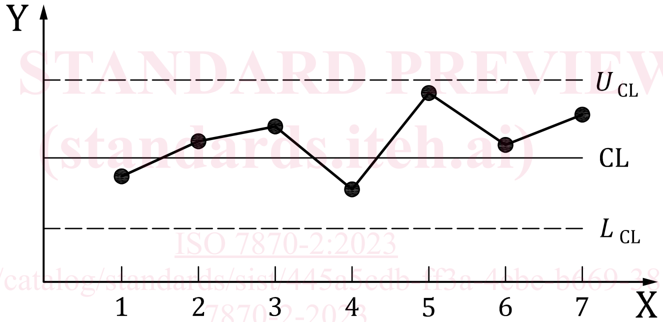
4 Concepts of Shewhart control charts

4.1 Shewhart control chart

A Shewhart control chart is a chart that is used to display a statistical measure (also called 'statistic') obtained from either variables or attribute data. The control chart requires data from rational subgroups (see 11.3) to be taken at approximately regular intervals from the process. The intervals may be defined in terms of time (for example hourly) or quantity (every lot). Usually, the data are obtained from the process in the form of samples or subgroups consisting of the same process characteristic, product or service with the same measurable units and the same subgroup size. From each subgroup, one or more statistical measures are calculated, such as average, $\bar{X}$, range, R, standard deviation, s, proportion of nonconforming items p, and number of nonconformities, c.

4.2 Control limits

Shewhart control chart is a chart on which some statistical measure of the values in each subgroup is plotted against subgroup number. It consists of centre line, CL, which is usually the average value of the statistical measure being considered or may be based on past experience, when the process is in state of statistical control. It may also be based on product or service target values. The control chart has two statistically determined limit lines, one on either side of the centre line, which are called the upper control limit, U_{CL}, and the lower control limit, L_{CL}, (see Figure 1).



Key

X subgroup number
Y statistic
CL centre line
L_{CL} lower control limit
U_{CL} upper control limit

Figure 1 — Outline of a control chart

4.3 Process in statistical control

4.3.1 The upper and lower control limits on the control chart, on each side of the centre line, are typically placed at a distance of three times the standard deviation of the statistic (3 σ) being plotted. If large number of observations from a process in statistical control are studied in form of frequency distribution, it often shows a bell shaped symmetrical pattern, which is well represented as normal distribution.

4.3.2 Placing the limits too close to the centre line will result in many searches for non-existing problems and yet placing the limits too far apart will increase the risk of not detecting process problems when they do exist. Under an assumption that the plotted statistic is approximately normally distributed 3 σ limits indicate that approximately 99,73 % of the values of the statistic will be included within the control limits, provided the process is in statistical control. Interpreted another way, there is a 0,27 % probability, or about three out of thousand plotted points will be out of the upper or lower control limit when the process is in control. The word “approximately” is used because deviations from underlying assumptions such as the distributional form of the data will affect the probability values. In fact, the choice of k σ limits, instead of 3 σ limits, depends on costs of investigation and taking appropriate action vis-à-vis consequences of not taking action.

4.4 Action limits

The possibility that a violation of the limits is really a chance event rather than a real signal is considered so small that when a point appears outside of the limits, action should be taken. Since action is required at this point, the 3 σ control limits are sometimes called the “action limits”.

4.5 Warning limits

Sometimes it is advantageous to mark 2 σ limits on the chart also. Then, any sample value falling beyond the 2 σ limits can serve as a warning of an impending out-of-control situation. As such, the 2 σ limits are sometimes called “warning limits”. While no action is required as a result of such a warning on the control chart, some users may wish to immediately select another subgroup of the same size to determine if corrective action is needed.

4.6 Type 1 error

When assessing the status of a process using control charts, two types of errors are possible. The first occurs when the process is actually in a state of control but a plotted point falls outside the control limits due to chance (Type 1 error). As a result, the chart has given a false signal resulting in an incorrect conclusion that the process is out of control. A cost is then incurred in an attempt to find the cause of a non-existent problem.

If normality is assumed and 3 σ control limits are used, the probability of Type 1 error is 0,27 %. In other words, this error will happen only about 3 times in 1 000 samples when the process is in control.

4.7 Type 2 error

4.7.1 The second error occurs when the process involved is not in control but the plotted point falls within the control limits due to chance (Type 2 error). In this case, the chart provides no signal and it is incorrectly concluded that the process is in statistical control. There may also be a substantial cost associated with failing to detect that a change in the process location or variability has occurred, the result of which might be the production of nonconforming output. The risk of this type of error occurring is a function of three things: the width of the control limits, the sample size, and the degree to which the process is out of control. In general, because the magnitude of the change in the process cannot be known, little can be determined about the actual size of the risk of this error.

4.7.2 Because it is generally impractical to make a meaningful estimate of probability of Type 2 error in any given situation, Shewhart control chart system is designed to control the risk (or probability) of Type 1 error.

4.8 Process not in control

When a plotted value falls outside of either control limit, or a series of values display an unusual pattern such as discussed in Clause 8 and Annex B, the state of statistical control can no longer be accepted. When this occurs, an investigation is initiated to locate the assignable cause, and the process may be stopped or adjusted. Once the assignable cause is determined and eliminated, the process is ready to continue. As discussed in 4.3.2, on rare occasions when no assignable cause can be found and it must be concluded that the point outside the limits represents the occurrence of a rare event, a false signal, which has resulted in a value outside of the control limits even though the process is in control.

NOTE Point on the control line is considered as point in control.

4.9 Phase 1 of statistical process control

When a process is to be studied for the first time with the objective of bringing the process in a state of statistical control, it is often found necessary to use historical data that has previously been obtained from the process or to undertake to obtain new data from a series of samples before attempting to establish the control chart. This retrospective stage during which the control chart parameters are being established is often referred to as Phase 1. Sufficient data will need to be found in order to obtain reliable estimates of the centre line and control limits for the control charts. The control limits established in Phase 1 are trial control limits as they are based upon data collected when the process may not be in control. The identification of the precise causes for signals given by the control chart at this stage may prove to be difficult because of the lack of information about the historical operating characteristics of the process. However, when special causes of variation can be identified and corrective action taken, the retrospective data from the process when under the influence of the special cause should be removed from consideration and the control chart parameters re-determined. This iterative procedure is continued until the final trial control chart shows no signals and the control limits then correspond to the process in control. Because some data may have to be removed from consideration during Phase 1, some additional data may have to obtained from the process to maintain the reliability of the parameter estimates.

4.10 Phase 2 of control charts

Once statistical control has been established, the revise control limits in Phase 1 are taken as the control limits for the ongoing monitoring of the process. The objective now, in what is referred to as Phase 2, is the maintenance of the process in a state of control as well as the rapid identification of special causes that may affect the process from time to time. It should be recognized that moving from Phase 1 to Phase 2 may be time consuming and difficult. However, it is critical, because failure to remove special causes of variation will result in overestimation of the process variation. In this case the control chart will have control limits that are set too wide apart resulting in a control chart that is not sufficiently sensitive for detecting the presence of special causes.

5 Types of control charts

5.1 Types of Shewhart control charts

5.1.1 Shewhart control charts are of following two types:

a) variables control charts;

b) attribute control charts.

5.1.2 For each of these control charts, there are two distinct situations:

a) when no pre-specified process parameters values are given;

b) when pre-specified process parameters values are given.

5.2 Control charts where no pre-specified values of process parameters are given

The purpose is to identify whether the values of the statistics, which are being plotted on the control charts for different subgroups, differ from the centre line by an amount greater than that can be attributed to chance causes only. Control charts will be constructed using only the data collected from samples from the process. The control charts are used for detecting those variations caused other than by chance with the purpose being to bring the process in a state of statistical control.

5.3 Control charts with respect to given pre-specified values of process parameters

5.3.1 The purpose is to identify whether the observed values of $\overline{X}$, s, etc., for several subgroups of n observations each, differ from the respective given values of μ_0, σ_0, etc. by amounts greater than that expected to be due to chance causes only. The difference between charts with given parameter values and those where no pre-specified values are given, is the additional requirement concerning the determination of the location of the centre and variation of the process. The pre-specified values may be based on experience obtained by using control charts with no prior information or specified values. They may also be based on economic values established upon consideration of the need for service and cost of production or be nominal values designated by the product specifications.

5.3.2 Preferably, the specified values should be determined through an investigation of preliminary data that is supposed to be typical of all future data. The specified values should be compatible with the inherent process variability for effective functioning of the control charts. Control charts based on such pre-specified values are used particularly during process operation to control processes and to maintain product or service uniformity at the desired level.

5.4 Types of variables and attribute control charts

5.4.1 Variables control charts

The following control charts for variables are considered when measurements are on continuous scales:

a) average, $\overline{X}$ chart, and range, R chart, or standard deviation, s chart;

b) individuals, X chart and moving range, R chart;

c) median, $\tilde{X}$ chart and range, R chart.

5.4.2 Attribute control charts

The following attribute control charts are used when items are classified as conforming and nonconforming or number of nonconformities are counted on the items:

a) p chart for proportion of nonconforming items, when sample size is not constant;

b) np chart for number of nonconforming items when the sample size is constant.

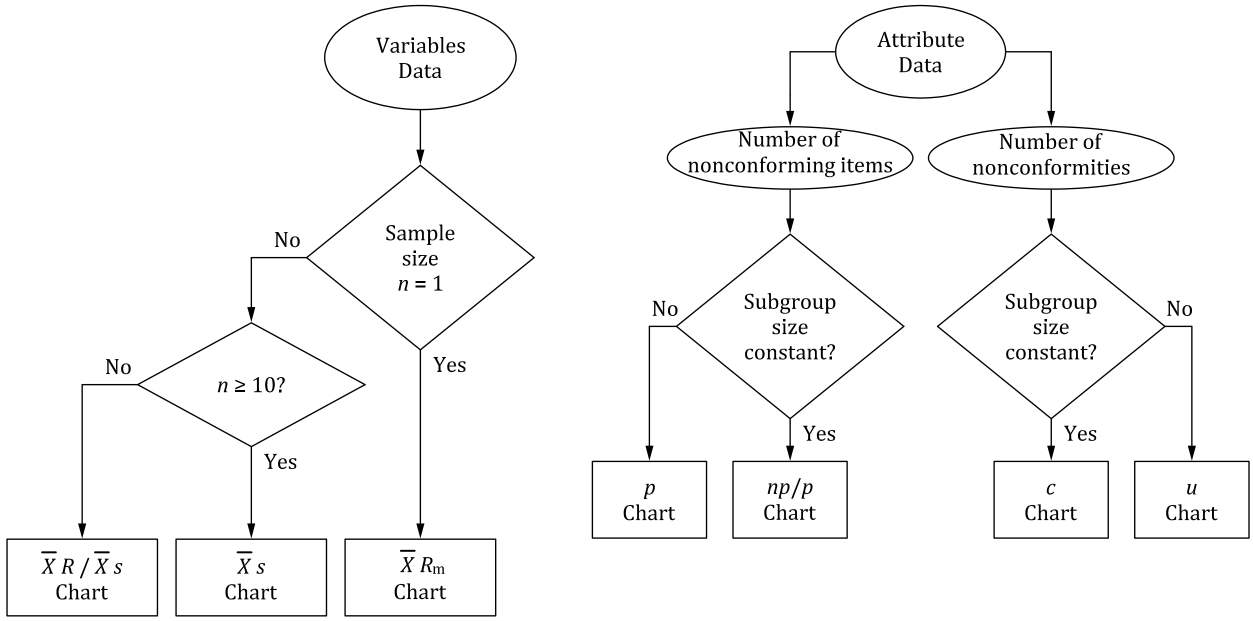
NOTE p chart can also be used in such a case. As it involves additional calculation to find p value for each subgroup for plotting them on p chart, and the result being the same as that of np chart; it is recommended to use np chart when sample size is constant.

c) c chart for number of nonconformities when the sample size is constant;

NOTE u chart can also be used in such a case. As it involves additional calculation to find u value for each subgroup for plotting them on u chart, and the result being the same as that of c chart; it is recommended to use c chart when sample size is constant.

d) u chart for the number of nonconformities per unit when the sample size is not constant.

Figure 2 shows a process of selecting an appropriate control chart for a given situation.



a) Variables data **b) Attribute data**

Figure 2 — Types of control charts

6 Variables control charts

6.1 Usefulness of variables control charts

Control charts for variables are particularly useful for several reasons including the following:

a) Most processes, and their output, have characteristics that are measurable, hence generate variables data, so the potential applicability is broad.

b) Variables charts are more informative than attribute charts since specific information about the process average and variation is obtained directly.

c) Although obtaining information on variables data for one item is more costly than that for attribute data, the subgroup sizes needed for variables data are smaller than those for attribute data, for an equivalent monitoring efficiency. This helps to reduce the total inspection cost and to shorten the time gap between the occurrence of a process problem and its corrective action.

d) Variables charts provide visual means to directly assess process performance regardless of the specifications.

6.2 Assumption of normality

For all variables control charts considered in this document, it is assumed that the distribution of the quality characteristic is normal. The factors used for computing control limits are derived using the assumption of normality. So, departure from this assumption will affect the performance of the charts. Since most control limits are used as empirical guides in making decisions, reasonably small departures from normality should not be of concern. In any case, because of the central limit theorem, averages tend to be normally distributed even when individual observations are not; this makes it reasonable for evaluating control to assume normality for $\overline{X}$ charts, even for sample sizes as small as 4 or 5. When dealing with individual observations for capability study purposes, the true form of the distribution is important. Periodic checks on the continuing validity of such assumptions are advisable, particularly for ensuring that only data from a single population are being used. It should be noted that the distributions of the ranges and standard deviations are not normal. Although normality is necessarily assumed in the determination of the constants for the calculation of control limits for the range or standard deviation chart, moderate deviations from normality of the process data should not be of major concern in the use of these charts as an empirical decision procedure.

6.3 Pair of control charts

6.3.1 As normality is assumed for variables type of data, and normal distribution has two parameters, namely, mean and standard deviation; a pair of control charts is prepared and analysed together, one for controlling variation of the process and the other for process mean. So, variables charts can describe process data in terms of both process variability (spread) and process average (location). Average, $\overline{X}$ chart is commonly used to control location and range, R chart to control inherent variability.

6.3.2 Each chart can be plotted using either estimated control limits, in which case limits are based on the information contained in the sample data plotted on the chart, or pre-specified control limits based on adopted specified values applicable to the statistical measures plotted on the chart.

6.3.3 The chart for spread is analysed first, since it provides the rationale and justification for the estimation of the process standard deviation. The resulting estimate of the process standard deviation is then be used in establishing control limits for the chart for location.

6.4 Average, $\overline{X}$ chart and range, R chart or average, $\overline{X}$ chart and standard deviation, s chart

$\overline{X}$ and R control charts can be used when subgroup sample size is small or moderately small, usually less than 10. $\overline{X}$ and s control charts are preferable in the case of large subgroup sample sizes ($n \geq 10$), since the range becomes increasingly less efficient in estimating the process standard deviation when the sample size gets larger. Where software is available to calculate process limits, standard deviation chart is preferable. Table 1 and Table 2 give the control limit formulae and the factors for each of these variables control charts.

Table 1 — Control limit formulae for average, range and standard deviation

Statistic	Estimated control limits		Pre-specified control limits	
	Centre line	U_{CL} and L_{CL}	Centre line	U_{CL} and L_{CL}
$\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{X}} \pm A_2\overline{R}$ and $\overline{\overline{X}} \pm A_3\overline{s}$	μ_0	$\mu_0 \pm A\sigma_0$
R	$\overline{R}$	$D_4\overline{R}, D_3\overline{R}$	$d_2\sigma_0$	$D_2\sigma_0, D_1\sigma_0$
s	$\overline{s}$	$B_4\overline{s}, B_3\overline{s}$	$c_4\sigma_0$	$B_6\sigma_0, B_5\sigma_0$
NOTE μ_0 and σ_0 are given values of parameters.				

Table 2 — Factors for computing control chart lines

subgroup size	Factors for control limits											Factors for centre line	
n	$\overline{X}$ chart			s chart				R chart[a]				Using s	Using R[a]
	A	A_2	A_3	B_3	B_4	B_5	B_6	D_1	D_2	D_3	D_4	c_4	d_2
2	2,121	1,880	2,659	0	3,267	0	2,606	0	3,686	0	3,267	0,798	1,128
3	1,732	1,023	1,954	0	2,568	0	2,276	0	4,358	0	2,575	0,886	1,693
4	1,500	0,729	1,628	0	2,266	0	2,088	0	4,698	0	2,282	0,921	2,059
5	1,342	0,577	1,427	0	2,089	0	1,964	0	4,918	0	2,114	0,940	2,326
6	1,225	0,483	1,287	0,030	1,970	0,029	1,874	0	5,079	0	2,004	0,952	2,534
7	1,134	0,419	1,182	0,118	1,882	0,113	1,806	0,205	5,204	0,076	1,924	0,959	2,704
8	1,061	0,373	1,099	0,185	1,815	0,179	1,751	0,388	5,307	0,136	1,864	0,965	2,847
9	1,000	0,337	1,032	0,239	1,761	0,232	1,707	0,547	5,394	0,184	1,816	0,969	2,970
10	0,949	0,308	0,975	0,284	1,716	0,276	1,669	0,686	5,469	0,223	1,777	0,973	3,078
11	0,905	0,285	0,927	0,321	1,679	0,313	1,637	0,811	5,535	0,256	1,744	0,975	3,173
12	0,866	0,266	0,886	0,354	1,646	0,346	1,610	0,923	5,594	0,283	1,717	0,978	3,258
13	0,832	0,249	0,850	0,382	1,618	0,374	1,585	1,025	5,647	0,307	1,693	0,979	3,336
14	0,802	0,235	0,817	0,406	1,594	0,399	1,563	1,118	5,696	0,328	1,672	0,981	3,407
15	0,775	0,223	0,789	0,428	1,572	0,421	1,544	1,203	5,740	0,347	1,653	0,982	3,472
16	0,750	0,212	0,763	0,448	1,552	0,440	1,526	1,282	5,782	0,363	1,637	0,984	3,532
17	0,728	0,203	0,739	0,466	1,534	0,458	1,511	1,356	5,820	0,378	1,622	0,985	3,588
18	0,707	0,194	0,718	0,482	1,518	0,475	1,496	1,424	5,856	0,391	1,609	0,985	3,640
19	0,688	0,187	0,698	0,497	1,503	0,490	1,483	1,489	5,889	0,404	1,596	0,986	3,689
20	0,671	0,180	0,680	0,510	1,490	0,504	1,470	1,549	5,921	0,415	1,585	0,987	3,735
21	0,655	0,173	0,663	0,523	1,477	0,516	1,459	1,606	5,951	0,425	1,575	0,988	3,778
22	0,640	0,167	0,647	0,534	1,466	0,528	1,448	1,660	5,979	0,435	1,565	0,988	3,819
23	0,626	0,162	0,633	0,545	1,455	0,539	1,438	1,711	6,006	0,443	1,557	0,989	3,858
24	0,612	0,157	0,619	0,555	1,445	0,549	1,429	1,759	6,032	0,452	1,548	0,989	3,895
25	0,600	0,153	0,606	0,565	1,435	0,559	1,420	1,805	6,056	0,459	1,541	0,990	3,931

[a] Not recommended for sample size $n \geq 10$.

6.5 Control chart for individuals, X, and moving ranges, R_m

6.5.1 In these charts, only one sample from each subgroup is drawn. This is applicable in situations, where it is either impossible or impractical to draw more samples, like, the characteristic to be controlled is destructive and the item being very costly. In some situations, where the material is homogeneous (liquid or powder form), it does not make sense to have rational subgroups of size more than one. In such situations, only one sample from each subgroup will be sufficient. It is then necessary to assess process control based on individual readings using X and R_m charts.

6.5.2 In the case of control charts for individuals, since each subgroup is of size one, it will not be able to provide an estimate of variability within each subgroup. Hence, a measure of variation is obtained from moving ranges of two consecutive observations. A moving range is the absolute value of the difference between two successive measurements; i.e. the absolute value of the difference between the first and second measurements, then between the second and third, and so on. From the moving ranges, the average moving range is calculated and used for the construction of control charts. Also, from the