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Geometrical product specifications (GPS) — Filtration —

Part 32: Robust profile filters: Spline filters

Spécification géométrique des produits (GPS) — Filtrage —

Partie 32: Filtres de profil robustes: Filtres splines

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Foreword

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This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This document cancels and replaces ISO/TS 16610-32:2009, which has been technically revised.

The main changes are as follows:

— conversion to a Technical Report;

— inclusion of spline filtration for non-uniform sampling points;

— addition of a generalized filter equation with a revision of the equation of the robust spline filter harmonizing the statistical estimator with that of ISO 16610-31;

— inclusion of a termination criterion of the iterations for the robust, therefore nonlinear, filter;

— addition of specifications of the tension parameter.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

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Introduction

This document develops the terminology and concepts for spline filters. Spline filters have the advantage of being implementable for non-uniform sampling positions and for closed profiles. An example of application of spline filters is given in Annex A.

Robust filters are tolerant against outliers. Spline filters offer one method for form removal.

For more detailed information of the relation of this document to the filtration matrix and the ISO GPS standards, see Annex B and Annex C.

Geometrical product specifications (GPS) — Filtration —

Part 32:
Robust profile filters: Spline filters

1 Scope

This document provides information on a generalized version of the linear spline filter for uniform and non-uniform sampling and the robust spline filters for surface profiles. It supplements ISO 16610-22, ISO 16610-30 and ISO 16610-31.

This document provides information on how to apply the robust estimation to the spline filter as specified in ISO 16610-22, as well as its generalized form for non-uniform sampling. The weight function chosen for the M-estimator is the Tukey biweight influence function as specified in ISO 16610-31.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
robust filter
filter that is insensitive against specific phenomena in the input data

Note 1 to entry: A robust filter is a filter that delivers output data with robustness.

Note 2 to entry: Robust filters are nonlinear filters.

[SOURCE: ISO 16610-31:2016, 3.1, modified — Definition revised and notes to entry added.]

3.2
spline
linear combination of piecewise polynomials, with a smooth fit between the pieces

[SOURCE: ISO 16610-22:2015, 3.1, modified — Note 1 to entry removed.]

3.3
spline filter
linear filter based on *splines* (3.2)

Note 1 to entry: An example of spline filter application is given in Annex A.

3.4
robust spline filter
robust filter (3.1) based on *splines* (3.2)

3.5
uniform sampling
sampling of data points at equidistant positions, i.e. with the width of spacing intervals between neighbouring probing points being constant

3.6
non-uniform sampling
sampling of data points with non-equidistant spacing points

3.7
robust statistical estimator
rule that indicates how to calculate an estimate based on sample data from a population that is insensitive against specific phenomena in the input data

Note 1 to entry: An example of specific phenomena is significant deviation of the distribution of the input data (amplitude distribution in the case of surface profiles) from a Gaussian distribution mostly in the form of long tails.

3.8
M-estimator
robust statistical estimator (3.7) which uses an influence function, i.e. a function which is asymmetric and scale invariant, to weight points according to their signed distance from the reference line

[SOURCE: ISO 16610-30:2015, 3.5, modified — Definition revised.]

3.9
Tukey's biweight influence function
influence function which supresses specific phenomena in the input data x and is defined by:

$$\psi(x)=\begin{cases} x\left(1-\left(\dfrac{x}{c}\right)^2\right)^2 & \text{for } |x|\le c \\ 0 & \text{for } |x|>c \end{cases}$$

where c is a scale parameter

4 Spline filter for uniform and non-uniform sampling

4.1 General

The following low-pass filter equation for spline profile filters is based on cubic splines with a regularization parameter depending on the nesting index, which complies with the cut-off wavelength in the case of linear filters, for the smoothness of the resultant waviness profile (low-passed signal) and a tension parameter influencing the slope of the transfer function.

4.2 Filter equation for cubic spline filter

4.2.1 General

The filter equation is given in Formula (1):

$$\boldsymbol{w}=\left(\boldsymbol{V}+\beta\alpha^2\boldsymbol{P}+(1-\beta)\alpha^4\boldsymbol{Q}\right)^{-1}\boldsymbol{V}\boldsymbol{z} \qquad (1)$$

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where

- $\boldsymbol{z}$ is the n-dimensional column vector of input data, e.g. the primary profile of n sampling points;
- $\boldsymbol{w}$ is the column vector of output data, e.g. the waviness profile or smoothed profile;
- $\boldsymbol{V}$ is the unity matrix in the case of the linear filter and the weighting matrix in the case of the robust filter;
- $\boldsymbol{P}$ and $\boldsymbol{Q}$ are the matrices for the discretized differentiation;
- β is the tension parameter (see also 4.2.3);
- α is the parameter (see 4.2.2) depending on the smoothness, the nesting index (cut-off wavelength in the case of linear filters) of the spline.

Formula (1) is obtained by minimization of the objective (cost) function J as indicated in Formula (2):

$$\min_{\boldsymbol{w}} J \tag{2}$$

with the objective function defined in Formula (3):

$$J = (\boldsymbol{z}-\boldsymbol{w})^T \boldsymbol{V} (\boldsymbol{z}-\boldsymbol{w}) + \beta \alpha^2 \boldsymbol{w}^\mathrm{T} \boldsymbol{P} \boldsymbol{w} + (1-\beta)\alpha^4 \boldsymbol{w}^\mathrm{T} \boldsymbol{Q} \boldsymbol{w} \tag{3}$$

where $\boldsymbol{Q} = \boldsymbol{P}^\mathrm{T} \boldsymbol{P}$.

A sufficient condition of a minimum is $\nabla_{\boldsymbol{w}} J = 0$ leading to the filter equation in Formula (1).

NOTE 1 After extending the matrices of Formula (1) to tensors, the filter is also applicable to areal data[11].

NOTE 2 Usually the objective function of smoothing splines is defined with a regularization parameter μ also fitted during the optimization process with an additional condition for the smoothness measured according to the deviations $z_i - s(x_i)$. Objective functions of the more common type of smoothing splines do not include non-zero tension $J = \sum_{i=1}^{n} (z_i - s(x_i))^2 + \mu \int_{x_1}^{x_n} \left(\frac{\partial}{\partial x} s(x) \right)^2 \mathrm{d}x$ with $s(x)$ being the spline polynomials and the regularization parameter μ determining the degree of smoothing and hence following the data points vs approximating them.

4.2.2 Regularization parameter

The parameter μ specifies the regularization, i.e. the degree of smoothing. In the case of minimum tension, it holds $\mu = \alpha^4$ and is therefore related to the nesting index n_i, which is in the case of linear filtration equal to the cut-off wavelength λ_c as given in Formula (4):

$$\alpha = \frac{1}{2 \sin\left(\frac{\pi \Delta}{n_\mathrm{i}} \right)} \tag{4}$$

where Δ is the sampling interval for uniformly sampled data and the average sampling interval as given in Formula (5):

$$\Delta = \frac{1}{n-1} \sum_{i=1}^{n-1} (x_{i+1} - x_i) \tag{5}$$

for data sampled non-uniformly at positions x_i with i = 1, ..., n−1.

NOTE 1 Formula (4) is derived in Reference [12].

NOTE 2 For sampling intervals $\Delta \ll n_\mathrm{i}$ the regularization parameter tends to infinity $\alpha^4 \to \infty$.

NOTE 3 For non-minimal tension the factor μ of the second order derivative term is also dependent on the tension parameter β : $\mu = (1-\beta)\alpha^4$.

4.2.3 Tension parameter

The product $\beta\alpha^2$ is the tension factor with parameter β lying between 0 and 1. The parameter β controls the degree of subsequent topography curvatures, where curvature means a local property of a curve or a surface, which is defined at every point quantifying second-order deviations of a curve from a straight line or a surface from a plane.

Following curvatures closely means optimal shape retainment of the low-pass result, the output data $\boldsymbol{w}$.

For $\beta = 0$ the characteristics of the transfer function conform to Formula (1) in ISO 16610-22, a minimum tension which is equivalent to the steepest slope of the transfer function and therefore a better shape retainment than for $\beta > 0$.

For β = 0,625 242 the characteristics of the transfer function is similar to that of the Gaussian filter[14] as specified in ISO 16610-21 and ISO 16610-61.

NOTE The shape retainment by the spline filter for $\beta = 0$ is global, while the shape retainment by the Gaussian regression with a parabolic regression ($p = 2$) is local.

4.2.4 Matrix *V* for linear cubic spline filter

Matrix $\boldsymbol{V}$ for linear filters is the $n \times n$-dimensional unity matrix as given in Formula (6):

$$\boldsymbol{V} = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix} \tag{6}$$

4.2.5 Matrix *V* for robust cubic spline filter

Matrix $\boldsymbol{V}$ contains the weights suppressing specific phenomena in the input data. They are derived from Tukey's biweight influence function as given in Formula (7):

$$\boldsymbol{V}^{(m)} = \begin{pmatrix} \delta_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \delta_n \end{pmatrix} \text{with } \delta_i = \begin{cases} \left(1 - \left(\dfrac{z_i - w_i^{(m)}}{c^{(m)}}\right)^2\right)^2 & \text{for } \left|z_i - w_i^{(m)}\right| \le c^{(m)} \\ 0 & \text{for } \left|z_i - w_i^{(m)}\right| > c^{(m)} \end{cases} \tag{7}$$

where

$i = 1, \dots, n$;

$\boldsymbol{z} = (z_1, \dots, z_n)^\mathrm{T}$;

$\boldsymbol{w}_{\boldsymbol{i}}^{(m)} = \left(w_1^{(m)}, \dots, w_n^{(m)}\right)^\mathrm{T}$;

superscript m denotes the iteration put into brackets, which is not to be confused with an exponent;

superscript T denotes transposed.

Furthermore, the parameter c is specified as given in ISO 16610-31:2016, Formula (8), and as shown in Formula (8):

$$c = a \operatorname{median}|z - \boldsymbol{w}| \text{ with } a \cong 4{,}447\,8 \tag{8}$$

NOTE The exact value of a is obtained by the inverse error function erf^{-1}:

$$a = \frac{3}{\sqrt{2} \quad \operatorname{erf}^{-1}(0{,}5)}$$

4.2.6 Termination of the iteration of robust estimation

The matrix $\boldsymbol{V}$ containing weights δ_i being dependent on output data w_i starts with w obtained by linear non-robust filtration, i.e. $\boldsymbol{V}^{(0)}$ is the unity matrix. The iteration process terminates if the condition given in Formula (9) is reached:

$$\frac{\left|c^{(m+1)} - c^{(m)}\right|}{c^{(m)}} \le 10^{-5} \text{ or } m \ge 12 \tag{9}$$

4.2.7 Matrices of differentiation *P* and *Q*

4.2.7.1 General

A profile can be sampled at lateral positions x_i that are not necessarily equidistant. The lateral positions are strictly monotonically increasing, i.e. $x_i < x_{i+1}$.

The samplings intervals are denoted $\Delta_{i,j} = x_i - x_j$ and the quotient of the average sampling interval Δ and the distance between sampling positions x_i and x_j is denoted by $D_{i,j} = \frac{\Delta}{\Delta_{i,j}}$.

4.2.7.2 Differentiation matrix for first discretized derivative

For open profiles, matrix $\boldsymbol{P}$ is tri-diagonal as shown in Formula (10):

$$\boldsymbol{P} = \begin{pmatrix} P_{1,1} & P_{1,2} & 0 & \cdots & & \\ P_{2,1} & P_{2,2} & P_{2,3} & 0 & \cdots & \\ 0 & P_{3,2} & P_{3,3} & P_{3,4} & 0 & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{10}$$

For rows $i = 2,\ldots,n-1$ the main diagonal has the elements: $P_{i,i} = D_{i,i-1}^2 + D_{i+1,i}^2$.

The two off-diagonals have the elements $P_{i,i-1} = -D_{i,i-1}^2$ and $P_{i,i+1} = -D_{i+1,i}^2$.

The first two elements of the first row and the last two elements of the last row are as follows:

$$P_{1,1} = D_{1,2}^2 \text{ and } P_{1,2} = -D_{1,2}^2 \text{ and } P_{n,n-1} = -D_{n,n-1}^2 \text{ and } P_{n,n} = D_{n,n-1}^2$$

For closed profiles the first row and the last row of the matrix differ, having additional non-zero entries at P_{1n} and at P_{n1} for the wrap around, i.e. the right neighbour of x_n is x_1 and the left neighbour of x_1 is x_n.