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Standard Practice for Life and Reliability Testing Based on the Exponential Distribution[1]

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ε[1] NOTE—Corrected 6.7.1.1 and 6.7.2.1 editorially in February 2013.

1. Scope

1.1 This practice presents standard sampling procedures and tables for life and reliability testing in procurement, supply, and maintenance quality control operations as well as in research and development activities.

1.2 This practice describes general procedures and definitions of terms used in life test sampling and describes specific procedures and applications of the life test sampling plans for determining conformance to established reliability requirements.

1.3 This practice is an adaptation of the Quality Control and Reliability Handbook H-108, "Sampling Procedures and Tables for Life and Reliability Testing (Based on Exponential Distribution)," U.S. Government Printing Office, April 29, 1960.

1.4 A system of units is not specified in this practice.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics

E2234 Practice for Sampling a Stream of Product by Attributes Indexed by AQL

E2555 Practice for Factors and Procedures for Applying the MIL-STD-105 Plans in Life and Reliability Inspection

3. Terminology

3.1 *Definitions:*

3.1.1 See Terminology E456 for a more extensive listing of terms in ASTM Committee E11 standards.

3.1.2 *consumer's risk, β, n*—probability that a lot having specified rejectable quality level will be accepted under a defined sampling plan. **E2555**

3.1.2.1 *Discussion*—In this practice, the consumer's risk is the probability of accepting lots with mean time to failure θ_1.

3.1.2.2 *Discussion*—For the procedures of 9.7 and 9.8, the consumer's risk may also be defined as the probability of accepting lots with unacceptable proportion of lot failing before specified time, p_1.

3.1.3 *life test, n*—process of placing one or more units of product under a specified set of test conditions and measuring the time until failure for each unit.

3.1.4 *mean time to failure, θ, n— in life testing*, the average length of life of items in a lot.

3.1.4.1 *Discussion*—Also known as mean life.

3.1.5 *number of failures, n*—number of failures that have occurred at the time the decision as to lot acceptability is reached.

3.1.5.1 *Discussion*—The expected number of failures required for decision is the average of the number of failures required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution.

3.1.6 *producer's risk, α, n*—probability that a lot having specified acceptable quality level will be rejected under a defined sampling plan.

3.1.6.1 *Discussion*—In this practice, the producer's risk is the probability of rejecting lots with mean time to failure θ_0.

3.1.6.2 *Discussion*—For the procedures of 9.7 and 9.8, the producer's risk may also be defined as the probability of rejecting lots with acceptable proportion of lot failing before specified time, p_0.

3.1.7 *sequential life test, n*—life test sampling plan whereby neither the number of failures nor the time required to reach a

[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.30 on Statistical Quality Control.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

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decision are fixed in advance but instead decisions depend on the accumulated results of the life test.

3.1.8 *unit of product, n*—that which is inspected to determine its classification as defective or nondefective or to count the number of defects. E2234

3.1.9 *waiting time, n—in life testing*, the time elapsed from the start of testing until a decision is reached as to lot acceptability.

3.1.9.1 *Discussion*—The expected waiting time required for decision is the average of the waiting times required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution.

4. Significance and Use

4.1 This practice was prepared to meet a growing need for the use of standard sampling procedures and tables for life and reliability testing in government procurement, supply, and maintenance quality control (QC) operations as well as in research and development activities where applicable.

4.2 A characteristic feature of most life tests is that the observations are ordered in time to failure. If, for example, 20 radio tubes are placed on life test, and t_i denotes the time when the ith tube fails, the data occur in such a way that $t_1 \leq t_2 \leq \ldots \leq t_n$. The same kind of ordered observations will occur whether the problem under consideration deals with the life of electric bulbs, the life of electronic components, the life of ball bearings, or the length of life of human beings after they are treated for a disease. The examples just given all involve ordering in time.

4.3 In destructive testing involving such situations as the current needed to blow a fuse, the voltage needed to break down a condenser, or the force needed to rupture a physical material, the test can often be arranged in such a way that every item in the sample is subjected to precisely the same stimulus (current, voltage, or stress). If this is done, then clearly the weakest item will be observed to fail first, the second weakest next, and so forth. While the random variable considered mostly in this guide is time to failure, it should be emphasized, however, that the methodology provided herein can be adapted to the testing situations mentioned above when the random variable is current, voltage, stress, and so forth.

4.4 Sections 6 and 7 describe general procedures and definitions of terms used in life test sampling. Sections 8, 9, and 10 describe specific procedures and applications of the life test sampling plans for determining conformance to established reliability requirements.

4.5 Whenever the methodology or choice of procedures in the practice requires clarification, the user is advised to consult a qualified mathematical statistician, and reference should be made to appropriate technical reports and other publications in the field.

5. Introduction

5.1 The theory underlying the development of the life test sampling plans of this section, including the operating characteristic curves, assumes that the measurements of the length of life are drawn from an exponential distribution. Statistical test procedures for determining the validity of the exponential distribution assumption have appeared in the technical statistical journals. Professor Benjamin Epstein published a comprehensive paper (in two parts) on this subject in the February and May 1960 issues of *Technometrics*.[3] Part I of the paper contains descriptions of the mathematical and graphical procedures as well as an extensive bibliography for reference purposes. Numerical examples illustrating the statistical procedures are included in Part II of the paper.

5.2 It is important to note that the life test sampling plans of this practice are not to be used indiscriminately simply because it is possible to obtain life test data. Only after the exponential assumption is deemed reasonable should the sampling plans be used.

5.3 Sections 6 and 7 describe general procedures and description of life test sampling plans. Section 8 describes specific procedures and applications of sampling plans when life tests are terminated upon the occurrence of a preassigned number of failures, and Section 9 provides sampling plans when life tests are terminated at a preassigned time. Section 10 describes sequential life test sampling plans. Section 8 covers: *(1)* acceptance procedures; *(2)* expected duration of life tests and cost considerations in selection of sample sizes; and *(3)* life test plans for certain specified values of α, β, and θ_1/θ_0. Section 9 covers: *(1)* acceptance procedures; *(2)* life test plans for certain specified values of α, β, θ_1/θ_0, and T/θ_0; and *(3)* life test plans based on proportion of lot failing before specified time. Section 10 covers: *(1)* acceptance procedures; *(2)* graphical acceptance procedures; and *(3)* expected number and waiting time required for decision.

5.4 Operating characteristic (OC) curves for the life test sampling plans of 8.1-8.5, 9.1-9.5, and Section 10 are shown in Fig. A1.1 for the corresponding sampling plans in these sections were matched with respect to their OC curves. The OC curves in Fig. A1.1 have been computed for the life test sampling plans of 8.1-8.5 but are equally applicable for the sampling plans of 9.1-9.5 and Section 10.

5.5 The procedures of this section are based on the premise that the life tests are monitored continuously. If the tests are monitored only periodically, the values obtained from the tables and curves are only approximations.

6. General Definitions of Life and Reliability Test Terms

6.1 *Discussion of Terms and Procedures:*

6.1.1 *Purpose*—This section provides definitions of terms required for the life test sampling plans and procedures of Sections 7 through 10.

6.1.2 *Life Test*—Life test is the process of placing the "unit of product" under a specified set of test conditions and measuring the time it takes until failure.

6.1.3 *Unit of Product*—The unit of product is the entity of product that may be placed on life test.

[3] Epstein, B., "Tests for the Validity of the Assumption that the Underlying Distribution of Life is Exponential," *Technometrics*, Vol 2, February and May 1960, pp. 83–101 and 167–183.

6.1.4 *Specifying Failure*—The state that constitutes a failure shall be specified in advance of the life test.

6.1.5 *Life Test Sampling Plan*—A life test sampling plan is a procedure that specifies the number of units of product from a lot that are to be tested and the criterion for determining acceptability of the lot.

6.1.6 *Life Test Terminated upon Occurrence of Preassigned Number of Failures*—Life test sampling plans whereby testing is terminated when a preassigned termination number of failures, *r*, occur are given in Section 8 of this practice.

6.1.7 *Life Test Terminated at Preassigned Time*—Life test sampling plans whereby testing is terminated when a preassigned termination time, *T*, is reached are given in Section 9 of this practice.

6.1.8 *Sequential Life Test*—Sequential life test is a life test sampling plan whereby neither the number of failures nor the time required to reach a decision are fixed in advance but, instead, decisions depend on the accumulated results of the life test. Information on the observed time to failure are accumulated over time and the results at any time determine the choice of one among three possible decisions: *(1)* the lot meets the acceptability criterion, *(2)* the lot does not meet the acceptability criterion, or *(3)* the evidence is insufficient for either decision *(1)* or *(2)* and the test must continue. Sequential life test sampling plans are given in Section 10 of this practice and have the advantage over the life test sampling plans mentioned in 6.1.6 and 6.1.7 in that, for the same OC curve, the expected waiting time and the expected number of failures required to reach a decision as to lot acceptability are less for the sequential life tests.

6.1.9 *Expected Number of Failures*—The number of failures required for decision is the number of failures that have occurred at the time the decision as to lot acceptability is reached. For the life test sampling plans mentioned in 6.1.6, this number of failures is known in advance of the life test; but, for the sampling plans mentioned in 6.1.7 and 6.1.8, this number cannot be predetermined. The expected number of failures required for decision is the average of the number of failures required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution. The expected number of failures can be predetermined for the sampling plans mentioned in 6.1.6-6.1.8.

6.1.10 *Expected Waiting Time*—The waiting time required for decision is the time elapsed from the start of the life test to the time decision is reached as to lot acceptability. The waiting time required for decision cannot be predetermined for any of the sampling plans mentioned in 6.1.6-6.1.8. The expected waiting time required for decision is the average of the waiting times required for decision when life tests are conducted on a large number of samples drawn at random from the same exponential distribution. The expected waiting time can be predetermined for the sampling plans mentioned in 6.1.6-6.1.8.

6.2 *Length of Life:*

6.2.1 *Length of Life*—The terms "length of life" and "time to failure" may be used interchangeably and shall denote the length of time it takes for a unit of product to fail after being placed on life test. The length of time may be expressed in any convenient time scale such as seconds, hours, days, and so forth.

6.2.2 *Mean Time to Failure*—The terms "mean time to failure" and "mean life" may be used interchangeably and shall denote the mean (or equivalently, the average) length of life of items in the lot. Mean life is denoted by θ.

6.2.3 *Acceptable Mean Life*—The acceptable mean life, θ_0, is the minimum mean time to failure that is considered satisfactory.

6.2.4 *Unacceptable Mean Life*—The unacceptable mean life, θ_1 ($\theta_1 < \theta_0$), is the mean time to failure such that lots having a mean life less than or equal to θ_1 are considered unsatisfactory. The interval between θ_0 and θ_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the mean life decreases from θ_0 to θ_1.

6.3 *Failure Rate:*

6.3.1 *Proportion of Lot Failing Before Specified Time*—The term "proportion of lot failing before specified time," *p*, denotes the fraction of the lot that fails before some specified time, *T*, that is:

$$p = 1 - \exp(-T/\theta) \tag{1}$$

6.3.2 *Failure Rate during Period of Time*—The "failure rate during period of time *T*," *G*, is given by:

$$G = \frac{1}{T}\{1 - \exp(T/\theta)\} = p/T \tag{2}$$

6.3.3 *Instantaneous Failure Rate*—The "instantaneous failure rate" or "hazard rate" is given by:

$$Z = 1/\theta \tag{3}$$

6.3.4 *Acceptable Proportion of Lot Failing Before Specified Time*—The "acceptable proportion of lot failing before specified time," p_0, is the maximum fraction of the lot that may fail before time, *T*, and still result in the lot being considered satisfactory.

6.3.5 *Unacceptable Proportion of Lot Failing Before Specified Time*—The "unacceptable proportion of lot failing before specified time," p_1, ($p_1 > p_0$), is the minimum fraction of the lot that may fail before time, *T*, and results in the lot being considered unsatisfactory. The interval between p_0 and p_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the fraction of the lot failing before time, *T*, increases from p_0 to p_1.

6.3.6 *Acceptable Failure Rate During Period of Time*—The "acceptable failure rate during period of time," G_0, is the maximum failure rate during the period of time that can be considered satisfactory.

6.3.7 *Unacceptable Failure Rate During Period of Time*—The "unacceptable failure rate during period of time," G_1, ($G_1 > G_0$), is the minimum failure rate during the period of time that results in the lot being considered unsatisfactory. The interval between G_0 and G_1 is a zone of indifference in which there is a progressively greater degree of dissatisfaction as the failure rate increases from G_0 to G_1.

6.3.8 *Life Test Sampling Plans Based on Failure Rates*—Life test sampling plans that are based on failure rates are given in 9.7 and 9.8.

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6.4 *OC Curves and Sampling Risks:*

6.4.1 *OC Curve*—The OC curve of a life test sampling plan is the curve that shows the probability that a submitted lot with given mean life would meet the acceptability criterion on the basis of that sampling plan.

6.4.2 *Producer's Risk*—The producer's risk, α, is the probability of rejecting lots with mean life, θ_0. For the procedures of 9.7 and 9.8, the producer's risk may also be defined as the probability of rejecting lots with acceptable proportion of lot failing before specified time, p_0.

6.4.3 *Consumer's Risk*—The consumer's risk, β, is the probability of accepting lots with mean life, θ_1. For the procedures of 9.7 and 9.8, the consumer's risk may also be defined as the probability of accepting lots with p_1 as the unacceptable proportion of lot failing before specified time.

6.5 *Submittal of Product:*

6.5.1 *Lot*—The term "lot" shall mean either an "inspection lot," that is, a collection of units of product manufactured under essentially the same conditions from which a sample is drawn and tested to determine compliance with the acceptability criterion or, a "preproduction lot," that is, one or more units of product submitted before the initiation of production for test to determine compliance with the acceptability criterion.

6.6 *Sample Selection:*

6.6.1 *Drawing of Samples*—A sample is one or more units of product drawn at random from a lot.

6.6.2 *Testing without Replacement*—Life test sampling without replacement is a life test procedure whereby failed units are not replaced.

6.6.3 *Testing with Replacement*—Life test sampling with replacement is a life test procedure whereby the life test is continued with each failed unit of product replaced by a new one, drawn at random from the same lot, as soon as the failure occurred. In the case of complex unit of product, this may be interpreted to mean replacement of the component that caused the failure by a new component drawn at random from the same lot of components. When the "sample sizes" are the same in both instances, the expected waiting time required for decision when testing with replacement is less than when testing without replacement.

6.6.4 *Sample Size*—The sample size, n, for a life test is the number of units of product placed on test at the start of a life test. When testing with replacement, the total number of units of product placed on test will, in general, be greater than the original sample size. The sample sizes for the life test plans of Sections 8 to 10 depend on the relative cost of placing large numbers of units of product on test and the expected length of time the life tests must continue to determine acceptability of the lots. Increasing the sample size will, on one hand, cut the average time required to determine acceptability but, on the other hand, will increase the cost because of placing more units of product on test.

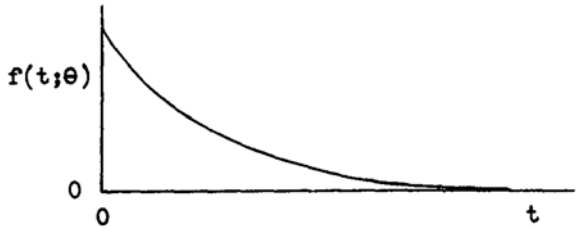
6.7 *Exponential Distribution:*

6.7.1 *Exponential Distribution with One Parameter*—The density function for the exponential distribution with one parameter is given by:

$$f(t;\theta) = 1/\theta\exp(-t/\theta) \qquad t \geq 0,\ \theta>0 \qquad (4)$$

$$= 0 \qquad t<0$$

6.7.1.1 The function has the following general graphical form:

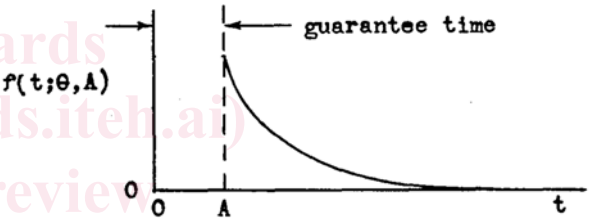


6.7.2 *Exponential Distribution with Two Parameters*—The density function for the exponential distribution with two parameters is given by:

$$f(t;\theta,A) = 1/\theta\exp[-(t-A)/\theta] \qquad t \geq A \geq 0 \qquad (5)$$

$$= 0 \qquad \text{elsewhere}$$

6.7.2.1 The function has the following general graphical form:



6.7.2.2 The quantity, A, is called "guarantee time" and the one parameter case is a special case of the two-parameter distribution with a guarantee time of zero.

6.7.3 *Exponential Distribution when Number of Parameters Is Unspecified*—In this practice, whenever the term "exponential distribution" is mentioned without specific mention of the number of parameters, it shall be assumed to mean the exponential distribution with one parameter.

7. General Description of Life Test Sampling Plans

7.1 *Scope:*

7.1.1 *Purpose*—Sections 7 through 10 of this practice establish life test sampling plans for determining acceptability of a product when samples are drawn at random from an exponential distribution.

7.1.2 *Specifying Acceptable Mean Life*—Before the start of the life test, the particular value of the acceptable mean life, θ_0, shall be specified except when using the procedures of 9.7 and 9.8.

7.1.3 *Specifying Unacceptable Mean Life*—The particular value of the unacceptable mean life, θ_1, shall be specified in advance of the life test when using the life test procedures of 8.6 and 9.6.

7.1.4 *Specifying Acceptable Proportion of Lot Failing before Specified Time*—The particular value, p_0, of the acceptable

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