International Standard

ISO 7870-6

Control charts —

Part 6: **EWMA control charts for the process mean**

Cartes de contrôle —

Partie 6: Cartes de contrôle EWMA pour la moyenne d'un processus

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Foreword

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in product and process management*.

This second edition cancels and replaces the first edition (ISO 7870-6:2016), which has been technically revised.

A list of all parts in the ISO 7870 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

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Introduction

Shewhart control charts are the most widespread statistical control methods used for controlling a process, but they are slow in signalling shifts of small magnitude in the process parameters. The exponentially weighted moving average[13] (EWMA) control chart makes possible faster detection of small to moderate shifts.

The Shewhart control chart is simple to implement and it rapidly detects shifts of major magnitude. However, it is fairly ineffective for detecting shifts of small or moderate magnitude. It happens quite often that the shift of the process is slow and progressive (in case of continuous processes in particular); this shift has to be detected very early in order to react before the process deviates seriously from its target value. There are two possibilities for improving the effectiveness of the Shewhart control charts with respect to small and moderate shifts.

— The simplest, but not the most economical possibility is to increase the subgroup size. This may not always be possible due to low production rate; time consuming or too costly testing. As a result, it may not be possible to draw samples of size more than 1.

— The second possibility is to take into account the results preceding the control under way in order to try to detect the existence of a shift in the production process. The Shewhart control chart takes into account only the information contained in the last sample observation and it ignores any information given by the entire sequence of points. This feature makes the Shewhart control chart relatively insensitive to small process shifts. Its effectiveness can be improved by taking into account the former results.

Where it is desired to detect slow, progressive shifts, it is preferable to use specific charts which take into account the past data and which are effective with a moderate control cost. Two very effective alternatives to the Shewhart control chart in such situations are

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a) Cumulative sum (CUSUM) control chart. This chart is described in ISO 7870-4. The CUSUM control chart reacts more sensitively than the X-bar chart to a shift of the mean value in the range of half to two sigma. If one plots the cumulative sum of deviations of successive averages from a specified target, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

b) Exponentially weighted moving average (EWMA) control chart which is covered by this document. This chart is presented like the Shewhart control chart; however, instead of placing on the chart the successive averages of the samples, one monitors a weighted average of the current average and of the previous averages.

EWMA control charts are generally used for detecting small shifts in the process mean. They will detect shifts of half sigma to two sigma much faster. They are, however, slower in detecting large shifts in the process mean. EWMA control charts can also be preferred when the subgroups are of size $n = 1$.

The joint use of an EWMA control chart with a small value of smoothing parameter (λ) and a Shewhart control chart has been recommended as a means of guaranteeing fast detection of both small and large shifts. The here considered EWMA control chart monitors only the process mean; monitoring the process variability requires the use of some other technique including special EWMA control charts.

The numbers in all tables and figures were calculated using the R-package SPC, (Knoth 2022), which makes use of the algorithm proposed by Crowder (1987).

The R-file containing the calculations can be downloaded on https://standards.iso.org/iso/7870/-6/ed-2/en.

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Control charts —

Part 6:
EWMA control charts for the process mean

1 Scope

This document covers EWMA control charts, originally proposed by Roberts (1959)[16], as a statistical process control technique to detect small shifts in the process mean. It makes possible the faster detection of small to moderate shifts in the process mean. In this chart, the process mean is evaluated in terms of exponentially weighted moving average of all previous observations or averages.

The EWMA control chart's application is worthwhile in particular when

— production rate is slow,

— a minor or moderate shift in the process mean is vital to be detected,

— sampling and inspection procedure is complex and time consuming,

— testing is expensive, and

— it involves safety risks.

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NOTE EWMA control charts are applicable for both variables and attributes data. The given examples illustrate both types (see 5.5, Annex A, Annex B and Annex C).

2 Normative references

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The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-2 apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
run length
number of samples taken up to the point at which a signal occurs

3.2
average run length
ARL
Mathematical expectation of the run length

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3.3
maximum run length
MAXRL
95 %-percentile of the run length

4 Symbols and abbreviated terms

μ_0	Target value for the mean of the process
U_μ, L_μ	Upper rejectable value of the mean, lower rejectable value of the mean
$\overline{x}_i$	Average of the sample *i*
R_i	Range of the sample *i*
n	Number of items in a sample (sample size)
z_i	EWMA value for the sample *i*
z_0	Initial value of EWMA series $\{z_i\}$
λ	The smoothing parameter
L_z	Factor to establish the control limit for z_i
s	Estimate of the standard deviation σ
σ	Standard deviation of the distribution of *X* (the random variable to be monitored)
σ_0	In-control standard deviation
$\sigma_{\overline{x}}$	Standard deviation of the averages of *n* individual observations; $\sigma_{\overline{x}} = \sigma / \sqrt{n}$
σ_z	Standard deviation of z_i when *i* tends towards infinity
δ	Shift in the mean from the target value μ_0, expressed in number of standard deviations
δ_1	Maximum acceptable shift in the mean from μ_0, expressed in number of standard deviations
p	Proportion of nonconforming items of the process
p_0	Target value for the proportion of nonconforming items of the process
p_1	Upper refusable value of the proportion of nonconforming items
p_i	Proportion of nonconforming items in the i^{th} sample
c	Mean number of nonconformities
c_0	Target value for the mean number of nonconformities
c_1	Refusable mean of nonconformities
c_i	Number of nonconformities in the i^{th} sample
U_{CL}	Upper control limit
L_{CL}	Lower control limit

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ARL Average Run Length

ARL_0 Average Run Length of the process in control

ARL_1 Average Run Length of the process with shift

CL Centre line

MAXRL Maximum Run Length

5 EWMA for inspection by variables

5.1 General

An EWMA control chart plots exponentially weighted moving averages of past and current data in which the values being averaged are assigned weights that decrease exponentially from the present into the past, see Figure 1. Consequently, the average values are influenced more by recent process performance. The exponentially weighted moving average is defined as Formula (1):

$$z_i = \lambda x_i + (1 - \lambda)\, z_{i-1} \tag{1}$$

Where $0 < \lambda \leq 1$ is a constant and the starting value (required with the first sample at $i = 1$) is the process target, so that $z_0 = \mu_0$.

NOTE 1 When the EWMA control chart is used with rational subgroups of size $n > 1$ then x_i is simply replaced with $\bar{x}_i$.

NOTE 2 μ_0 can be estimated by the average of preliminary data.

The EWMA control chart becomes an $\bar{X}$ chart for $\lambda = 1$.

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5.2 Weighted average explained

To demonstrate that the EWMA is a weighted average of all previous observations or averages, the right-hand side of Formula (1) in 5.1 can be substituted with z_{i-1} to obtain Formula (2):

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$$\begin{aligned} z_i &= \lambda x_i + (1-\lambda)\left[\lambda x_{i-1} + (1-\lambda) z_{i-2}\right] \\ &= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2} \end{aligned} \tag{2}$$

Continuing to substitute recursively for z_{i-j}, where $j = 2, 3, \ldots$, we obtain Formula (3):

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j\, x_{i-j} + (1-\lambda)^i\, z_0 \tag{3}$$

For $i = 1$, $z_1 = \lambda x_1 + (1 - \lambda)\mu_0$.

The weights, $\lambda(1 - \lambda)^j$, decrease geometrically with the age of the observation or average. Furthermore, the weights sum to unity:

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j + (1-\lambda)^i = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)} \right] + (1-\lambda)^i = 1 \tag{4}$$

If $\lambda = 0{,}25$, then the weight assigned to the current average is 0,25 and the weights given to the preceding averages are 0,187 5; 0,140 6; 0,105 5 and so forth. These weights are shown in Figure 1. Because these weights decline geometrically, the EWMA is sometimes called a geometric moving average (GMA), which is the original name of the control chart[16].

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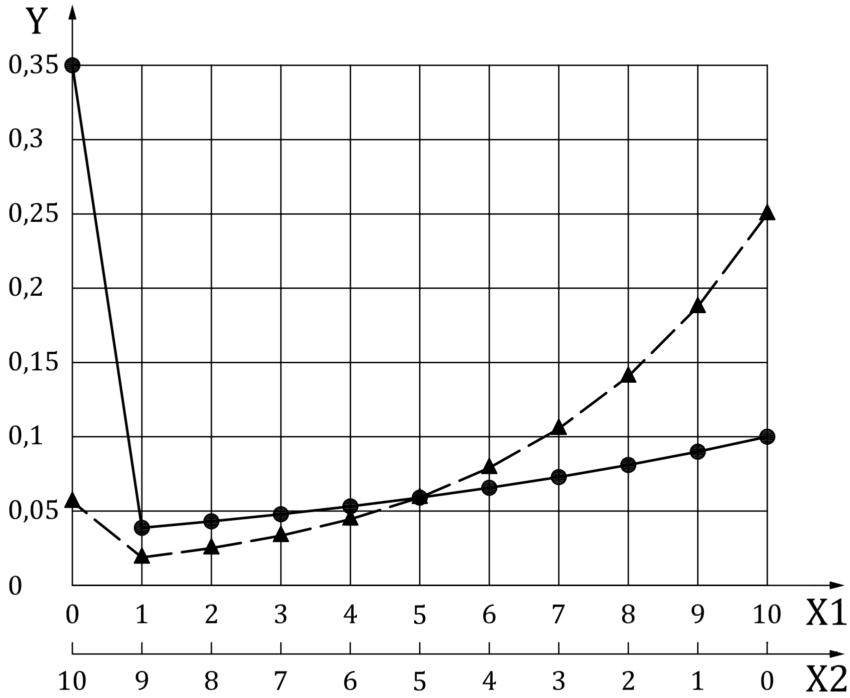

Key

X1 sample number

X2 age of the observation or average

Y weights $\lambda(1-\lambda)^{10-X1}$

—●— $\lambda = 0{,}1$

– –▲– – $\lambda = 0{,}25$

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Figure 1 — Weights of all 10 averages after having incorporated sample 10

Since the EWMA value can be viewed as a weighted average of all past and current observations, it is very insensitive to the normality assumption. It is, therefore an ideal control chart to use with individual observations.

5.3 Control limits for EWMA control chart

If the observations x_i are independent random variables with variance σ^2, then the variance of z_i is represented by Formula (5):

$$\sigma_{z_i}^2 = \frac{\sigma^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right] \quad (5)$$

Therefore, the EWMA control chart would be constructed by plotting z_i versus the sample number i (or time). The centre line and control limits for the EWMA control chart are as follows:

Centre line = μ_0

$$U_{\text{CL}} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \quad (6)$$

$$L_{\text{CL}} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \quad (7)$$

The factor L_z refers to the distance of the control limits from the centre line and can be derived by setting an appropriate in-control ARL (average run length) value. It is well-known that the common Shewhart

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control chart with 3 sigma limits exhibits an in-control ARL of 370,4. To achieve the same value for an EWMA control chart with control limits, see Formula (6) and Formula (7), one chooses factor L_z =2,715 and 2,864 for λ = 0,1 and 0,2, respectively. For other choices of λ see Table 4. Note that using just the Shewhart factor 3 (or other values accordingly) provides a quick and dirty approach to set EWMA limits, where the detection performance is mostly better than that of the older Shewhart control chart.

No action is taken as long as z_i falls between these limits, and the process is considered to be out of control as soon as z_i overshoots the control limits. In this case, an investigation is initiated to locate the assignable cause, and the process can be stopped or adjusted. In the latter case, resume the EWMA control chart after reinitializing it, i.e. by not taking into account the results obtained prior to this resetting, but by taking z_0 as the initial value.

The term $[1 - (1 - \lambda)^{2i}]$ approaches unity as i gets larger. This means that after the EWMA control chart has been running for several time periods, the control limits will approach steady state values obtained using Formula (8) and Formula (9):

Centre line = μ_0

$$U_{CL} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{8}$$

$$L_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{9}$$

However, it is strongly recommended to use the exact control limits. This will greatly improve the performance of the control chart in detecting an off-target process immediately after the EWMA control chart is initiated.

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NOTE For practical purposes, use the estimate of σ, denoted by s, estimated from the data.

5.4 Construction of EWMA control chart

To illustrate the construction of an EWMA control chart, a process with the following parameters calculated from historical data (individual observations, i.e. sample size n = 1) is considered:

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$\mu_0 = 50$

$\sigma = 2{,}053\,9$

with λ chosen to be 0,3; so that

$$\sqrt{\frac{\lambda}{(2-\lambda)}} = \sqrt{\frac{0{,}3}{1{,}7}} = 0{,}420\,1 \tag{10}$$

The control limits at steady-state are given, obtained using Formula (11) and Formula (12):

$$U_{CL} = 50 + 2{,}925\ (0{,}420\,1)\ (2{,}053\,9) = 52{,}523\,8 \tag{11}$$

$$L_{CL} = 50 - 2{,}925\ (0{,}420\,1)\ (2{,}053\,9) = 47{,}476\,2 \tag{12}$$

The factor L_z is picked from Table 2. The data consisting of 20 points as given in Table 1 are considered.

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