International Standard

ISO 21771-1

First edition

Cylindrical involute gears and gear pairs —

Part 1: Concepts and geometry

Roues et engrenages cylindriques à développante —

Partie 1: Concepts et géométrie

PROOF/ÉPREUVE

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771-1 cancels and replaces ISO 21771:2007, which has been technically revised.

The main changes are as follows:

— the sign convention for internal gears used in the ISO 6336 series[8] has been adopted. The negative value for the number of teeth of an internal gear is applied to the diameters and centre distance, so these dimensions of internal gears have negative values;

— flank direction has been renamed as hand of helix and sign (+/-) of helix angle is used;

— a definition of normal surface has been added and this is used rather than normal plane;

— the annex on tooth thickness was removed because it is now addressed in ISO 21771-2.

Additional material has been added to cover:

— calculation of form diameters for tooth tip corner radius and tooth root fillet radius in the transverse plane for an involute cylindrical gear (Clauses 10, 11 and Annex B);

— calculation of the tooth tip corner radius for a specified form diameter and tip diameter of an involute cylindrical gear;

— calculation of a radius tangent to the involutes of adjacent teeth at root or tip diameter (Annex A);

— generated tooth root fillet shape for individual involute cylindrical gears (Annex B);

— concepts and parameters for involute cylindrical gear pairs with crossed axes (Clause 6 and Annex C);

— geometry of surfaces in contact (Annex D);

— projection of a transverse plane profile of a tooth onto another plane (Annex E);

— interface to ISO 10828 for involute worm gear geometry (Annex F).

A list of all parts in the ISO 21771 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Cylindrical involute gears and gear pairs —

Part 1:
Concepts and geometry

1 Scope

This document specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included. The formulae in this document apply to all pressure angles.

It also covers the concepts and parameters for involute cylindrical gear pairs with parallel or crossed axes, and a constant gear ratio. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

2 Normative references

There are no normative references in this document.

3 Terms, definitions, symbols, subscripts and units

3.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1.1
basic rack tooth profile for involute gear teeth
tooth profile of a normal section through the teeth of a basic rack which corresponds to an external spur gear with number of teeth $z = \infty$ and diameter $d = \infty$

Note 1 to entry: The tooth of the basic rack tooth profile is bounded by the tip line at the top and by the parallel root line at the bottom. The fillet between the straight part of the profile and the root line is a circular arc with a radius equal to ρ_{fP}.

3.1.2
counterpart rack tooth profile
rack tooth profile symmetrical to the basic rack tooth profile about the datum line P-P and displaced by half a pitch relative to it

3.1.3
nominal involute flank
pure involute flank prior to any modifications

Note 1 to entry: See 4.4 for more information on a gear tooth involute flank.

3.1.4
tip alteration coefficient
k
change to the addendum relating to the standard addendum of one normal module, it is made non-dimensional by dividing by the normal module

Note 1 to entry: Tip alteration is also known as addendum modification, tip shortening or truncation.

3.1.5
generating gear pair
generating tool (rack, hob, pinion-type cutter, or grinding wheel) and the gear being machined during gear tooth machining process

3.2 Symbols

Table 1 provides all the symbols used in this document.

Table 1 — Symbols

Symbol	Description	Subclause
A	pinion lower end point of meshing (near pinion root)	5.5.6.1
a_w	working centre distance of a cylindrical gear pair	5.3.3, 5.3.5, 6.3.12
a_{w0}	centre distance in the generating process with pinion-type cutter	11.2
B	pinion lower point of single tooth contact (LPSTC)	5.5.6.1
b	facewidth	4.3.9
b_{Ea}	length of relief near tip	8.5.2
b_{Ef}	length of relief near root	8.5.2
b_F	usable facewidth	4.3.9
b_w	active facewidth (the facewidth used)	5.5.9.2
C	working pitch point with subscript: amount of relief for modifications	5.3.5 8
C_{ay}	modification of the profile by function	8.6
C_{Ea}	amount of triangular end relief modification at tip	8.5.2
C_{Ef}	amount of triangular end relief modification at root	8.5.2
$C_{H\alpha}$	amount of transverse profile slope modification	8.3.3
$C_{H\beta}$	amount of flank line slope modification	8.4.2
$C_{i,j}$	amount of modification at point (i,j)	8.5.1
C_α	amount of profile crowning (barrelling)	8.3.4
$C_{\alpha a}$	amount of tip relief	8.3.2
$C_{\alpha f}$	amount of root relief	8.3.2
C_β	amount of flank line crowning	8.4.3
$C_{\beta I}$, $C_{\beta II}$	amount of end relief	8.4.1
$C_{\beta y}$	modification of the flank line by function	8.6
$C_{\Sigma y}$	modification of the flank surface by function	8.6
c	tip clearance	5.3.8, 6.3.15
c_F	form over dimension	5.5.5
D	pinion highest point of single tooth contact (HPSTC)	5.5.6.1
d	reference diameter	4.3.4
d_a	tip diameter	4.6.5
d_{a0}	tip diameter of pinion-type cutter	11.1

Table 1 *(continued)*

Symbol	Description	Subclause
d_b	base diameter	4.4.2
d_{b0}	base diameter of the pinion-type cutter	11.1
d_{bK}	chamfering base diameter	8.2.2
$d_{C\alpha a}$	tip relief start diameter	8.3.2
$d_{C\alpha f}$	root relief start diameter	8.3.2
d_{Cf}	Profile control diameter	8.3.2
d_{Ea}	diameter for start of triangular end relief at tooth tip	8.5.2
d_{Ef}	diameter for start of triangular end relief at tooth root	8.5.2
d_{Fa}	tip form diameter	9.7
d_{Ff}	root form diameter	5.5.2, 9.7
d_f	root diameter (nominal dimension)	4.6.7
d_{fE}	root diameter produced	9.6
d_{Na}	active tip diameter	5.5.2, 6.4.3
d_{Nf}	start of active profile diameter (SAP diameter, active root diameter)	5.5.2, 6.4.3
d_v	V-circle diameter	4.6.2
d_w	working pitch diameter	5.3.5
d_y	Y-circle diameter	4.6.3
d_0	reference diameter of pinion-type cutter	11.1.1
E	pinion upper end point of meshing (near pinion tip)	5.5.6.1
E_{sni}	lower tooth thickness limit deviation for normal tooth thickness at reference cylinder	9.4
E_{sns}	upper tooth thickness limit deviation for normal tooth thickness at reference cylinder	9.4
e_n	normal space width on the reference cylinder	4.8.7
e_P	space width of the basic rack tooth profile	4.3.3
e_t	transverse space width on the reference cylinder	4.8.4
e_{yn}	normal space width on the y-cylinder	4.8.7
e_{yt}	transverse space width on the y-cylinder	4.8.4
F	force	D.3
g_a	length of addendum path of contact	5.5.6.2, 6.4.5.2
g_f	length of dedendum path of contact	5.5.6.2, 6.4.5.2
g_α	length of path of contact	5.5.6.2
$g_{\alpha y}$	distance of a point Y from working pitch point C	5.7.5
g_β	overlap roll length (arc of contact)	5.5.9.4
h	tooth depth (between tip line and root line)	4.7.1
h_a	addendum from reference pitch circle	4.7.2
h_{aP}	addendum of the basic rack tooth profile	4.3.2
h_{aP0}	addendum of the counterpart of the basic rack tooth profile	9.2.2
h_{aw}	addendum from working pitch circle	4.7.2
h_{a0}	addendum of pinion-type cutter	11.1.1
h_{FaP0}	straight part of tip flank of tool-generating profile	9.2.2
h_{FfP}	portion of the dedendum to root form line, of the basic rack tooth profile	4.3.2
h_{FfP0}	portion of the dedendum to root form line of the counterpart rack tooth profile	9.2.1
h_{fP0}	dedendum of the counterpart rack tooth profile and rack tool	9.2.2
h_f	dedendum from reference circle	4.7.2
h_{fP}	dedendum of the basic rack tooth profile	4.3.3

Table 1 *(continued)*

Symbol	Description	Subclause
h_{fw}	dedendum from working pitch circle	4.7.2
h_K	height of the tip corner chamfering or tip corner rounding	8.2.2
h_P	tooth depth of basic rack tooth profile	4.3.2
h_w	working depth of teeth in a gear pair	5.3.7, 6.3.14
i	transmission ratio of a gear pair	5.3.2
inv	involute function (not a variable)	4.4.5
j_{bn}	normal base backlash	5.6.4, 6.5.2
j_{bt}	transverse backlash	5.6.2
j_r	radial backlash	5.6.5
j_t	circumferential backlash at the reference circle	5.6.3, 6.5.3
j_{wn}	working normal backlash	6.5.2
j_{wt}	circumferential backlash at the working pitch circle	5.6.3, 6.5.3
K_g	sliding factor	5.7.6
K_{ga}	sliding factor at tooth tip	5.7.6
K_{gf}	sliding factor at tooth root	5.7.6
k	tip alteration coefficient	3.1.3, 4.6.4, 5.3.9, 6.3.16
L_{Ca}	tip relief roll length	8.3.2
L_{Cf}	root relief roll length	8.3.2
L_{Cl}, L_{Cll}	length of end relief	8.4.1
L_{Ea}	tip roll length of triangular end relief modification	8.5.2
L_{Ef}	root roll length of triangular end relief modification	8.5.2
L_y	length of roll to y-cylinder	4.4.10
L_{yt}	length of involute profile to y-cylinder	4.4.11
l_{max}	maximum length of a contact line	5.5.10
M_y	a point on a tooth flank where radius of curvature is calculated	7.1
m	module	4.3.7
m_n	normal module	4.3.7
m_t	transverse module	4.3.7
m_x	axial module	4.3.7
N	number of tooth or pitch	4.2.6
n_a	rotational speed of driving gear (rpm)	5.3.2
n_b	rotational speed of driven gear (rpm)	5.3.2
O	centre of a circle	6.3.4
P_d	diametral pitch	4.3.8
P_{nd}	normal diametral pitch	4.3.8
p_{bn}	normal base pitch	4.5.5.1
p_{bt}	transverse base pitch	4.5.5.1
p_{en}	normal base pitch on the path of contact	4.5.5.3
p_{et}	transverse base pitch on the path of contact	4.5.5.2
p_n	normal pitch on the reference cylinder	4.5.2.2
p_t	transverse pitch on the reference cylinder	4.5.2.1
p_{wn}	normal pitch at the working diameter	6.3.6, 6.5.2
p_x	axial pitch	4.5.4

Table 1 *(continued)*

Symbol	Description	Subclause
p_{yn}	normal pitch on the y-cylinder	4.5.3
p_{yt}	transverse pitch on the y-cylinder	4.5.3
p_z	lead	4.4.6
pr	protuberance of the tool (as seen in ISO 6336-3)	9.2.1
q	machining allowance on tooth flank	9.3
q_{Fs}	magnitude (amount) of undercut in transverse plane	8.2.1
R'	first principal radius of curvature of surface	7.4
R''	second principal radius of curvature of surface	7.4
R_c	radius of curvature of trochoid at point M	10.4
R_{fp}	radius of curvature of the basic rack profile at point Q	10.4
$R_{tro\text{-}y}$	radius of the fillet at point Q	10.4
r_{a0}	tip radius of the pinion-type cutter	11.1.1
r_{b0}	base radius of the pinion-type cutter	11.2
r_{ea}	x axis of ellipse	10.4
r_{eb}	y axis of ellipse	10.4
r_{Fa0}	tip form radius of the pinion-type cutter	11.1.1
r_{Ff}	root form radius	10.3
r_{inv}	radius to point on involute	10.1, 11.4.4.1
r_{M0}	radius for the centre of the tool tooth tip rounding of the pinion-type cutter	11.1.1
r_{tro}	radius to point on trochoid	10.1
r_w	manufacturing pitch circle radius of the gear	11.4.3
r_{w0}	manufacturing pitch circle radius of the pinion-type cutter	11.4.3
$r_{ya0}(\theta_M)$	radial polar coordinate of point M on tip radius of pinion-type cutter	Figure B.1
$r_{yft}(\theta_M)$	radial polar coordinate of point M on the gear fillet generated with pinion-type cutter	
S_α	twist of the transverse profile	8.5.3
S_β	twist of the flank line	8.5.3
s_{aK}	tip transverse tooth thickness when tip chamfering or tip rounding	8.2.2
s_n	normal tooth thickness at the reference diameter	4.8.6
s_{ni}	minimum normal tooth thickness at the reference diameter	9.4
s_{ns}	maximum normal tooth thickness at the reference diameter	9.4
s_P	tooth thickness of the basic rack tooth profile	4.3.3
s_{pr}	residual fillet undercut (on normal surface)	10.1
s_{prt}	residual fillet undercut (transverse plane)	11.4.1
s_t	transverse tooth thickness at the reference diameter	4.8.2
s_{wn}	normal tooth thickness at working diameter	6.3.6, 6.5.2
s_{wt}	transverse tooth thickness at working diameter	6.3.8
s_{yn}	normal tooth thickness at the Y circle diameter	4.8.6
s_{yt}	transverse tooth thickness at the Y circle diameter	4.8.2
T	tangent point on base circle of line normal to involute	4.4.9
T_1	point of contact between the line of action and the base circle of pinion	5.5.6.1
T_2	point of contact between the line of action and the base circle of gear wheel	5.5.6.1
T_1	unit vector of reference helix	4.10
T_2	unit vector of generator	4.10

Table 1 *(continued)*

Symbol	Description	Subclause
T_{M1}	tangency point between the base cylinder of the pinion and a line normal to a contact line through point M_y	7.1
T_{M2}	tangency point on the base cylinder of the gear wheel and a line normal to a contact line through point M_y	7.1
U	involute point of origin	4.4.9
u	gear ratio	5.3.1
v	circumferential velocity at reference diameter	5.7.2
v_b	circumferential velocity at base diameter	5.7.2
v_g	sliding velocity	5.7.5
v_{ga}	sliding velocity at the active addendum	5.7.5
v_{gf}	sliding velocity at the active dedendum	5.7.5
v_{Mg}	sliding velocity at point M	6.6.4
v_n	normal velocity	5.7.3
v_{ry}	rolling velocity at diameter d_y	5.7.4
v_w	circumferential velocity of the working pitch circles	5.7.2, 5.7.6
v_y	circumferential velocity at diameter d_y	5.7.2
x	profile shift coefficient	4.3.10
x_E	generating profile shift coefficient	9.2.1
x_{Ei}	lower limit generating profile shift coefficient	9.5
x_{Es}	upper limit generating profile shift coefficient	9.5
x_{EsV}	profile shift coefficient for rough machining, upper limit	9.5
x_{EiV}	profile shift coefficient for rough machining, lower limit	9.5
x_{Eu}	generating profile shift coefficient at undercut limit	9.8
x_0	profile shift coefficient of the pinion-type cutter	11.1.1
X_1	point on x axis of coordinate system of surface 1	D.1
X_2	point on x axis of coordinate system of surface 2	D.1
Y	any point on a tooth flank or involute	4.4.3
Y_1	point on y axis of coordinate system of surface 1	D.1
Y_2	point on y axis of coordinate system of surface 2	D.1
z	number of teeth	4.2.5
z_a	number of teeth of driving gear	5.3.2
z_b	number of teeth of driven gear	5.3.2
z_0	number of teeth of pinion-type cutter	11.1.1
Z_1	point on z axis of coordinate system of surface 1	D.1
Z_2	point on z axis of coordinate system of surface 2	D.1
α_a	pressure angle at tip circle	5.5.8.3
α_{Fa}	pressure angle at the tip form diameter d_{Fa}	A.2.1
α_{Fa0}	pressure angle on the radius r_{Fa0} of pinion-type cutter	11.1.1
α_{Ff}	pressure angle at root form circle	9.7
α_{KP0}	normal chamfering pressure angle of the counterpart rack tooth profile	8.2.2
α_{KP0t}	transverse chamfering pressure angle of the counterpart rack tooth profile	8.2.2
α_{Mt0}	transverse pressure angle for the radius at the point M of the pinion-type cutter	11.1.1
α_{NP}	pressure angle at start of active profile	5.5.2.2
α_n	normal pressure angle	4.4.4
α_P	pressure angle of the basic rack tooth profile	4.4.4