Standard Practice for
Definition and Determination of Thermionic Constants of Electron Emitters[1]

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INTRODUCTION

Cathode materials are often evaluated by an emission test which in some ways measures the temperature-limited emission. A more basic approach to this problem is to relate the emission to fundamental properties of the emitter, in particular, the work function. Comparisons are conveniently made between emitters using the thermionic constants, that is, the work function, the emission constant, and the temperature dependence of the work function. These quantities are independent of geometry and field effects when properly measured. Although referred to as "constants" these quantities show variations under different conditions. Considerable confusion exists over the definition, interpretation, and usage of these terms and, hence, there is a need for at least a general agreement on nomenclature.

1. Scope

1.1 This practice covers the definition and interpretation of the commonly used thermionic constants of electron emitters **(1, 2, 3)**,[2] with appended standard methods of measurement.

2. Referenced Documents

2.1 *ASTM Standards:*
F 8 Practice for Testing Electron Tube Materials Using Reference Triodes[3]

3. Terminology

3.1 *Definitions:*

3.1.1 *effective work function,* ϕ—the work function obtained by the direct substitution of experimentally determined values of emission current density and temperature into the Richardson-Dushman equation of electron emission of the form:

$$J = AT^2 e^{-e\phi/kT} \tag{1}$$

For direct calculation of the work function, this is conveniently put in the form:

$$\phi = (kT/e) \ln (AT^2/J) \tag{2}$$

where:

J = emission current density in A/cm^2 measured under specified field conditions except zero field. (J_0 = emission current density in A/cm^2 measured under zero field conditions.)

A = the theoretical emission constant, which is calculated from fundamental physical constants, with its value generally taken as 120 $A/cm^2 \cdot K^2$. A more exact calculation **(3)** gives 120.17 which is used in determining the effective work function.

T = cathode temperature, K.

e = electronic charge, C.

e = natural logarithmic base.

k = Boltzmann's constant.

ϕ = work function, V.

The form of Eq 1 is a simplified form of the emission equation which assumes zero reflection coefficient for electrons with energy normally sufficient for emission at the emitter surface. The effective work function is an empirical quantity and represents an average of the true work function, giving the maximum information obtainable from a single measurement of the thermionic emission.

3.1.2 *Richardson work function,* ϕ_0—the work function usually obtained graphically from a Richardson plot, which is a plot of $\ln (J/T^2)$ versus l/T using data of emission measurements at various temperatures. It is the work function obtained from Eq 1, with the value of A determined graphically, instead of using the theoretical value. For better visualization of the Richardson plot, Eq 1 may be put in the form:

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[2] The boldface numbers in parentheses refer to references at the end of this practice.
[3] Discontinued, see *1971 Annual Book of ASTM Standards*, Part 43.

$$\ln (J/T^2) = \ln A - (e/kT)\phi_0 \tag{3}$$

It can be seen (Fig. X1.4) that the Richardson work function ϕ_0 is obtained from the slope of the graph, and the emission constant A from the intercept ($1/T = 0$) on the $\ln (J/T^2)$ axis. The Richardson work function is also an empirical quantity. Its value is found with reasonable accuracy from the graph. However, large errors in the value of A may be expected (4). Considering only one factor, a slight inaccuracy in the measurement of temperature introduces a large error in the value of A. Values of A obtained on practical emitters can range from about 0.1 to 200 A/cm^2·K^2.

3.1.3 *true work function,* ϕ_t—the difference between the Fermi energy and the surface potential energy, which is the maximum potential energy of an electron at the surface of the emitter, or the energy just necessary to remove an electron from the emitter. The true work function, ϕ_t, is expressed in volts or sometimes as $e\phi_t$ in electron volts. For a polycrystalline surface, the true work function will vary with position on the surface. It will also be a function of temperature. The true work function is primarily a theoretical concept used in analysis involving a theoretical model of the surface.

4. Interpretation and Relation of Terms

4.1 Both the effective (ϕ) and the Richardson (ϕ_0) work functions are derived from the same basic equation for electron emission. They differ in the manner of applying the equation. The effective work function represents a direct computation using the theoretical value of the emission constant A of the equation. The Richardson work function is based on a plot of emission data at different temperatures from which both the work function and emission constant were obtained. Work function varies slightly with temperature. If this variation is approximately linear, it can be expressed as a simple temperature coefficient of the work function, α, V/K. Under these conditions, the emission data yield a straight-line Richardson plot and, also, result in a straight-line plot of effective work function with temperature. These and other relations can be seen by introducing α into the Richardson-Dushman equation (Eq 1) and considering the Richardson work function as

representing the value at 0 K. The effective work function at temperature T is then equal to $\phi_0 + \alpha T$. Substituting this into the equation gives:

$$J = AT^2 e^{-(e/kT)(\phi_0 + \alpha T)} \tag{4}$$

which can be put in the form:

$$J = (Ae^{-e\alpha/k})T^2 e^{-e\phi_0/kT} \tag{5}$$

It can be seen from Eq 5 that a Richardson plot slope would determine ϕ_0 and a value of the emission constant $e^{-e\alpha/k}$ times the theoretical value A. The form of Eq 4 is that used for calculation of the effective work function, with $\phi_0 + \alpha T$ substituted for the effective work function ϕ. It can be seen that ϕ_0, the value at zero temperature, is what would be obtained from a straight-line Richardson plot. These observations are summarized in the following equations:

$$\phi = \phi_0 + \alpha T \tag{6}$$

$$(\text{Theoretical } A/\text{Richardson } A) = e^{e\alpha/k} \tag{7}$$

$$\alpha(k/e) \ln (\text{Theoretical } A/\text{Richardson } A) \tag{8}$$

The above expressions are useful in equating and interpreting the effective and Richardson constants. For example, if the thermionic constants of an emitter are specified by the effective work function and temperature coefficient, the equivalent Richardson work function and emission constant may be calculated from the equation. Although α as determined here serves the purpose of relating the work functions, it should not be regarded as a true measure of the temperature coefficient. Other methods, such as the cathode cooling effect of electron emission, are available for a more valid determination (4). The temperature dependence of the effective work function involves many factors such as the presence of a reflection coefficient, the effects of averaging over a nonuniform surface, a temperature dependence of Fermi energy and any errors in measuring the temperature (including gradients) and effective area of the cathode; on aged cathodes interface impedance may be a factor.

5. Keywords

5.1 electron emitters; electron tube materials; thermionic constants; work function

APPENDIX

(Nonmandatory Information)

X1. EXAMPLES FOR DETERMINING THERMIONIC CONSTANTS OF CATHODES

X1.1 The following examples illustrate two customary methods for determining the thermionic constants of cathodes including procedures for establishing the emission current at zero field. Other methods are discussed in the literature (1, 2, 3, 4).

X1.1.1 *Example 1—The Retarding Potential Method* (4)— To determine the emission at zero field, the emission current from a cathode is measured by varying the collecting voltage

from 2 or 3 V negative to 2 to 5 V positive. The logarithm of the measured emission current is plotted as a function of the applied voltage for a given cathode temperature (Fig. X1.1). An extrapolation of the two straight portions of the curve leads to an intersection. At the intersection the retarding field is zero and, hence, this point determines the zero field emission, J_0. The effective work function at temperature T is obtained by substituting the values of J_0 and T in Eq 2. For purposes of