



Designation: **E481 – 10 E481 – 15**

Standard Test Method for Measuring Neutron Fluence Rates by Radioactivation of Cobalt and Silver¹

This standard is issued under the fixed designation E481; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers a suitable means of obtaining the thermal neutron fluence rate, or fluence, in well moderated nuclear reactor environments where the use of cadmium, as a thermal neutron shield as described in Test Method E262, is undesirable because of potential spectrum perturbations or of temperatures above the melting point of cadmium.

1.2 This test method describes a means of measuring a Westcott neutron fluence rate (**Note 1**) by activation of cobalt- and silver-foil monitors (See Terminology **E170**). The ~~reaction~~ reaction $^{59}\text{Co}(n,\gamma)^{60}\text{Co}$ results in a well-defined gamma emitter having a half-life of 1925.28 days (**1**).² The ~~reaction~~ reaction $^{109}\text{Ag}(n,\gamma)^{110\text{m}}\text{Ag}$ results in a nuclide with a complex decay scheme which is well known and having a half-life of 249.76 days (**1**). Both cobalt and silver are available either in very pure form or alloyed with other metals such as aluminum. A reference source of cobalt in aluminum alloy to serve as a neutron fluence rate monitor wire standard is available from the National Institute of Standards and Technology (NIST) as Standard Reference Material 953.³ The competing activities from neutron activation of other isotopes are eliminated, for the most part, by waiting for the short-lived products to die out before counting. With suitable techniques, thermal neutron fluence rate in the range from $10^9 \text{ cm}^{-2} \cdot \text{s}^{-1}$ to $3 \times 10^{15} \text{ cm}^{-2} \cdot \text{s}^{-1}$ can be measured. For this method to be applicable, the reactor must be well moderated and be well represented by a Maxwellian low-energy distribution and an ($1/E$) epithermal distribution. These conditions are usually met in positions surrounded by hydrogenous moderator without nearby strongly absorbing materials. Otherwise the true spectrum must be calculated to obtain effective activation cross sections over all energies.

NOTE 1—Westcott fluence rate $= v \int_0^\infty n(v) dv$.

1.3 The values stated in SI units are to be regarded as the standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*⁴

E170 Terminology Relating to Radiation Measurements and Dosimetry

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E181 Test Methods for Detector Calibration and Analysis of Radionuclides

E261 Practice for Determining Neutron Fluence, Fluence Rate, and Spectra by Radioactivation Techniques

E262 Test Method for Determining Thermal Neutron Reaction Rates and Thermal Neutron Fluence Rates by Radioactivation Techniques

3. Significance and Use

3.1 ~~The pertinent data for these two reactions are given in Table 1.~~ This test method uses one monitor (cobalt) with a nearly $1/v$ absorption cross-section curve and a second monitor (silver) with a large resonance peak so that its resonance integral is large

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² The boldface numbers in parentheses refer to references listed at the end of this test method.

³ Standard Reference Material 953 is available from National Institute of Standards and Technology, U.S. Dept. of Commerce, Washington, DC 20234.

⁴ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

compared to the thermal cross section. The pertinent data for these two reactions are given in Table 1. The equations are based on the Westcott formalism ((2, 3) and Practice E261) and determine a Westcott 2200 m/s neutron fluence rate $\nu\nu_0$ and the Westcott epithermal index parameter $k\sqrt{T/T_0}\sqrt{T/T_0}$. References 4, 5, and 6 contain a general discussion of the two-reaction test method. In this test method, the absolute activities of both cobalt and silver monitors are determined. This differs from the test method in the references wherein only one absolute activity is determined.

3.2 The advantages of this test method are the elimination of three difficulties associated with the use of cadmium: (1) the perturbation of the field by the cadmium; (2) the inexact cadmium cut-off energy; (3) the low melting temperature of cadmium. In addition, the reactivity changes accompanying the rapid insertion and removal of cadmium may prohibit the use of the cadmium-ratio method. However, the self-shielding corrections remain important unless the concentrations of cobalt and silver are small. Studies indicate that the accuracy of the two-reaction method for determination of thermal neutron fluence is comparable to the cadmium-ratio method (14).

3.3 The long half-lives of the two monitors permit the determination of fluence for long-term monitoring.

4. Apparatus

4.1 *NaI(Tl) or Germanium Gamma-Ray Spectrometer* (using a multichannel analyzer)—For the NaI(Tl) technique and the germanium technique, see Method Test Methods E181.

4.2 *Precision Balance.*

4.3 *Digital Computer.*

5. Materials and Manufacture

5.1 The two monitors required for this test method are cobalt and silver. Although these two materials are available commercially in very pure form, they have been used (15) alloyed with aluminum ($\leq 1\%$ cobalt and $\leq 1\%$ silver) to minimize the self-shielding effect and to permit insertion into a high thermal-neutron fluence rate ($>10^{15} \text{ cm}^{-2}\text{s}^{-1}$) facility (6, 16). Typical alloys contain 0.1 % silver or cobalt in aluminum) see 6.1 and 8.1).

TABLE 1 Recommended Constants

Symbol	Parameter	Cobalt (^{60}Co)		Silver ($^{110\text{m}}\text{Ag}$)	
		Value ^A	Reference	Value ^A	Reference
$t_{1/2}$	Half-life	1925.28 (14) days	(1)	249.76 (4) days	(1)
A	Abundance of parent isotope	100 % (^{59}Co)	(†)	48.161 (8) % (^{109}Ag)	(†)
Δ	Abundance of parent isotope	100 % (^{59}Co)	(1)	48.161 (8) % (^{109}Ag)	(1)
Δ	Mass excess of residual isotope (scaled to $\Delta(^{12}\text{C}) = 0$) (1 amu = 931.494 MeV) ^B	-61.64904 MeV	(†)	-87.3424 MeV	(†)
σ_a	Absorption 2200 m/s cross section for target ^{59}Co and ^{109}Ag	37.233 b \pm 0.16 %	C,D	91.0 b \pm 1 %	(7)
σ_a	Absorption 2200 m/s cross section for target ^{59}Co and ^{109}Ag	37.233 b \pm 0.16 %	B,C	91.0 b \pm 1 %	(7)
σ_0	2200 m/s cross section for formation of ^{60}Co and $^{110\text{m}}\text{Ag}$	37.233 b \pm 0.16 %	C,D	4.12 b \pm 2.54 %	(8)
σ_0	2200 m/s cross section for formation of ^{60}Co and $^{110\text{m}}\text{Ag}$	37.233 b \pm 0.16 %	B,C	4.12 (10)	(8)
S_0	Correction factor which describes the departure of the cross section from the 1/v law in the epithermal region	1.80 [$^{59}\text{Co}(n,\gamma)^{60}\text{Co}$]	E	18.13 \pm 4 % [$^{109}\text{Ag}(n,\gamma)^{110\text{m}}\text{Ag}$] 17.76 [$^{109}\text{Ag}(n,\gamma)^{110\text{m}}+^{110\text{g}}\text{Ag}$]	(8)
S_0	Correction factor which describes the departure of the cross section from the 1/v law in the epithermal region	1.80 [$^{59}\text{Co}(n,\gamma)^{60}\text{Co}$]	D	18.1(7) [$^{109}\text{Ag}(n,\gamma)^{110\text{m}}\text{Ag}$]	(8)
I_0	Resonance Integral	75.421 b \pm 0.77 % [$^{59}\text{Co}(n,\gamma)^{60}\text{Co}$]	(9) ^E	67.9 b \pm 4.5 % [$^{109}\text{Ag}(n,\gamma)^{110\text{m}}\text{Ag}$]	(8)
I_0	Resonance Integral	75.421 b \pm 0.77 % [$^{59}\text{Co}(n,\gamma)^{60}\text{Co}$]	(9) ^E	67.9 (31) b [$^{109}\text{Ag}(n,\gamma)^{110\text{m}}\text{Ag}$]	(8)
σ_2	Effective absorption cross section for product nuclide (reactor spectrum)	2 b	(10)	82 b	(11)
G_{th}	Thermal neutron self-shielding factor	Table 3	(12)	$\cong 1 - 4/3 R \Sigma_a$	(4)
G_{res}	Resonance neutron self-shielding factor	Table 3	(12)	Fig. 1 ^G	
G_{res}	Resonance neutron self-shielding factor	Table 3	(12)	Fig. 1 ^F	
g	Correction factor which describes the departure of the cross section from 1/v law in thermal region	1.0	(2)	See Table 4	(2)

^A The numbers in parenthesis following given values are the uncertainty in the last digit(s) of the value; 0.729 (8) means 0.729 \pm 0.008, 70.8(1) means 70.8 \pm 0.1.

^B Isotopic masses may be calculated as $A + 931.49432/\Delta$, where A is the atomic mass number.

^C A 2200 m/s cross section (E = 0.0253 eV, T = 20°C) was taken from the sources indicated in Ref (9).

^D Cross section uncertainty data is taken from Ref (7), the cross section comes from the other reference.

^E Calculated using Eq 510.

^F Cross section uncertainty comes from covariance data provided in the cross section source. The other reference indicates the source of the cross section.

^G In Fig. 1, $\Theta = 4E_kT/AT^2 = 0.2$ corresponds to the value for ^{109}Ag for $T = 293 \text{ K}$, $\Sigma_r = N_0\sigma_{r, \text{max}} - \sigma_{r, \text{max}} = 29999 - 29999 \text{ barn}$ at 5.19 eV (13).

5.2 The uncertainties and nonuniformity of alloy concentrations must be established by one or more different test methods. These might include chemical and activation analysis, or spectrometry. The purity of the aluminum matrix should also be established.

5.3 Whenever possible, the alloys should be tested for interfering impurities by neutron activation.

5.4 The method of encapsulating the monitors for irradiation depends upon the characteristics of the facility in which the measurements are to be made. The monitors have essentially the same chemical characteristics as pure aluminum; therefore, an environment in which aluminum would not be adversely affected would be generally satisfactory for the alloys. However, the low mechanical strength of the monitors requires in many instances that it be encapsulated or shielded from physical disturbances by some type of container. Aluminum cans or tubing are satisfactory for many cases of interest, but for hostile environments, stainless steel or vanadium may be preferable. Perturbation due to the presence of the container must be accounted for, especially in the case of stainless steel. The container should be constructed in such a manner that it will not create a significant flux perturbation and that it may be opened easily, especially if the monitors must be removed remotely.

6. Westcott Neutron Fluence Convention

6.1 The Westcott thermal neutron fluence convention is designed primarily for calculations involving reactions rather than those involving scattering or diffusion. It states that the reaction rate per atom present, R , is equal to the product of an effective cross section, $\hat{\sigma}$, with the Westcott neutron fluence $\phi_w = nv_0$, where n = the neutron density, including both thermal and epithermal neutrons, cm^{-3} , and $v_0 = 2200$ m/s.

so that:

$$R = \phi_w \hat{\sigma} = nv_0 \hat{\sigma} \quad (1)$$

The true equation for reaction rate is given by the equation:

$$R = \int_0^{\infty} n(v) v \sigma(v) dv \quad (2)$$

where:

$n(v)$ = neutron density per unit velocity,

v = neutron velocity, and

$\sigma(v)$ = cross section for neutrons of velocity v .

Therefore, the effective cross section is defined by the equation:

$$\hat{\sigma} = \int_0^{\infty} n(v) v \sigma(v) dv / nv_0 \quad (3)$$

The neutron spectrum assumed by Westcott has the form: $n(v) = n(1-f)P_m(v) + n f P_e(v)$, where P_m and P_e are the Maxwellian and epithermal density distribution functions normalized so that: $\int_0^{\infty} P_m(v) dv = \int_0^{\infty} P_e(v) dv = 1$. The quantity f is the fraction of the total density, n , in the epithermal distribution. The epithermal distribution is assumed proportional to $1/E$ per unit energy interval. This distribution is terminated by a cut-off function at a suitable lower limit of energy. Based on the above spectrum, one obtains the following relation for the effective cross section:

$$\hat{\sigma} = \sigma_0(g + rs) \quad (4)$$

where:

σ_0 = cross section of 2200 m/s neutrons,

g = a measure of the departure of the cross section from $1/v$ dependence in the thermal region,

s = $S_0 \sqrt{T/T_0}$, a factor which describes the departure of the cross section from the $1/v$ law in the epithermal region, including resonance effects, and

r = a measure of the proportion of epithermal neutrons in the reactor spectrum.

More specifically:

$$r = f \sqrt{\pi \mu_n} / 4 \quad (5)$$

where:

f = fraction of the total density in the epithermal distribution, and

μ_n = a factor chosen to give the proper normalization to the epithermal density distribution. A suitable factor for water moderated systems is 5 (2).

6.2 *Limitation of the Westcott Convention*—Sufficient conditions for the applications of the Westcott convention are that:

$$\sum_s / \xi \sum_s < 0.1 \quad (6)$$

and:

$$T/T_m < 1.07 \tag{7}$$

where:

- Σ_a = macroscopic absorption cross section averaged over all materials affecting spectrum,
- ξ = average logarithmic energy decrement per collision,
- Σ_s = macroscopic scattering cross section averaged over all materials affecting spectrum,
- T = neutron temperature, K, and
- T_m = temperature of the moderator, K.

If as a result of neutron captures (for example, in the fuel) the quantity $\Sigma_a/\xi\Sigma_s$ becomes too great or if the neutron temperature T is too great relative to the moderator temperature T_m , the Maxwell spectrum hypothesis fails and the true spectrum must be calculated and the effective cross section determined with this spectrum.

6.3 The conventional 2200 m/s thermal neutron-fluence rate, ϕ_0 , and the epithermal fluence-rate parameter, ϕ_e , as defined in Test Method E262, can be obtained from the Westcott neutron-fluence rate, ϕ_w , and the Westcott epithermal index, $r\sqrt{T/T_0}$, by means of equations Eq 8 and Eq 9:

$$\phi_0 = \left(1 - \frac{4r}{\sqrt{\pi\mu_n}}\right) \phi_w \tag{8}$$

$$\phi_e = \frac{2}{\sqrt{\pi}} r \sqrt{\frac{T}{T_0}} \phi_w \tag{9}$$

6.4 In Eq 8, it is necessary to estimate the neutron temperature, T , in order to obtain the value of r from the index $r\sqrt{T/T_0}$. Provided inequality (Eq 7) is satisfied, only slight error is introduced by assuming $T = T_m$, the moderator temperature.

6.5 Although the $Ag^{109}(n,\lambda)Ag^{110m}$ S_0 value in Table 1 is a measured value, S_0 can be calculated by the following equation:

$$S_0 = \frac{2}{\sqrt{\pi}} \frac{I''_0}{\sigma_0} = \frac{2}{\sqrt{\pi}} \left(\frac{I_0}{\sigma_0} - 2g \sqrt{\frac{E_0}{E_{Cd}}} \right) \tag{10}$$

where:

- I''_0 = resonance integral excess over the $1/v$ cross section value, cm^2 ,
- σ_0 = 2200 m/s cross-section value, cm^2 ,
- I_0 = resonance integral, $\int_{E_{Cd}}^{\infty} \frac{\sigma(E)}{E} dE$
- E_0 = 0.0253 eV, and
- E_{Cd} = 0.55 eV.

7. Procedure

7.1 Decide on the size and shape of the monitors to be irradiated, taking into consideration the size and shape of the irradiation space. The mass and exposure time are parameters which can be varied to obtain a desired disintegration rate for a given neutron fluence rate level. To facilitate the convergence of the two activity equations for the fluence rate and the epithermal index in Section index, 7, the concentration of the alloys should be chosen so that the ratio of the disintegration rates is on the order of one.

7.2 Weigh the samples to a precision of $\pm 1.0\%$ (1S %) as defined in Practice E177.

7.3 Irradiate the samples for the predetermined time period. Record the power level and any changes in power during the irradiation, the time at the beginning and end of the irradiation, and the relative position of the monitors in the irradiation facility.

7.4 A waiting period is necessary between termination of the exposure and start of counting when using Co-Al and Ag-Al monitors. This allows the 0.62356 days (17) half-life ^{24}Na which is formed by fast-neutron reactions on ^{27}Al or by thermal-neutron captures by ^{23}Na impurities to decay below levels at which its radiations may cause interferences. It is sometimes advisable to count the samples periodically and follow the decay of the portions of the activities due to the ^{24}Na . The length of the waiting period can be reduced by the use of a germanium detector.

7.5 With the gamma-ray spectrometer, analyze the silver sample for ^{110m}Ag and the cobalt sample for ^{60}Co . Obtain the net count rate in each full-energy gamma-ray peak of interest, that is, 657.7623 keV or 884.684 keV for ^{110m}Ag ; 1332.501 keV for ^{60}Co (see Method-Test Methods E181). See Table 2 for gamma radiations of ^{110m}Ag .

8. Calculation

8.1 Calculate the activities of ^{110m}Ag and ^{60}Co in disintegrations per second.

8.2 A Westcott 2200 m/s neutron fluence rate, $n\nu_0$, or ϕ_w and the Westcott epithermal index parameter, $r\sqrt{T/T_0}$ are related to the measured activities of the silver and cobalt monitors by the following equation:

$$A = N_0 \lambda BFG \delta_i \phi_w t_i \tag{11}$$

TABLE 2 Gamma Radiations of ^{110m}Ag (17,18)

	Energy of Gamma ^A (keV)	Relative ^{B,A} Emission Probability-(%)
-1.	657.7600 (11)	100
-2.	884.6781 (13)	77.1 (3)
-3.	937.485 (3)	36.3 (6)
-4.	1384.2931 (20)	26.4 (8)
-5.	763.9424 (17)	23.98 (21)
-6.	706.6760 (15)	17.31 (5)
-7.	1505.0280 (20)	14.42 (19)
-8.	677.6217 (12)	11.20 (2)
-9.	818.0244 (18)	7.78 (8)
10.	687.0091 (18)	6.83 (5)
11.	744.2753 (18)	5.06 (9)
12.	1562.294 (18)	1.319 (17)

TABLE 2 Gamma Radiations of ^{110m}Ag (18)^A

	Energy of Gamma ^B (keV)	Intensity ^B (%)
1.	657.7600 (11)	95.61
2.	884.6781 (13)	75.00 (11)
3.	937.485 (3)	35.0 (3)
4.	1384.2931 (20)	25.1 (5)
5.	763.9424 (17)	22.60 (7)
6.	706.6760 (15)	16.69 (7)
7.	1505.0280 (20)	13.33 (15)
8.	677.6217 (12)	10.70 (5)
9.	818.0244 (18)	7.43 (4)
10.	687.0091 (18)	6.53 (3)
11.	744.2753 (18)	4.77 (3)
12.	1562.294 (18)	1.22 (3)

^A The number of parentheses following some given values is the uncertainty in the last digit(s) of the value: 0.729 (8) means 0.729 ± 0.008, 80.8 (1) means 70.8 ± 0.1.

^B For absolute intensity (17) multiply emission probabilities by 0.943 ± 0.004 for an alternate source of data.

TABLE 3 Self-Shielding Factors for Cobalt Wires (12)

Wire Diameter in. (mm)	Cobalt Content, (mass %)	G _{res} (132 eV)	G _{th}
0.050 (1.27)	0.104	1.00	1.00
0.050 (1.27)	0.976	0.95 ± 0.04	0.99 ± 0.01
0.001 (0.03)	100	0.81 ± 0.03	0.99 ± 0.02
0.005 (0.13)	100	0.52 ± 0.02	0.97 ± 0.01
0.010 (0.25)	100	0.42 ± 0.02	0.94 ± 0.01
0.015 (0.38)	100	0.38 ± 0.01	0.92 ± 0.02
0.020 (0.51)	100	0.34 ± 0.01	0.90 ± 0.02
0.025 (0.64)	100	0.32 ± 0.01	0.88 ± 0.03

where:

A = measured activity at the end of the exposure time, disintegrations/s,

N₀ = number of target atoms of ⁵⁹Co or ¹⁰⁹Ag at start of irradiation,

λ = disintegration constant of product nuclide, s⁻¹,

B = Self-absorption factor of the decay gamma ray in the monitor material,

F = burnup and decay correction factor,

G = self-shielding factor (see Eq 4, Table 3 and Fig. 1).

Ḡ = self-shielding factor (see Eq 15, Table 3 and Fig. 1).

σ̂₁ = Westcott's effective absorption cross section for production of the product nuclide, cm²,

φ_w = a 2200 m/s neutron fluence rate in which n is the neutron density (including both thermal and epithermal neutrons) and (σ_wv₀) t_i is 2200 m/s, and

φ_w = a 2200 m/s neutron fluence rate in which n is the neutron density (including both thermal and epithermal neutrons) and v₀ is 2200 m/s, and

t_i = exposure time.

8.3 The self-absorption factor, if not known for the gamma rays being measured, can be approximated by the following equation: equation (20):

$$B \approx 1 - (4/3)(\mu_a R) \tag{12}$$

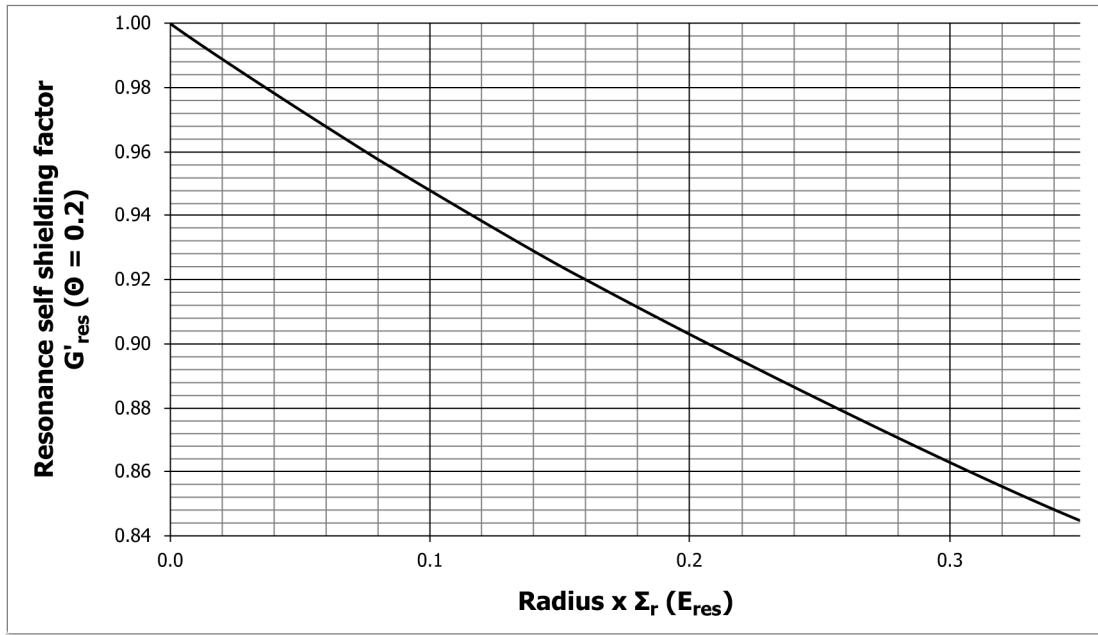


FIG. 1 Resonance Self-Shielding Factor for Cylinders ($\theta = 0.2$) as a Function of Radius \times Macroscopic Absorption Cross Section at the Peak of the Resonance, for ^{109}Ag at 5.19 eV, $\sigma_{\gamma, \max} = 29999$ barn (13). Calculations used shielding curve for cylinders from (19). To convert σ_r to Σ_r , use $\Sigma_r = N_0 \sigma_r$, where N_0 is the number density of ^{109}Ag atoms in the wire.

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where:

- μ_a = linear absorption coefficient in monitor, cm^{-1} , and
- $\underline{\mu}_a$ = linear absorption coefficient in monitor, cm^{-1} (21), and
- R = radius of monitor wire, cm.

8.4 The burnup and decay correction factor is given by:

$$F = \frac{\exp(-\hat{\sigma}_a \phi_w t_i) - \exp(-(\lambda + \hat{\sigma}_2 \phi_w) t_i)}{(\lambda + \hat{\sigma}_2 \phi_w - \hat{\sigma}_a \phi_w) t_i} \tag{13}$$

<https://standards.itih.ai/catalog/standards/sist/d0400912-703a-4a53-93c7-ca2e534e2c56/astm-e481-15>

where:

- $\hat{\sigma}_a$ = Westcott's effective absorption cross section for target nuclide, cm^2 , and
- $\hat{\sigma}_2$ = Westcott's effective absorption cross section for the product nuclide, cm^2 .

8.5 The self-shielding factor is given by:

$$G = \frac{gG_{th} + (r\sqrt{T/T_0})S_0G'_{res}}{g + (r\sqrt{T/T_0})S_0} \tag{14}$$

$$G = \frac{gG_{th} + (r\sqrt{T/T_0})S_0G'_{res}}{g + (r\sqrt{T/T_0})S_0} \tag{14}$$

where:

- g = correction factor which describes the departure of the cross section from the $1/v$ law in the thermal region (see Table 4 for silver "g" factors),
- G_{th} = thermal neutron self-shielding factor,
- G'_{res} = resonance neutron self-shielding factor,
- r = a measure of the proportion of epithermal neutrons in the reactor spectrum,
- T = neutron temperature, K,
- T_0 = 293.6 K, and
- S_0 = correction factor which describes the departure of the cross section from the $1/v$ law in the epithermal region.

7.6 Although the $\text{Ag}^{109}(n,\lambda)\text{Ag}^{110m} S_0$ value in Table 1 is a measured value, S_0 can be calculated by the following equation:

$$S_0 = \frac{2}{\sqrt{\pi}} \frac{I''_0}{\sigma_0} = \frac{2}{\sqrt{\pi}} \left(\frac{I_0}{\sigma_0} - 2g \sqrt{\frac{E_0}{E_{cd}}} \right) \tag{5}$$