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Standard Practice for
Process Capability and Performance Measurement[1]

This standard is issued under the fixed designation E2281; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice provides guidance for determining process capability and performance under several common scenarios of use including: (*a*) normal distribution based capability and performance indices such as C_p, C_{pk}, P_p, and P_{pk}; (*b*) process capability using attribute data for non-conforming units and non-conformities per unit type variables, and (*c*) additional methods in working with process capability or performance.

2. Referenced Documents

2.1 *ASTM Standards:*[2]
E456 Terminology Relating to Quality and Statistics
E2334 Practice for Setting an Upper Confidence Bound For a Fraction or Number of Non-Conforming items, or a Rate of Occurrence for Non-conformities, Using Attribute Data, When There is a Zero Response in the Sample

2.2 *Other Document:*
MNL 7 Manual on Presentation of Data and Control Chart Analysis[3]

3. Terminology

3.1 *Definitions*—Unless otherwise noted, all statistical terms are defined in Terminology E456.

3.1.1 *long term standard deviation*, σ_{LT}, *n*—sample standard deviation of all individual (observed) values taken over a long period of time.

3.1.1.1 *Discussion*—A long period of time may be defined as shifts, weeks, or months, etc.

3.1.2 *process capability, PC, n*—statistical estimate of the outcome of a characteristic from a process that has been demonstrated to be in a state of statistical control.

3.1.3 *process capability index*, C_p, *n*—an index describing process capability in relation to specified tolerance.

3.1.4 *process performance, PP, n*—statistical measure of the outcome of a characteristic from a process that may not have been demonstrated to be in a state of statistical control.

3.1.5 *process performance index*, P_p, *n*—index describing process performance in relation to specified tolerance.

3.1.6 *short term standard deviation*, σ_{ST}, *n*—the inherent variation present when a process is operating in a state of statistical control, expressed in terms of standard deviation.

3.1.6.1 *Discussion*—This may also be stated as the inherent process variation.

3.1.7 *stable process, n*—process in a state of statistical control; process condition when all special causes of variation have been removed.

3.1.7.1 *Discussion*—Observed variation can then be attributed to random (common) causes. Such a process will generally behave as though the results are simple random samples from the same population.

3.1.7.2 *Discussion*—This state does not imply that the random variation is large or small, within or outside of specification, but rather that the variation is predictable using statistical techniques.

3.1.7.3 *Discussion*—The process capability of a stable process is usually improved by fundamental changes that reduce or remove some of the random causes present or adjusting the mean towards the preferred value, or both.

3.1.7.4 *Discussion*—Continual adjustment of a stable process will increase variation.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *lower process capability index*, C_{pkl}, *n*—index describing process capability in relation to the lower specification limit.

3.2.2 *lower process performance index*, P_{pkl}, *n*—index describing process performance in relation to the lower specification limit.

3.2.3 *minimum process capability index*, C_{pk}, *n*—smaller of the upper process capability index and the lower process capability index.

3.2.4 *minimum process performance index*, P_{pk}, *n*—smaller of the upper process performance index and the lower process performance index.

3.2.5 *special cause, n*—variation in a process coming from source(s) outside that which may be expected due to chance causes (or random causes).

3.2.5.1 *Discussion*—Sometimes "special cause" is taken to be synonymous with "assignable cause." However, a distinction should be recognized. A special cause is assignable only when it is specifically identified. Also, a common cause may be assignable.

3.2.5.2 *Discussion*—A special cause arises because of specific circumstances which are not always present. As such, in a process subject to special causes, the magnitude of the variation from time to time is unpredictable.

3.2.6 *upper process capability index, C_{pku}, n*—index describing process capability in relation to the upper specification limit.

3.2.7 *upper process performance index, P_{pku}, n*—index describing process performance in relation to the upper specification limit.

4. Significance and Use

4.1 *Process Capability*—Process capability can be defined as the natural or inherent behavior of a stable process that is in a state of statistical control (**1**).[4] A "state of statistical control" is achieved when the process exhibits no detectable patterns or trends, such that the variation seen in the data is believed to be random and inherent to the process. Process capability is linked to the use of control charts and the state of statistical control. A process must be studied to evaluate its state of control before evaluating process capability.

4.2 *Process Control*—There are many ways to implement control charts, but the most popular choice is to achieve a state of statistical control for the process under study. Special causes are identified by a set of rules based on probability theory. The process is investigated whenever the chart signals the occurrence of special causes. Taking appropriate actions to eliminate identified special causes and preventing their reappearance will ultimately obtain a state of statistical control. In this state, a minimum level of variation may be reached, which is referred to as common cause or inherent variation. For the purpose of this standard, this variation is a measure of the uniformity of process output, typically a product characteristic.

4.3 *Process Capability Indices*—The behavior of a process (as related to inherent variability) in the state of statistical control is used to describe its capability. To compare a process with customer requirements (or specifications), it is common practice to think of capability in terms of the proportion of the process output that is within product specifications or tolerances. The metric of this proportion is the percentage of the process spread used up by the specification. This comparison becomes the essence of all process capability measures. The manner in which these measures are calculated defines the different types of capability indices and their use. Two process capability indices are defined in 5.2 and 5.3. In practice, these indices are used to drive process improvement through continuous improvement efforts. These indices may be used to identify the need for management actions required to reduce common cause variation, compare products from different sources, and to compare processes.

4.4 *Process Performance Indices*—When a process is not in a state of statistical control, the process is subject to special cause variation, which can manifest itself in various ways on the process variability. Special causes can give rise to changes in the short-term variability of the process or can cause long-term shifts or drifts of the process mean. Special causes can also create transient shifts or spikes in the process mean. Even in such cases, there may be a need to assess the long-term variability of the process against customer specifications using process performance indices, which are defined in 6.2 and 6.3. These indices are similar to those for capability indices and differ only in the estimate of variability used in the calculation. This estimated variability includes additional components of variation due to special causes. Since process performance indices have additional components of variation, process performance usually has a wider spread than the process capability spread. These measures are useful in determining the role of measurement and sampling variability when compared to product uniformity.

4.5 Attribute capability applications occur where attribute data are being used to assess a process and may involve the use of non-conforming units or non-conformities per unit.

4.6 Additional measures and methodology to process assessments include the index C_{pm}, which incorporates a target parameter for variable data, and the calculation of Rolled Throughput Yield (RTY), that measures how good a series of process steps are.

5. Process Capability Analysis

5.1 It is common practice to define process behavior in terms of its variability. Process capability, PC, is calculated as:

$$PC = 6\sigma_{ST} \qquad (1)$$

where σ_{ST} is the inherent variability of a controlled process (**2, 3**). Since control charts can be used to achieve and verify control for many different types of processes, the assumption of a normal distribution is not necessary to affect control, but complete control is required to establish the capability of a process (**2**). Thus, what is required is a process in control with respect to its measures of location and spread. Once this is achieved, the inherent variability of the process can be estimated from the control charts. The estimate obtained is an estimate of variability over a short time interval (minutes, hours, or a few batches). From control charts, σ_{ST} may be estimated from the short-term variation within subgroups depending on the type of control chart deployed, for example, average-range ($\bar{X} - R$) or individual-moving range ($\bar{X} - MR$). The estimate is:

$$\hat{\sigma}_{ST} = \frac{\bar{R}}{d_2} \text{ or } \frac{\overline{MR}}{d_2} \qquad (2)$$

where, $\bar{R}$ is the average range, $\overline{MR}$ is the average moving range, d_2 is a factor dependent on the subgroup size, *n*, of the

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[4] The boldface numbers in parentheses refer to the list of references at the end of this standard.

control chart (see ASTM MNL 7, Part 3). If an average-standard deviation ($\bar{X} - s$) chart is used, the estimate becomes:

$$\hat{\sigma}_{ST} = \frac{\bar{s}}{c_4} \qquad (3)$$

where $\bar{s}$ is the arithmetic average of the sample standard deviations, and c_4 is a factor dependent on the subgroup size, n, of the control chart (see ASTM MNL 7, Part 3).

5.1.1 Therefore, PC is estimated by:

$$6 \hat{\sigma}_{ST} = \frac{6\bar{R}}{d_2} \text{ or } \frac{6\bar{s}}{c_4} \qquad (4)$$

5.2 *Process Capability Index, C_P:*

5.2.1 The process capability index relates the process capability to the customer's specification tolerance. The process capability index, C_p, is:

$$C_p = \frac{\text{Specification Tolerance}}{\text{Process Capability}} = \frac{USL - LSL}{6\sigma_{ST}} \qquad (5)$$

where USL = upper specification limit and LSL = lower specification limit. For a process that is centered with an underlying normal distribution, Fig. 1, Fig. 2, and Fig. 3 denotes three cases where PC, the process capability, is wider than (Fig. 1), equal to (Fig. 2), and narrower than (Fig. 3) the specification tolerance.

5.2.2 Since the tail area of the distribution beyond specification limits measures the proportion of product defectives, a larger value of C_p is better. The relationship between C_p and the percent defective product produced by a centered process (with a normal distribution) is:

C_p	Percent Defective	Parts per Million	C_p	Percent Defective	Parts per Million
0.6	7.19	71900	1.1	0.0967	967
0.7	3.57	35700	1.2	0.0320	318
0.8	1.64	16400	1.3	0.0096	96
0.9	0.69	6900	1.33	0.00636	64
1.0	0.27	2700	1.67	0.00006	0.57

5.2.3 From these examples, one can see that any process with a $C_p < 1$ is not as capable of meeting customer requirements (as indicated by % defectives) as a process with values of $C_p \geq 1$. Values of C_p progressively greater than 1 indicate more capable processes. The current focus of modern quality is on process improvement with a goal of increasing product uniformity about a target. The implementation of this focus is to create processes with $C_p > 1$. Some industries consider $C_p = 1.33$ (an $8\sigma_{ST}$ specification tolerance) a minimum with a $C_p = 1.66$ preferred (4). Improvement of C_p should

depend on a company's quality focus, marketing plan, and their competitor's achievements, etc.

5.3 *Process Capability Indices Adjusted For Process Shift, C_{pk}:*

5.3.1 The above examples depict process capability for a process centered within its specification tolerance. Process centering is not a requirement since process capability is independent of any specifications that may be applied to it. The amount of shift present in a process depends on how far the process average is from the center of the specification spread. In the last part of the above example ($C_p > 1$), suppose that the process is actually centered above the USL. The C_p has a value >1, but clearly this process is not producing as much conforming product as it would have if it were centered on target.

5.3.2 For those cases where the process is not centered, deliberately run off-center for economic reasons, or only a single specification limit is involved, C_p is not the appropriate process capability index. For these situations, the C_{pk} index is used. C_{pk} is a process capability index that considers the process average against a single or double-sided specification limit. It measures whether the process is capable of meeting the customer's requirements by considering:

5.3.2.1 The specification limit(s),

5.3.2.2 The current process average, and

5.3.2.3 The current $\hat{\sigma}_{ST}$.

5.3.3 Under the assumption of normality,[5] C_{pk} is calculated as:

$$C_{pk} = \min[C_{pku}, C_{pkl}] \qquad (6)$$

and is estimated by:

$$\hat{C}_{pk} = \min[\hat{C}_{pku}, \hat{C}_{pkl}] \qquad (7)$$

where the estimated upper process capability index is defined as:

$$\hat{C}_{pku} = \frac{USL - \bar{X}}{3 \hat{\sigma}_{ST}} \qquad (8)$$

and the estimated lower process capability index is defined as:

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[5] Testing for the normality of a set of data may range from simply plotting the data on a normal probability plot (2) to more formal tests, for example, Anderson-Darling test (which can be found in many statistical software programs, for example, Minitab).
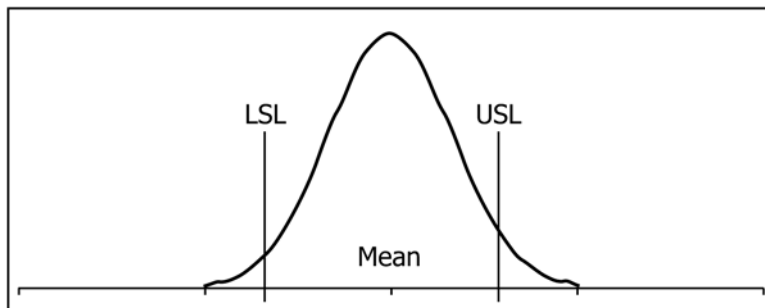

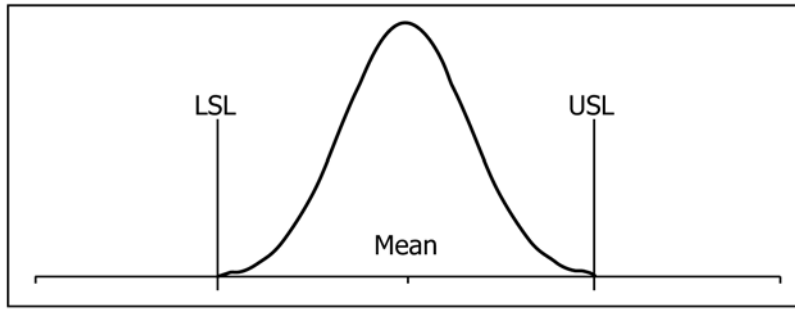FIG. 1 Process Capability Wider Than Specifications, $C_p < 1$

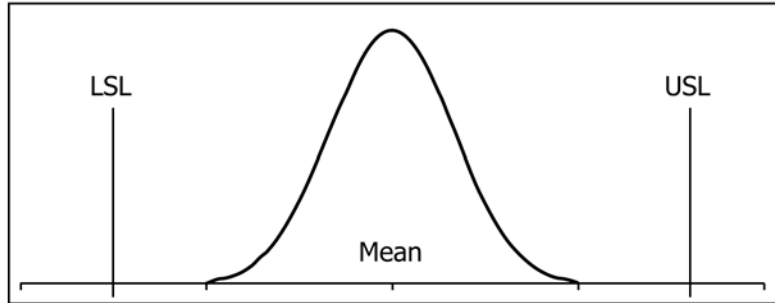**FIG. 2 Process Capability Equal to Specification Tolerance, $C_p = 1$**

FIG. 3 Process Capability Narrower Than Specifications, $C_p > 1$

$$\hat{C}_{pkl} = \frac{\bar{X} - LSL}{3\ \hat{\sigma}_{ST}} \tag{9}$$

5.3.4 These one-sided process capability indices (C_{pku} and C_{pkl}) are useful in their own right with regard to single-sided specification limits. Examples of this type of use would apply to impurities, by-products, bursting strength of bottles, etc. Once again, the meaning of C_{pk} is best viewed pictorially in Fig. 4.

5.3.5 The relationship between C_p and C_{pk} can be summarized (**2**) as:

5.3.5.1 C_{pk} can be equal to but never larger than C_p,

5.3.5.2 C_p and C_{pk} are equal only when the process is centered on target,

5.3.5.3 If C_p is larger than C_{pk}, then the process is not centered on target,

5.3.5.4 If both C_p and C_{pk} are >1, the process is capable and performing within the specifications,

5.3.5.5 If both C_p and C_{pk} are <1, the process is not capable and not performing within the specifications, and

5.3.5.6 If C_p is >1 and C_{pk} is <1, the process is capable, but not centered and not performing within the specifications.

5.4 *Caveats on the Practical Use of Process Capability Indices:*

5.4.1 One must keep the theoretical aspects and assumptions underlying the use of process capability indices in mind when calculating and interpreting the corresponding values of these indices. To review:

5.4.1.1 For interpretability, C_{pk} requires a Gaussian (normal or bell-shaped) distribution or one that can be transformed to a normal. Definition of C_{pk} requires a normal distribution with a spread of three standard deviations on either side of the mean (**2, 5**).

5.4.1.2 The process must be in a state of statistical control (stable over time with constant short-term variability).

5.4.1.3 Large sample sizes (preferably >200 or a minimum of 100) are required to estimate C_{pk} with a high level of confidence (at least 95 %).
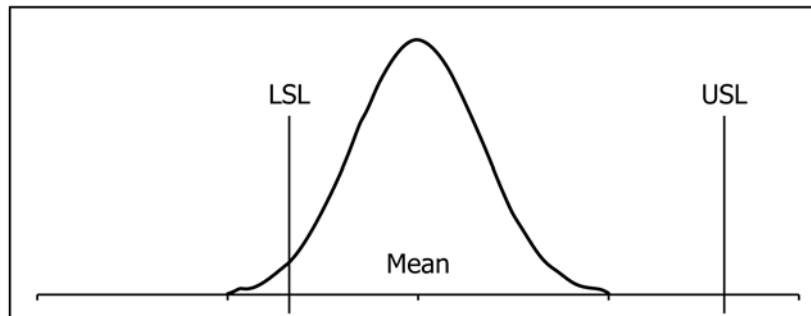

FIG. 4 Noncentered Process, $C_p > 1$ and $C_{pk} < 1$

5.4.1.4 C_p and C_{pk} are affected by sampling procedures, sampling error, and measurement variability. These effects have a direct bearing on the magnitude of the estimate for inherent process variability, the main component in estimating these indices.

5.4.1.5 C_p and C_{pk} are statistics and as such are subject to uncertainty (variability) as found in any statistic.

5.4.2 For additional information about process capability and process capability indices, see Refs (**2, 5, 6**).

6. Process Performance Analysis

6.1 *Process Performance:*

6.1.1 Process performance represents the actual distribution of product and measurement variability over a long period of time, such as weeks or months. In process performance, the actual performance level of the process is estimated rather than its capability when it is in control.

6.1.2 As in the case of process capability, it is important to estimate correctly the process variability. For process performance, the long-term variation, σ_{LT}, (**2, 3**) is estimated. Thus, the accumulated individual production measurements from a process over a long time period, $X_1, X_2, …, X_n$, has an overall sample standard deviation estimated as:

$$\hat{\sigma}_{LT} = \sqrt{\frac{\Sigma\left(X_i - \bar{X}\right)^2}{n - 1}} \quad (10)$$

6.1.3 This standard deviation contains the following "components" of variability: (**6**)

6.1.3.1 Lot-to-lot variability over the long term,

6.1.3.2 Within-lot variability over the short term,

6.1.3.3 Measurement system variability over the long term, and

6.1.3.4 Measurement system variability over the short term.

6.1.4 If the process were in the state of statistical control, one would expect the estimate of σ_{LT}, $\hat{\sigma}_{LT}$, to be very close to the estimate of σ_{ST}, $\hat{\sigma}_{ST}$. One would expect that the two estimates would be almost identical if a perfect state of control were achieved. According to Ott, Schilling, and Neubauer (**2**) and Gunter (**5**), this perfect state of control is unrealistic since control charts may not detect small changes in a process. Such changes give rise to values of $\hat{\sigma}_{LT}$ that are nearly equal but slightly larger than $\hat{\sigma}_{ST}$.

6.1.5 Process performance or process spread is:

$$PP = 6\sigma_{LT} \quad (11)$$

6.2 *Process Performance Index:*

6.2.1 Comparisons of process performance to specification spread result in performance indices that are analogous to process capability indices. The simplest process performance index is P_p, where:

$$P_p = \frac{\text{Specification Tolerance}}{\text{Process Performance}} \quad (12)$$

and is estimated by:

$$\frac{USL - LSL}{6\,\hat{\sigma}_{LT}}$$

6.2.2 The interpretation of P_p is similar to that of C_p. That is, a $P_p \geq 1$ represents a process that has no trouble meeting customer requirements in the long term. A process with $P_p < 1$ cannot meet specifications all the time. In either case, there is no assumption that the process is in the state of statistical control or centered.

6.3 *Process Performance Indices Adjusted For Process Shift:*

6.3.1 For those cases where the process is not centered, deliberately run off-center for economic reasons, or only a single specification limit is involved, P_{pk} is the appropriate process performance index. P_{pk} is a process performance index adjusted for location (process average). It measures whether the process is actually meeting the customer's requirements by considering:

6.3.1.1 The specification limit(s),

6.3.1.2 The current process average, and

6.3.1.3 The current value of $\hat{\sigma}_{LT}$.

6.3.2 Under the assumption of normality, P_{pk} is calculated as:

$$P_{pk} = \min[P_{pku}, P_{pkl}] \quad (13)$$

and is estimated by:

$$\hat{P}_{pk} = \min\left[\hat{P}_{pku}, \hat{P}_{pkl}\right] \quad (14)$$

where:

$$\hat{P}_{pku} = \frac{USL - \bar{X}}{3\,\hat{\sigma}_{LT}} \quad (15)$$

and

$$\hat{P}_{pkl} = \frac{\bar{X} - LSL}{3\,\hat{\sigma}_{LT}} \quad (16)$$

which are the estimates of the one-sided process performance indices.

6.3.3 Values of P_{pk} have an interpretation similar to those for C_{pk}. The difference is that P_{pk} represents how the process is running with respect to customer requirements over a specified long time period. One interpretation is that P_{pk} represents what the producer *makes* and C_{pk} represents what the producer *could make* if its process were in a state of statistical control. The relationship between P_p and P_{pk} are also similar to that of C_p and C_{pk}.

6.4 *Interpretation of Process Performance Indices:*

6.4.1 The caveats around process performance indices are similar to those for capability indices. Of course, two obvious differences pertain to the lack of statistical control and the use of long-term variability estimates.

7. Confidence Bounds for Process Capability Indices

7.1 Capability indices are based on sample statistics and should not be considered as absolute measures of process capability or performance. All of the indices discussed in this standard are based on sample estimates, and are therefore subject to sampling error. The sampling error will be a function of the sample size, n. Generally, the larger the sample size, the more accurate will be the sample estimates $\hat{C}_p$, $\hat{C}_{pk}$, $\hat{P}_p$, or $\hat{P}_{pk}$. It is recommended that some measure of the sampling error be
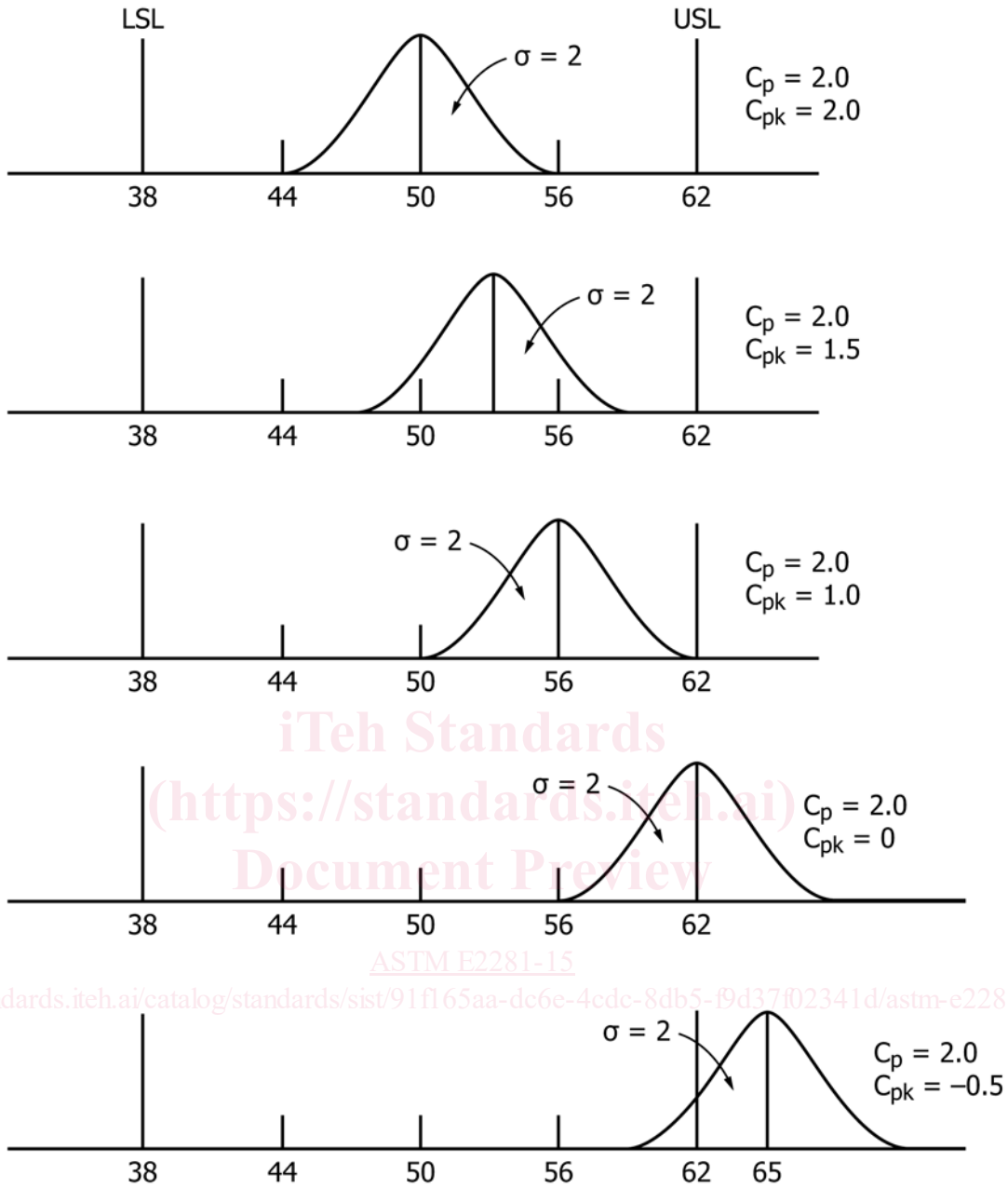
FIG. 5 Theoretical Process Capability Scenarios

calculated whenever these indices are used. Either a standard error of the estimate or a lower confidence bound is the preferred method. These statistics give the user of a capability index some idea of the resulting uncertainty for a given sample size. A lower confidence bound for a process capability index is a statistic that one can claim as the smallest value for the process index, with some stated confidence, say 95 %. It is the lower bound that is of primary interest since it favors the consumer. A consumer is usually interested in the question, "How small might the true process index be?" For example, suppose a consumer requires a P_{pk} of at least 1.33 for a large batch of product. Based on a sample, the supplier shows that the lower 95 % confidence bound for P_{pk} is 1.38. The consumer then has 95 % assurance that the accepted product meets the

process index requirement of 1.33. In accepting the product, the consumer is willing to take a 5 % risk that the true P_{pk} is really less than 1.38; however, this risk is minimal and manageable. To claim that the process index is at least some derived quantity with a high degree of confidence is the assurance that the process is not worse that being claimed as the lower bound.

7.2 When a process is in good statistical control, the short term capability estimates $\hat{C}_p$ and $\hat{C}_{pk}$ and their long term performance equivalent estimates, and will give similar results for any fixed sample size. The results stated below are cast in terms of the long term measures $\hat{P}_p$ and $\hat{P}_{pk}$, but they could just as well be applied to the short term measures when the process