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Standard Guide for Selecting Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Buildings and Building Systems¹

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1. Scope

1.1 This guide covers techniques for treating uncertainty in input values to an economic analysis of a building investment project. It also recommends techniques for evaluating the risk that a project will have a less favorable economic outcome than what is desired or expected.²

1.2 The techniques include breakeven analysis, sensitivity analysis, risk-adjusted discounting, the mean-variance criterion and coefficient of variation, decision analysis, simulation, and stochastic dominance.

1.3 The techniques can be used with economic methods that measure economic performance, such as life-cycle cost analysis, net benefits, the benefit-to-cost ratio, internal rate of return, and payback.

2. Referenced Documents

2.1 ASTM Standards:³

- E631 Terminology of Building Constructions
- E833 Terminology of Building Economics
- E917 Practice for Measuring Life-Cycle Costs of Buildings and Building Systems
- E964 Practice for Measuring Benefit-to-Cost and Savings-to-Investment Ratios for Buildings and Building Systems
- E1057 Practice for Measuring Internal Rate of Return and Adjusted Internal Rate of Return for Investments in Buildings and Building Systems
- E1074 Practice for Measuring Net Benefits and Net Savings

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² For an extensive overview of techniques for treating risk and uncertainty, see Marshall, H.E., *Techniques for Treating Uncertainty and Risk in the Economic Evaluation of Building Investments*, National Institute of Standards and Technology, Special Publication 757, 1988.

³ For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

- for Investments in Buildings and Building Systems
- E1121 Practice for Measuring Payback for Investments in Buildings and Building Systems
- E1185 Guide for Selecting Economic Methods for Evaluating Investments in Buildings and Building Systems
- E1946 Practice for Measuring Cost Risk of Buildings and Building Systems and Other Constructed Projects
- E2204 Guide for Summarizing the Economic Impacts of Building-Related Projects

2.2 Adjuncts:

- Discount Factor Tables Adjunct to Practices E917, E964, E1057, E1074, and E1121⁴

3. Terminology

3.1 *Definitions*—For definitions of general terms related to building construction used in this guide, refer to Terminology E631; and for general terms related to building economics, refer to Terminology E833.

4. Summary of Guide

4.1 This guide identifies related ASTM standards and adjuncts. It describes circumstances when measuring uncertainty and risk may be helpful in economic evaluations of building investments. This guide defines uncertainty, risk exposure, and risk attitude. It presents nonprobabilistic and probabilistic techniques for measuring uncertainty and risk exposure. This guide describes briefly each technique, gives the formula for calculating a measure where appropriate, illustrates the techniques with a case example, and summarizes its advantages and disadvantages.

4.2 Since there is no best technique for measuring uncertainty and risk in every economic evaluation, this guide concludes with a discussion of how to select the appropriate technique for a particular problem.

4.3 This guide describes in detail how risk exposure can be measured by probability functions and distribution functions

⁴ Available from ASTM International Headquarters. Order Adjunct No. ADJE091703. Original adjunct produced in 1984.

(see [Annex A1](#)). It also describes how risk attitude can be incorporated using utility theory and other approaches (see [Annex A2](#)).

5. Significance and Use

5.1 Investments in long-lived projects such as buildings are characterized by uncertainties regarding project life, operation and maintenance costs, revenues, and other factors that affect project economics. Since future values of these variable factors are generally not known, it is difficult to make reliable economic evaluations.

5.2 The traditional approach to project investment analysis has been to apply economic methods of project evaluation to best-guess estimates of project input variables as if they were certain estimates and then to present results in single-value, deterministic terms. When projects are evaluated without regard to uncertainty of inputs to the analysis, decision makers may have insufficient information to measure and evaluate the risk of investing in a project having a different outcome from what is expected.

5.3 Risk analysis is the body of theory and practice that has evolved to help decision makers assess their risk exposures and risk attitudes so that the investment that is the best bet for them can be selected.

NOTE 1—The decision maker is the individual or group of individuals responsible for the investment decision. For example, the decision maker may be the chief executive officer or the board of directors.

5.4 Uncertainty and risk are defined as follows. Uncertainty (or certainty) refers to a state of knowledge about the variable inputs to an economic analysis. If the decision maker is unsure of input values, there is uncertainty. If the decision maker is sure, there is certainty. Risk refers either to risk exposure or risk attitude.

5.4.1 Risk exposure is the probability of investing in a project that will have a less favorable economic outcome than what is desired (the target) or is expected.

5.4.2 Risk attitude, also called risk preference, is the willingness of a decision maker to take a chance or gamble on an investment of uncertain outcome. The implications of decision makers having different risk attitudes is that a given investment of known risk exposure might be economically acceptable to an investor who is not particularly risk averse, but totally unacceptable to another investor who is very risk averse.

NOTE 2—For completeness, this guide covers both risk averse and risk taking attitudes. Most investors, however, are likely to be risk averse. The principles described herein apply both to the typical case where investors have different degrees of risk aversion and to the atypical case where some investors are risk taking while others are risk averse.

5.5 No single technique can be labeled the best technique in every situation for treating uncertainty, risk, or both. What is best depends on the following: availability of data, availability of resources (time, money, expertise), computational aids (for example, computer services), user understanding, ability to measure risk exposure and risk attitude, risk attitude of decision makers, level of risk exposure of the project, and size of the investment relative to the institution's portfolio.

6. Procedures

6.1 The recommended steps for carrying out an evaluation of uncertainty or risk are as follows:

6.1.1 Determine appropriate economic measure(s) for evaluating the investment (see [Guide E1185](#)).

6.1.2 Identify objectives, alternatives, and constraints (see [Practices E917, E964, E1057, E1074, and E1121](#)).

6.1.3 Decide whether an uncertainty and risk evaluation is needed, and, if so, choose the appropriate technique (see [Sections 5, 7, 8, and 10](#)).

6.1.4 Compile data and establish assumptions for the evaluation.

6.1.5 Determine risk attitude of the decision maker (see [Section 7 and Annex A2](#)).

6.1.6 Compute measures of worth⁵ and associated risk (see [Sections 7 and 8](#)).

6.1.7 Analyze results and make a decision (see [Section 9](#)).

6.1.8 Document the evaluation (see [Section 11](#)).

7. Techniques: Advantages and Disadvantages

7.1 This guide considers in detail three nonprobabilistic techniques (breakeven analysis, sensitivity analysis, and risk-adjusted discounting) and four probabilistic techniques (mean-variance criterion and coefficient of variation, decision analysis, simulation, and stochastic dominance) for treating uncertainty and risk. This guide also summarizes several additional techniques that are used less frequently.

7.2 Breakeven Analysis:

7.2.1 When an uncertain variable is critical to the economic success of a project, decision makers frequently want to know the minimum or maximum value that variable can reach and still have a breakeven project; that is, a project where benefits (savings) equal costs. For example, the breakeven value of an input *cost* variable is the maximum amount one can afford to pay for the input and still break even compared to benefits earned. A breakeven value of an input *benefit* variable is the minimum amount the project can produce in that benefit category and still cover the projected costs of the project.

NOTE 3—Benefits and costs are treated throughout this guide on a discounted cash-flow basis, taking into account taxes where appropriate. (See [Practice E917](#) for an explanation of discounted cash flows considering taxes.)

7.2.2 To perform a breakeven analysis, an equation is constructed wherein the benefits are set equal to the costs for a given investment project, the values of all inputs except the breakeven variable are specified, and the breakeven variable is solved algebraically.

7.2.3 Suppose a decision maker is deciding whether or not to invest in a piece of energy conserving equipment for a government-owned building. The deviation of the formula for computing breakeven investment costs for the equipment is as follows:

⁵ The NIST Building Life-Cycle Cost (BLCC) Computer Program helps users calculate measures of worth for buildings and building components that are consistent with ASTM standards. The program is downloadable from <http://energy.gov/eere/femp/building-life-cycle-cost-programs>.

$$S = C \tag{1}$$

$$C = I + O\&M + R$$

$$S = I + O\&M + R$$

$$I = S - O\&M - R$$

where:

- S = savings (benefits) in reduced energy costs from using the equipment,
- C = all costs associated with the equipment,
- I = initial investment costs of the equipment,
- $O\&M$ = operation and maintenance costs of the equipment, and
- R = replacement costs required to keep the equipment functional over the study period, and where all cost and benefit cash flows are discounted to present values.

7.2.4 By rearranging terms, the breakeven investment unknown is isolated on the left side of the equation. Substitution of known values for the terms on the right side allows the analyst to solve for the breakeven value. For example, if $S = \$20\,000$, $O\&M = \$2500$, and $R = \$1000$, then

$$I = \$20\,000 - \$2500 - \$1000 \tag{2}$$

or

$$I = \$16\,500 \tag{3}$$

7.2.5 This means that \$16 500, the breakeven value, is the maximum amount that can be paid for the energy-conserving equipment and still recover all costs through energy savings.

7.2.6 An advantage of breakeven analysis is that it can be computed quickly and easily with limited information. It also simplifies project evaluation in that it gives just one value to decision makers to use as a benchmark for comparison against the predicted performance of that uncertain variable. Breakeven analysis helps decision makers assess the likelihood of achieving the breakeven value and thereby contributes implicitly to the analysis of project risk.

7.2.7 A disadvantage is that it provides no probabilistic picture of input variable uncertainty or of project risk exposure. Furthermore, it includes no explicit treatment of risk attitude.

7.3 Sensitivity Analysis:

7.3.1 Sensitivity analysis measures the impact on project outcomes of changing a key input value about which there is uncertainty. For example, choose a pessimistic, expected, and optimistic value for an uncertain variable. Then do an economic analysis for each of the three values to see how the outcome changes as they change, with other things held the same.

7.3.2 Sensitivity analysis also applies to different combinations of input values. That is, alter several variables at once and then compute a measure of worth. For example, one scenario might include a combination of all pessimistic values, another all expected values, and a third all optimistic values; or a combination might include optimistic values for some variables in conjunction with pessimistic or expected values for

others. Examining different combinations is required if the uncertain variables are interrelated.

7.3.3 The following illustration of sensitivity analysis treats an accept/reject decision. Consider a decision on whether or not to install a programmable time clock to control heating, ventilating, and air conditioning (HVAC) equipment in a building. The time clock reduces electricity consumption by turning off that part of the HVAC equipment that is not needed during hours when the building is unoccupied. Using the benefit-to-cost ratio (BCR) as the economic method, the time clock is acceptable on economic grounds if its BCR is greater than 1.0. The energy reduction benefits from the time clock, however, are uncertain. They are a function of three factors: the initial price of energy, the rate of change in energy prices over the life cycle of the time clock, and the number of kilowatt hours saved. Assume that the initial price of energy and the number of kilowatt-hours saved are relatively certain, and that the sensitivity of the BCR is being tested with respect to the following three values of energy price change: a low rate of energy price escalation (slowly increasing benefits from energy savings); a moderate rate of escalation (moderately increasing benefits); and a high rate of escalation (rapidly increasing benefits). These three assumed values of energy price change might correspond to our projections of pessimistic, expected, and optimistic values. Three BCR estimates result from repeating the BCR computation for each of the three energy price escalation rates. For example, BCRs of 0.8, 2.0, and 4.0 might result. Whereas a deterministic approach might have generated a BCR estimate of 2.0, now it is apparent that the BCR *could be* significantly less than 2.0, and even less than 1.0. Thus accepting the time clock could lead to an inefficient outcome.

7.3.4 There are several advantages of sensitivity analysis. First, it shows how significant a single input variable is in determining project outcomes. Second, it recognizes the uncertainty associated with the input. Third, it gives information about the range of output variability. And fourth, it does all of these when there is little information, resources, or time to use more sophisticated techniques.

7.3.5 Disadvantages of sensitivity analysis in evaluating risk are that it gives no explicit probabilistic measure of risk exposure and it includes no explicit treatment of risk attitude. The findings of sensitivity analysis are ambiguous. How likely is a pessimistic or expected or optimistic value, for example, and how likely is the corresponding outcome value? Sensitivity analysis can in fact be misleading if all pessimistic assumptions or all optimistic assumptions are combined in calculating economic measures. Such combinations of inputs are unlikely in the real world.

7.3.6 Sensitivity results can be presented in text, tables, or graphs. One type of graph that is useful in showing the sensitivity of project worth to a critical variable is illustrated in Fig. 1. Net benefits (NB) for Projects A and B decrease as the discount rate increases. The slopes of the functions show that NB is more sensitive to discount rate changes for Project A than for Project B, assuming other variables remain unchanged. These functions also help in making comparisons as to which project is more cost effective. At a discount rate below 7 %, for

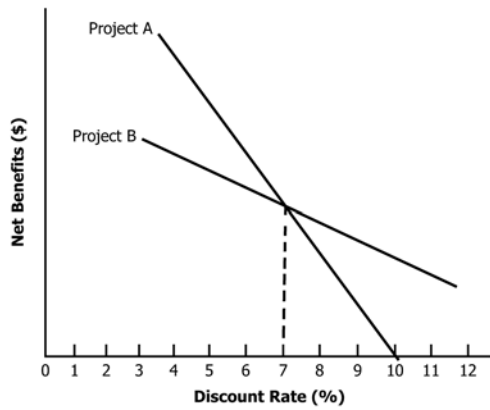
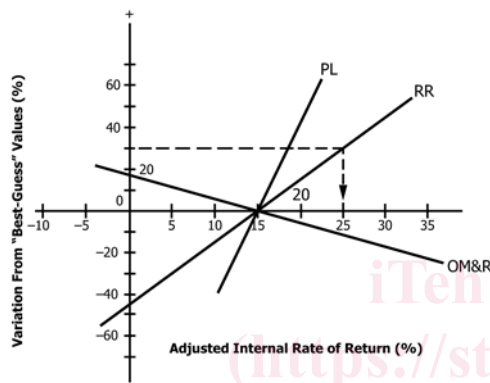


FIG. 1 Sensitivity of Net Benefits of Projects A and B to Discount Rate



NOTE 1—PL = project life, RR = reinvestment rate, and OM&R = operation, maintenance, and replacement costs.

FIG. 2 Spider Diagram Showing Sensitivity of the Adjusted Internal Rate of Return to Variations in Uncertain Variables

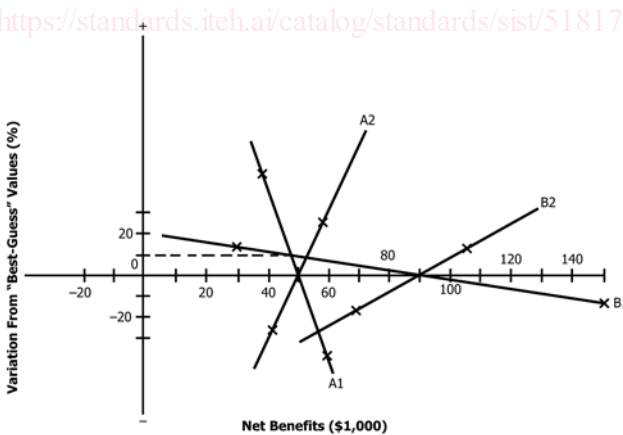


FIG. 3 Spider Diagrams for Competing Projects

these two projects. The sensitivity graph in this sense contributes to an implicit description of risk exposure. Yet the graph fails to provide a quantitative measure of the probability of any given NB occurring.

7.3.8 Another special graph for sensitivity analysis that presents a snapshot of potential impacts of uncertain input variables on project outcomes is the spider diagram. The one illustrated in Fig. 2 shows for a prospective commercial building investment the sensitivity of the adjusted internal rate of return (AIRR) to three variables: operation, maintenance, and replacement costs (OM&R); project life (PL); and the reinvestment rate (RR). Each variable is represented by a labeled function that shows what AIRR values would result from different values of the uncertain variable. For example, the downward-sloping OM&R function indicates that the AIRR is inversely proportional to OM&R costs. By design, the OM&R function (as well as the other two functions) passes through the horizontal axis at the best-guess estimate of the AIRR (15 % in this case), based on the best-guess estimates of the three uncertain variables. Since each of the variables is measured by different units (money, years, and percent), the vertical axis is denominated in positive and negative percent changes from the best-guess values fixed at the horizontal axis. The AIRR value corresponding to any given percent variation indicated by a point on the function is found by extending a line perpendicular to the horizontal axis and reading directly the AIRR value. Thus a 30 % increase in the best-guess reinvestment rate would yield a 25 % AIRR, assuming other values remain unchanged.

7.3.9 The contribution of the spider diagram is its picture of the relative importance of the different uncertain variables. It shows immediately that the lesser the slope of a function, the more sensitive is the AIRR to that variable. For example, any given percent change in OM&R will have a greater impact on the AIRR than will an equal percent change in RR or PL.

7.3.10 Spider diagrams can be helpful when comparing competing projects as long as the decision maker keeps in mind that extreme values of the measure of worth reflect variations in one variable only. For example, look at the spider diagram for Projects A and B in Fig. 3. The NB of Project A is a function of variables A1 and A2, and the NB of Project B is a function of variables B1 and B2. The horizontal axis suggests that Project B has a higher present value net benefits (\$90 000) than Project A (\$50 000). That is, if only best-guess values were used in a single-value, deterministic approach, Project B would be the preferred project. However, if we assign, say a 50 % confidence interval about the uncertain variables A1, A2, B1, and B2, as shown by X's on the functions, there appears the possibility that Project A could yield a higher NB than Project B. That is, within that confidence interval, if the extreme B1 value to the left were to occur, Project B would yield a lesser NB than would Project A for A1 or A2 extreme values to the left. Furthermore, if A1 and B1 were the same input variable, we would know that Project A would be preferred at values of A1 and B1 above 10 % over the best-guess value, and Project B would be preferred at values of A1 and B1 below 10 %.

example, Project A has the greater NB. At a rate above 7 %, Project B yields the greater NB. And at 7 %, the two projects provide identical NB.

7.3.7 Note that the functions indicate the potential values of NB if different values of the discount rate occur. If decision makers have some idea as to the likelihood of specific discount rates, the graph will help them evaluate the NB implications for

7.3.11 Once again, however, sensitivity analysis gives no indication of the probability of any given value of NB. Furthermore, because only one variable is allowed to change at a time, and NB is a function of more than one variable, sensitivity analysis gives an incomplete description of the possible outcomes.

7.4 Risk-Adjusted Discounting:

7.4.1 One technique used by the business community to account for risk is the risk-adjusted discount rate (RADR). The objective of using the RADR technique is to raise the likelihood that the investor will earn a return over time sufficient to compensate for extra risk associated with specific projects.

7.4.2 Projects with anticipated high variability in distributions of project worth have their net benefits or returns discounted at higher rates than projects with low variability. Thus in computing net benefits or the benefit-to-cost ratio, the discount rate is higher for benefit streams of risky projects than for those with certain outcomes. Or when applying rate-of-return methods, the minimum acceptable rate of return (MARR) is raised above the risk-free rate to compensate for the higher variability of returns in risky projects.

7.4.3 Calculate the RADR as follows:

$$\text{RADR} = \text{RF} + \text{AR1} + \text{AR2} \quad (4)$$

where:

- RF = risk-free rate,
- AR1 = adjustment for normal risk encountered in the firm's operations, and
- AR2 = adjustment for extra risk above or below normal risk.

All terms are expressed as percents.

7.4.4 The risk-free rate (RF) component accounts for the time value of money. It is what might be earned, for example, on government treasury bills, the closest thing to a riskless investment available to most investors. The adjustment for normal risk (AR1) is the risk premium that a firm might impose to cover the average riskiness of its normal operations. The sum of RF and AR1 should equal the MARR the firm requires on typical investments. The AR2 component adjusts for projects with more or less risk than what is normally associated with the firm. The adjustment can be positive or negative.

7.4.5 For discounting *benefit* streams, AR2 is an increasing function of (1) the perceived variability in project outcomes (risk exposure) and (2) the degree to which the decision maker is risk averse (risk attitude). For *cost* streams, AR2 is a decreasing function of those same risk factors.

7.4.6 For computing the RADR, each benefit and cost stream should be discounted with a unique RADR that includes AR1 and AR2 values that describe that stream's uncertainty. For benefit or savings streams, AR1 and AR2 are adjusted upwards as perceived risk increases; that is, as future benefits become more uncertain, the RADR technique requires raising the discount rate to make the project look less desirable. For cost streams, AR1 and AR2 are adjusted downwards as perceived risk increases; that is, as future costs become more uncertain, the correct application of the RADR technique requires lowering the discount rate to make the project look less cost effective. It follows then that the appropriate adjustment for risk when using life cycle cost (LCC) analysis is a

decrease in the discount rate for each cost stream to make project costs appear higher. Otherwise LCC analysis will be biased in favor of projects with a greater risk of higher-than-anticipated costs.

7.4.7 Let us look once again at the BCR of the time clock for an illustration of the RADR when making an accept/reject decision. If no unusual risk is associated with the time clock, the discount rate is equal to the sum of RF and AR1 as shown under Eq 4. Let us suppose that the BCR for the time clock is 1.1 in this case. Thus it appears economically sound.

7.4.8 Now let us assume instead that the economic performance of the time clock is more risky than average. This might arise, for example, from the impact of uncertain kilowatt-hour reductions or uncertain future energy prices. Furthermore, let us assume that the decision maker is risk averse. Using the RADR technique, we raise the discount rate for evaluating energy cost savings by some positive value of AR2. If the resulting BCR falls below 1.0, the project no longer appears economically acceptable.

7.4.9 Advantages of the RADR technique are that it is relatively simple to understand; it is easy to compute; and it accounts to some extent for uncertainty of inputs, risk exposure, and risk attitude.

7.4.10 A major limitation in using the RADR is the lack of any accepted procedure for establishing the RADR value. It is typically estimated based on the decision maker's best judgment. One common approach is to simply lump projects into risk categories, each of which has an assigned RADR. There is little fine tuning. Furthermore, there is no distinction between adjustments for handling risk exposure and risk attitude.

7.4.11 A common mistake in application is to use a constant AR2 over the entire study period. This distorts risk adjustment when there are periods for which no special adjustment is necessary above or below what is considered normal risk. A constant AR2 also distorts risk adjustment because it implies in effect that returns become exponentially more uncertain over time, which is often not the case. Thus a discount rate that includes a constant AR2 severely reduces the weight of net benefits accrued in later years, regardless of the certainty of their occurrence. This biases selection towards projects with early payoffs. To avoid this common mistake in application and its resulting bias, use a variable AR2.

7.5 Mean-Variance Criterion and Coefficient of Variation:

7.5.1 Comparing mean values and standard deviations of measures of project worth can help decision makers evaluate returns and risk exposure of one project versus another and determine stochastic dominance. If two projects competing for limiting funds are compared on the basis of BCRs, for example, the mean-variance criterion dictates that the one with the higher mean (that is, expected value) and lower standard deviation be chosen. This presumes that decision makers prefer higher BCRs to lower BCRs and less risk to more risk.

7.5.2 If one project has a higher mean and higher standard deviation of the measure of project worth, then the choice is not clear with the mean-variance criterion. In this case, the coefficient of variation can be computed to determine the

relative risk of the two projects. The coefficient of variation is found by dividing the standard deviation by the mean as follows:

$$CV = \sigma/\mu, \tag{5}$$

where:

- CV = coefficient of variation,
- σ = standard deviation, and
- μ = mean or expected value.

7.5.3 The project with the lower coefficient of variation has the lesser risk per unit of return or project worth. It will be preferred by risk-averse decision makers. Risk-taking decision makers, on the other hand, will prefer the project with the higher coefficient.

7.5.4 An advantage of the coefficient of variation is that it provides an explicit measure of relative risk exposure. Another is that risk attitude is considered when the decision maker evaluates the coefficients of variation to choose among alternative projects. The major limitation is in acquiring the σ and μ values for the measure of project worth.

7.6 Decision Analysis:

7.6.1 Decision analysis is one of the few techniques for making economic decisions in an uncertain environment that treats formally both risk exposure and risk attitude. It provides a methodology that allows a decision maker to include alternative outcomes, risk attitudes, and subjective impressions about uncertain events in an evaluation of investments.

7.6.2 Decision analysis typically uses decision trees to represent all possible outcomes, costs, and probabilities associated with a given decision problem. A decision tree is a decision-flow diagram that serves as a road map to clarify possible alternatives and outcomes of sequential decisions. A decision tree is used in this section to illustrate how it helps bring order to complex decisions about risky investments.

7.6.3 In general, the decision analysis approach has three steps. The first is to structure the problem. This includes defining variables, describing with models their relationships, assigning values to possible outcomes, and measuring the importance of variables through sensitivity analysis. The second step is to assign subjective probabilities to important variables and possible outcomes, and to find the best bet alternative. This includes describing uncertainty with subjective probability distributions, describing risk attitude with a utility function (see Annex A2), and finding the alternative that is expected to yield the greatest economic return (or utility if the decision maker is not risk neutral). The third step, which is not always taken, is to determine whether obtaining additional information is worth the cost. If it is, then the information is collected, and steps 1 and 2 are repeated.

NOTE 4—Subjective probability distributions are developed by the decision analyst asking the decision maker or an expert(s) designated by the decision maker a series of probing questions designed to reveal the best judgments available on the likelihood of uncertain events.

7.6.4 Decision Analysis of Energy Conservation Investment:

7.6.4.1 This illustration examines an energy investment problem facing a state energy office. The office has been directed to make a choice regarding an energy conservation project from among six possibilities for retrofitting two public

buildings. The purpose of the conservation project is to demonstrate to private companies that energy conservation is profitable. The objective of the decision analysis exercise is to choose the retrofit package that yields the maximum expected net benefits (NB), that is, shows the greatest profit potential. If none of the packages yields a positive NB, the choice will be not to invest at all.

7.6.4.2 Two types of retrofit costs are considered. The first is a fixed retrofit investment cost that is incurred for energy conservation work in each building regardless of which retrofit package is chosen. The second is the cost of implementing the individual retrofits in each package. The present value fixed investment (F1 and F2) costs and retrofit package (R1 through R6) costs are shown in Table 1. All costs are assumed to be certain.

7.6.4.3 The predicted benefit outcomes (dollar energy savings in present value terms) are uncertain for the different retrofit packages. Table 2 shows estimates of these possible benefit outcomes with their respective probabilities of occurrence. Since the state is assumed to be risk neutral and act so as to maximize the expected monetary value of its investments, there is no need to consider risk attitude and the corresponding utility measures of outcomes. Furthermore, since the state pays no taxes, they are not included in the analysis.

7.6.4.4 The decision tree in Fig. 4 clarifies the possible alternatives and outcomes listed in Table 1 and Table 2. The following explanation describes the potential paths of the decision tree starting from the left side.

NOTE 5—The procedure for finding the package that yields maximum net benefits requires starting from the right side of the tree, as will be shown later. It is easier to explain the tree structure, however by starting from the left.

7.6.4.5 The basic alternative of not investing is indicated by the top line segment coming out of the box on the left side of Fig. 4. The fixed investment of \$500 000 in Building I is shown by the next line, and the investment of \$800 000 in Building II is shown by the bottom line. Each box in a decision tree represents a decision juncture or node, and the line segments represent alternative branches on the decision tree. The state energy office will select that branch sequence that they expect will maximize the present value of net benefits from conservation.

7.6.4.6 Associated with each building is another decision node, requiring a decision regarding a specific set of retrofit choices, R1 through R3 or R4 through R6. The known costs of each retrofit package are shown under each alternative branch.

7.6.4.7 The benefit outcomes (dollar energy savings) are uncertain for the different retrofit packages. Thus at the end of each retrofit package branch is a chance node or juncture followed by alternative outcomes. Retrofit package R1, for example, is followed by a chance node, indicated by a circle,

TABLE 1 Fixed Investment and Retrofit Package Cost for Buildings I and II (Cost in Millions of Dollars)

	Building I			Building II				
	F1	R1	R2	R3	F2	R4	R5	R6
	0.5	2.0	3.0	4.0	0.8	3.0	4.0	4.5