Designation: ~~D4015 – 07~~ D4015 – 15

Standard Test Methods for Modulus and Damping of Soils by ~~Resonant-Column Method~~Fixed-Base Resonant Column Devices[1]

This standard is issued under the fixed designation D4015; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope*

1.1 These test methods cover the determination of shear ~~modulus, shear damping, rod modulus (commonly referred to as Young's modulus), and rod damping~~ modulus and shear damping as a function of shear strain amplitude for solid cylindrical specimens of soil in ~~the undisturbed~~ intact and remolded conditions by vibration using ~~the~~ resonant ~~column.~~ column devices. The vibration of the specimen may be superposed on a controlled ~~ambient~~static state of stress in the specimen. The vibration apparatus and specimen may be enclosed in a triaxial chamber and subjected to an all-around pressure and axial load. In addition, the specimen may be subjected to other controlled conditions (for example, pore-water pressure, degree of saturation, temperature). These test methods of modulus and damping determination are considered nondestructive when the shear strain amplitudes of vibration are less than 10~~$^{-4}$~~$^{-2}$ ~~rad~~ % (10~~$^{-4}$~~$^{-4}$ in./ ~~in./in.),~~ in.), and many measurements may be made on the same specimen and with various states of ~~ambient~~static stress.

1.2 Two device configurations are covered by these test methods: Device Type 1 where a known torque is applied to the top of the specimen and the resulting rotational motion is measured at the top of the specimen, and Device Type 2 where an uncalibrated torque is applied to the top of the specimen and the torque transmitted through the specimen is measured by a torque transducer at the base of the specimen. For both devices, the torque is applied to the active end (usually top) of the specimen and the rotational motion also is measured at the active end of the specimen.

1.3 These test methods ~~cover only~~ are limited to the determination of the shear modulus and shear damping, the necessary vibration, and specimen preparation procedures related to the vibration, etc., and do not cover the application, measurement, or control of the ~~ambient stress.~~ axial and lateral static normal stresses. The latter procedures may be covered by, but are not limited to, Test ~~Methods~~Method ~~D2166~~D2850, D3999/D3999M, D4767, D5311/D5311M, or ~~D2850~~D7181.

1.4 *Significant Digits*—All recorded and calculated values shall conform to the guide for significant digits and rounding established in Practice D6026.

1.4.1 The procedures used to specify how data are collected/recorded and calculated in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that should generally be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objectives; and it is common practice to increase or reduce significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis methods for engineering design.

1.4.2 Measurements made to more significant digits or better sensitivity than specified in this standard shall not be regarded a nonconformance with this standard.

1.5 *Units*—The values stated in SI units are to be regarded as ~~the standard. Reporting~~ standard. The values given in parentheses are mathematical conversions to inch-pound units, which are provided for information only and are not considered standard. Reporting of test results in units other than SI shall not be regarded as ~~conformance~~nonconformance with these test methods.

1.5.1 The converted inch-pound units use the gravitational system of units. In this system, the pound (lbf) represents a unit of force (weight), while the unit for mass is slugs. The converted slug unit is not given, unless dynamic (F = ma) calculations are involved.

1.5.2 It is common practice in the engineering/construction profession to concurrently use pounds to represent both a unit of mass (lbm) and of force (lbf). This implicitly combines two separate systems of units; that is, the absolute system and the gravitational system. It is scientifically undesirable to combine the use of two separate sets of inch-pound units within a single

---

[1] These test methods are under the jurisdiction of ASTM Committee D18 on Soil and Rock and are the direct responsibility of Subcommittee D18.09 on Cyclic and Dynamic Properties of Soils.

Current edition approved ~~Sept. 1, 2007~~Oct. 1, 2015. Published ~~October 2007~~November 2015. Originally approved in 1981. Last previous edition approved in ~~2000~~2007 as ~~D4015 – 92~~D4015 – 07. ~~(2000).~~ DOI: ~~10.1520/D4015-07.~~10.1520/D4015-15.

***A Summary of Changes section appears at the end of this standard**

standard. As stated, this standard includes the gravitational system of inch-pound units and does not use/present the slug unit for mass. However, the use of balances or scales recording pounds of mass (lbm) or recording density in lbm/ft^3 shall not be regarded as nonconformance with this standard.

1.6 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

D653 Terminology Relating to Soil, Rock, and Contained Fluids

~~D2166~~D2166/D2166M Test Method for Unconfined Compressive Strength of Cohesive Soil

D2216 Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass

D2850 Test Method for Unconsolidated-Undrained Triaxial Compression Test on Cohesive Soils

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

D3999/D3999M Test Methods for the Determination of the Modulus and Damping Properties of Soils Using the Cyclic Triaxial Apparatus

D4753 Guide for Evaluating, Selecting, and Specifying Balances and Standard Masses for Use in Soil, Rock, and Construction Materials Testing

D4767 Test Method for Consolidated Undrained Triaxial Compression Test for Cohesive Soils

D5311/D5311M Test Method for Load Controlled Cyclic Triaxial Strength of Soil

D6026 Practice for Using Significant Digits in Geotechnical Data

D7181 Test Method for Consolidated Drained Triaxial Compression Test for Soils

3. Terminology

3.1 *Definitions*—For definitions of other terms used in ~~this Test Method,~~these test methods, see Terminology D653.

3.2 *Definitions of Terms Specific to This Standard:*

~~3.2.1 *ambient stress*—stresses applied to the specimen, during the test, that do not result from the vibration strains. These test methods do not cover the application and measurement of ambient stresses; however, the ambient stress at the time of measurement of the system resonant frequency and system damping shall be measured and recorded in accordance with the Report section of these test methods.~~

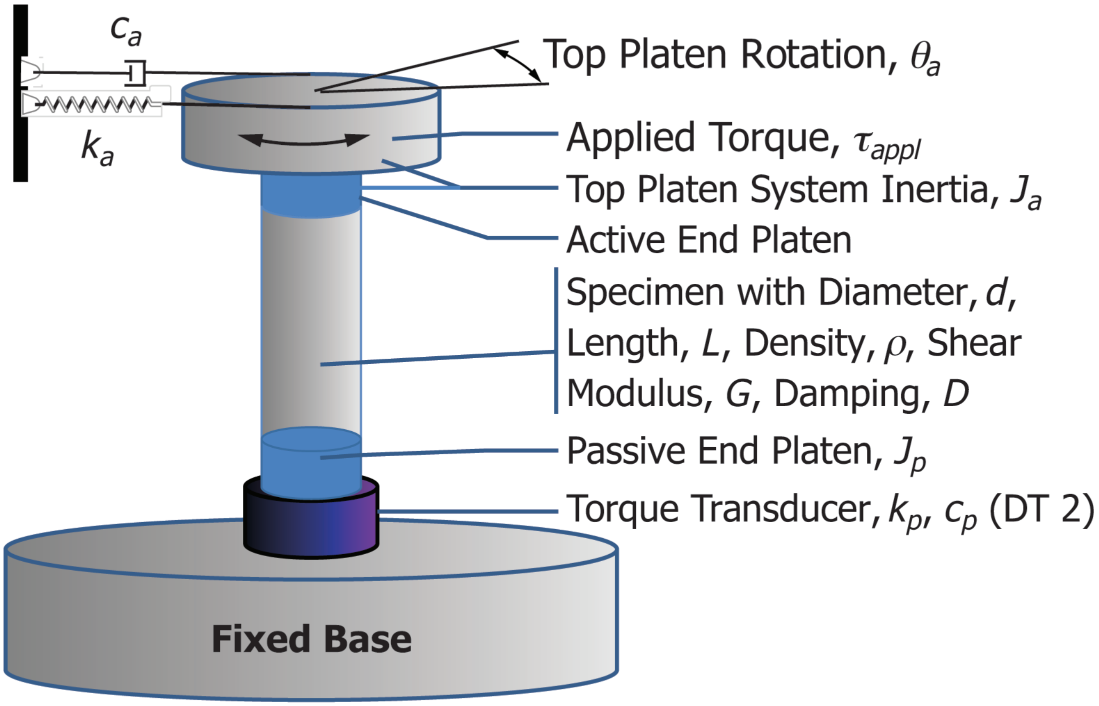
3.2.1 ~~*apparatus model and*~~*damping capacity D* ~~constants—~~[unitless, typically expressed in %],*n*—~~the rigidity and mass distribution of the resonant column shall be as required in the following section in order for the resonant-column system to be accurately represented by the model shown in Fig. 1. The apparatus constants are the mass of the passive-end platen, *M*~~*in*~~$_P$, *including the mass*~~ *resonant column systems,* ~~of all attachments rigidly connected to it; the rotational inertia of the passive-end platen,~~is related to the component of the dynamic shear ~~J~~modulus~~$_P$, including the rotational inertia of all attachments rigidly connected to it; similar mass,~~ that lags the ~~M_A, and rotational inertia, J_A, for the active-end platen and all attachments rigidly connected to it, such as portions of the vibration excitation device; the spring and damping constants for both longitudinal and torsional springs and dashpots (K_{SL}, K_{ST}, ADC_L, ADC_T); the apparatus resonant frequencies for longitudinal vibration, f_{oL}, and torsional vibration, f_{oT}; the force/current constant, FCF, relating applied vibratory force to the current applied to the longitudinal excitation device; the torque/current constant,~~ applied shear stress by 90° degrees.~~ TCF, relating applied vibratory torque to the current applied to the torsional excitation device; and the motion transducer calibration factors (LCF_A, RCF_A, LCF_P, RCF_P) relating the transducer outputs to active- and passive-end longitudinal and rotational motion.~~

~~3.2.3 *moduli and damping capacities*—Young's modulus (herein called rod modulus), *E*, is determined from longitudinal vibration, and the shear modulus, *G*, is determined from torsional vibration. The rod and shear moduli shall be defined as the elastic moduli of a uniform, linearly viscoelastic (Voigt model) specimen of the same mass density and dimensions as the soil specimen necessary to produce a resonant column having the measured system resonant frequency and response due to a given vibratory force or torque input. The stress-strain relation for a steady-state vibration in the resonant column is a hysteresis loop. These moduli will correspond to the slope of a line through the end points of the hysteresis loop. The section on calculations provides for computation of rod and shear moduli from the measured system longitudinal and torsional resonant frequencies. The energy dissipated by the system is a measure of the damping of the soil. Damping will be described by the rod damping ratio, D_L, and the shear damping ratio, D_T, which are analogous to the critical viscous damping ratio, c/c_r, for a single-degree-of-freedom system. The damping ratios shall be defined by:~~

~~$D_L = 0.5(\eta\omega/E)$~~ ~~(1)~~

---

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.



For Device Type 1, no torque transducer is needed and the Passive End Platen is connected to the Fixed Base.

FIG. 1 ~~Resonant-column Schematic~~Resonant-Column Schematic for Both Device Types 1 and 2

~~where:~~

~~η = viscous coefficient for rod motion, N·s/m^2,~~
~~ω = circular resonant frequency, rad/s, and~~
~~E = rod modulus, Pa.~~

~~and by:~~

~~$D_T = 0.5(\mu\omega/G)$ (2)~~

~~where:~~

~~μ = viscous coefficient for torsional motion, N-s/m^2, and~~
~~G = shear modulus, Pa.~~

~~3.2.3.1 Values of damping determined in this way will correspond to the area of the stress-strain hysteresis loop divided by 4π times the elastic strain energy stored in the specimen at maximum strain. Methods for determining damping ratio are prescribed later. In viscoelastic theory, it is common to use complex moduli to express both modulus and damping. The complex rod modulus is given by:~~

~~$E^* = E(1+2iD_L)$ (3)~~

~~and the complex shear modulus is given by:~~

~~$G^* = G(1+2iD_T)$ (4)~~

~~where $i=\sqrt{-1}$.~~

3.2.2 ~~*resonant-column system—*~~*Device Type 1, DT1, n—~~a system consisting of a cylindrical specimen or column of~~in resonant column systems,* ~~soil that has platens attached to each end~~ a resonant column system as shown in Fig. 1~~. A sinusoidal vibration excitation device is attached to the active-end platen. The other end is the passive-end platen. It may be rigidly fixed (the criterion for establishing fixity is given later) or its mass and rotational inertia must be known. The vibration excitation device may incorporate springs and dashpots~~ where the passive end platen is directly connected to the ~~active-end platen, where the spring constants and viscous damping coefficients are known. Vibration excitation may be longitudinal or torsional. A given apparatus may have the capability of applying one or the other, or both. The mass and rotational inertia of the active-end platen and portions of the vibration excitation device moving with it must be known. Transducers are used to measure the vibration amplitudes for each type of motion~~ Fixed Base (no torque transducer), a calibrated vibratory torque is applied to the active end, and rotation is measured at the active ~~end and also at the passive end if it is not rigidly fixed. The frequency of excitation will be adjusted to produce resonance of the system, composed of the specimen and its attached platens and vibration excitation device.~~end.

3.2.2.1 *Discussion—*

The vibration excitation device may incorporate springs and dashpots connected to the active-end platen, where the spring constants and viscous damping coefficients must be known. The rotational inertia of the active-end platen and portions of the vibration excitation device moving with it must be known.

3.2.3 *Device Type 2, DT2, n—in resonant column systems*, a resonant column system as shown in Fig. 1 where the passive end platen is connected to a torque transducer, an uncalibrated torque is applied to the active end, torque is measured by the torque transducer at the passive end, and rotation is measured at the active end.

3.2.3.1 *Discussion—*

The vibration excitation device may incorporate springs and dashpots connected to the active-end platen, but the spring constants and viscous damping coefficients are not needed. The rotational inertia of the active-end platen and portions of the vibration excitation device moving with it also are not needed.

3.2.4 *dynamic shear modulus, G^* [FL^{-2}], n—in resonant column systems*, is the ratio of shear stress to shear strain under vibratory conditions (also known as complex shear modulus).

3.2.5 *equivalent elastic shear modulus G [FL^{-2}], n—in resonant column systems*, is the component of the dynamic shear modulus that is in-phase with the applied shear stress.

3.2.6 *resonant-column system, n*—a system as shown in Fig. 1 consisting of a cylindrical specimen or column of soil enclosed with a flexible membrane that has platens attached to each end and where a sinusoidal vibration excitation device is attached to the active-end platen and where the other end is the passive-end platen that is rigidly fixed.

3.2.7 *specimen ~~strain—~~shear strain γ,* [unitless, frequently expressed as %],*n—*~~for longitudinal motion, the strain, ε, is the average axial strain in the entire specimen. For torsional motion, the strain, γ,~~ in resonant column systems, is the average shear strain in the ~~specimen. In the case of torsion,~~ specimen where the shear strain in each cross section varies from zero along the axis of rotation to a maximum at the perimeter of the ~~specimen, and the average shear strain for each cross section occurs at a radius equal to 80 percent the radius of the specimen. Methods for calculating specimen strain are given later in the calculations section.~~specimen.

3.2.7.1 *Discussion—*

The radius for calculating average shear strains vary depending on soil type, strain level, confining stress, etc. The default value of the radius for calculating average strain is 0.4*diameter but values in the range of 0.33 to 0.40*diameter may be used if the value is documented in the report.

3.2.8 *system resonant ~~frequency—~~frequency f_r*[s^{-1}],*n—~~the definition of system~~in resonant column systems,* ~~resonance depends on both apparatus and specimen characteristics. For the case where the passive-end platen is fixed, motion~~ for Device Type 1 is the lowest frequency at which the rotational velocity at the active end is ~~used to establish resonance, which is defined as the lowest frequency for which the sinusoidal excitation force (or moment) is~~ in phase with the ~~velocity of the active-end platen. For the case where the passive-end platen mass (or passive end platen rotational inertia) is greater than 100 times the corresponding value of the specimen and is not rigidly fixed, resonance is~~ sinusoidal excitation torque and for Device Type 2, is the lowest frequency ~~for which the sinusoidal excitation force (or moment) is 180° out of phase with the velocity of the active-end platen. Otherwise,~~ at which the rotational motion at the ~~passive end is used to establish resonance, which is the second lowest frequency for which the sinusoidal excitation force (or moment) is in phase with the velocity of the passive-end platen. (The lowest frequency for this condition is not used because it does not produce significant strains in the specimen.) In general, the system resonant frequency for torsional excitation will be different from the system resonant frequency for longitudinal excitation.~~active end is a maximum.

4. Summary of Test Method

4.1 The resonant column device is shown schematically in Fig. 1. In the resonant column test, a cylindrical soil ~~specimen~~ specimen, usually enclosed with a thin membrane, is subjected to an imposed ~~ambient stress condition. Once equilibrium at the imposed stress condition is achieved, torsional or longitudinal, or both,~~ static axial and lateral stress condition. Torsional sinusoidal vibrations are applied ~~to the~~ at the top of the soil specimen and the ~~specimen motions (strains) resulting from the imposed vibrations are~~ rotational response is measured. The frequency of excitation is varied until ~~resonance~~ the system resonant frequency is achieved as described in ~~Section 3.2.6~~3.2.8. Given the geometry, mass and system parameters, the equivalent elastic shear ~~and longitudinal moduli and material (hysteretic) damping may~~ modulus and damping capacity can be determined at a measured ~~strain value.~~ level of excitation vibration. The amplitude of vibration ~~is~~ (which is related to shear strain) is typically varied to measure the variation of ~~moduli~~modulus and damping as a function of shear strain. ~~Since the~~ The test is usually conducted at ~~strain levels between 0.00001 and 0.5% strain leaving the specimen relatively intact, the~~ levels of shear strain between 0.00001 % and 0.2 %.

(The upper limit of shear strain is dependent on the specimen stiffness and the maximum torque capability of the excitation system.) For specimens where the maximum shear strain measured is of the order of 0.01 %, the test is often conducted at several different sets of ~~ambient~~ static axial and lateral stress conditions to measure the variation of moduli and damping with ~~ambient stress.~~static stress states. The test results are dependent on sample quality/specimen disturbance which are beyond the scope of this standard.

~~NOTE 1—The quality of the results produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.~~

5. Significance and Use

5.1 The equivalent elastic shear modulus and damping capacity of a given soil, as measured by the ~~resonant-column~~ resonant column technique herein described, depend upon the strain amplitude of vibration, the ~~ambient~~ state of effective stress, and the void ratio of the soil, temperature, time, etc. Since the application and control of the ~~ambient~~ static axial and lateral stresses and the void ratio are not prescribed in these methods, the applicability of the results to field conditions will depend on the degree to which the application and control of the ~~ambient~~ static axial and lateral stresses and the void ratio, as well as other parameters such as soil structure, duplicate field conditions. The techniques used to simulate field conditions depend on many factors and it is up to the engineer to decide on which techniques apply to a given situation and soil type. The results of these tests are useful for calculations involving soil-structure interaction and seismic response of soil deposits.

NOTE 1—The quality of the results produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities. Agencies that meet the criteria of Practice D3740 are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice D3740 does not in itself assure reliable results. Reliable results depend on many factors; Practice D3740 provides a means of evaluating some of those factors.

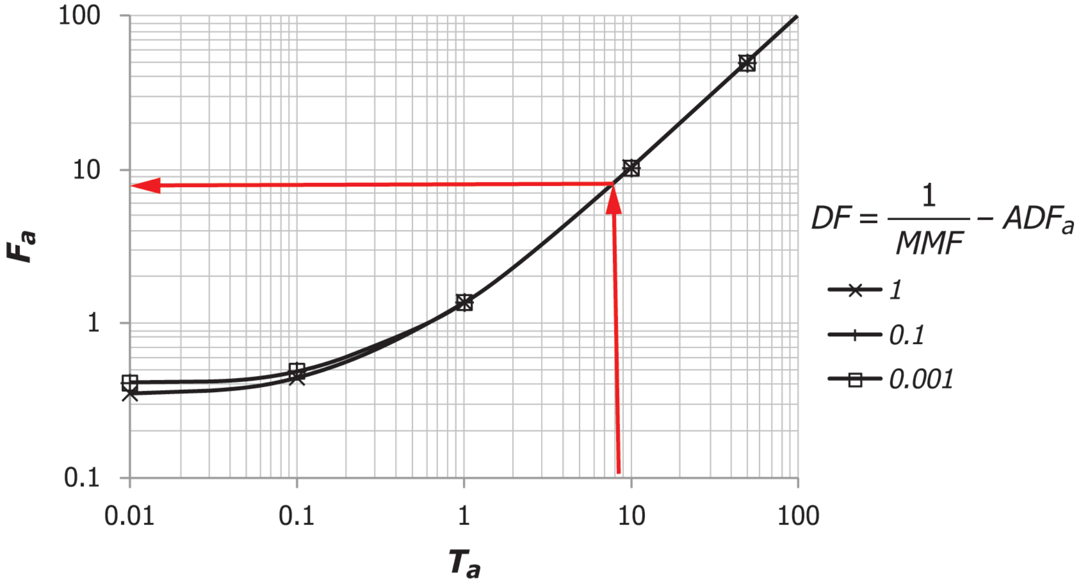
6. Apparatus

6.1 *General*—The complete test apparatus is shown schematically in Fig. 1 and includes the platens for holding the specimen in the pressure cell, the vibration excitation ~~device,~~ device (torque motor) , transducers for measuring the response, the control and readout instrumentation, and auxiliary equipment for specimen preparation. The theory for the resonant column is provided in Annex A1. The entire apparatus is generally enclosed within a pressure chamber (commonly referred to as a triaxial cell). For some apparatus that can apply an axial load to the specimen, the pressure chamber lid may be fitted with a piston passing through the top.

6.2 *Specimen Platens*—Both the active-end and passive-end platens shall be constructed of noncorrosive material having a modulus at least ten times the modulus of the material to be tested. Each platen shall have a circular cross section and a plane surface of contact with the specimen, except that the plane surface of contact may be roughened to provide for more efficient coupling with the ends of the specimen. Roughening and flow of fluids into or from the specimen may be accomplished by rigidly fastening porous disks to the platens. The diameter of platens shall be equal to or greater than the diameter of the specimen. The construction of the platens shall be such that their stiffness is at least ten times the stiffness of the specimen. ~~The active-end platen may have a portion of the excitation device, transducers, springs, and dashpots connected to it. The transducers and moving portions of the excitation device must be connected to the platen in such a fashion that they are to be considered part of the platen and have the same motion as the platen for the full range of frequencies to be encountered when testing soils. The theoretical model used for the resonant-column system represents the active-end platen, with all attachments, as a rigid mass that is attached to the specimen; this mass may also have massless springs and dashpots attached to it as shown in Fig. 1. If springs are used, the excitation device and active-end platen (without the specimen in place) form a two degree-of-freedom system (one-degree-of-freedom system for devices designed for only longitudinal or only torsional motion) having undamped natural frequencies for longitudinal motion,~~ ~~f_{oL},~~ ~~and torsional motion,~~ ~~f_{o-T}.~~ ~~The device shall be constructed such that these modes of vibration are uncoupled. The passive-end platen may have a mass and transducers rigidly attached to it or it may be rigidly fixed. The passive-end platen may be assumed to be rigidly fixed when the inertia of it and the mass(es) attached to it and the stiffness of the support of the mass(es) provide a dimensionless frequency factor within 1 % of the dimensionless frequency factor for the passive-end inertia ratio equal to infinity. (Use Fig. 2 and the calculations section to get the dimensionless frequency factor.)~~

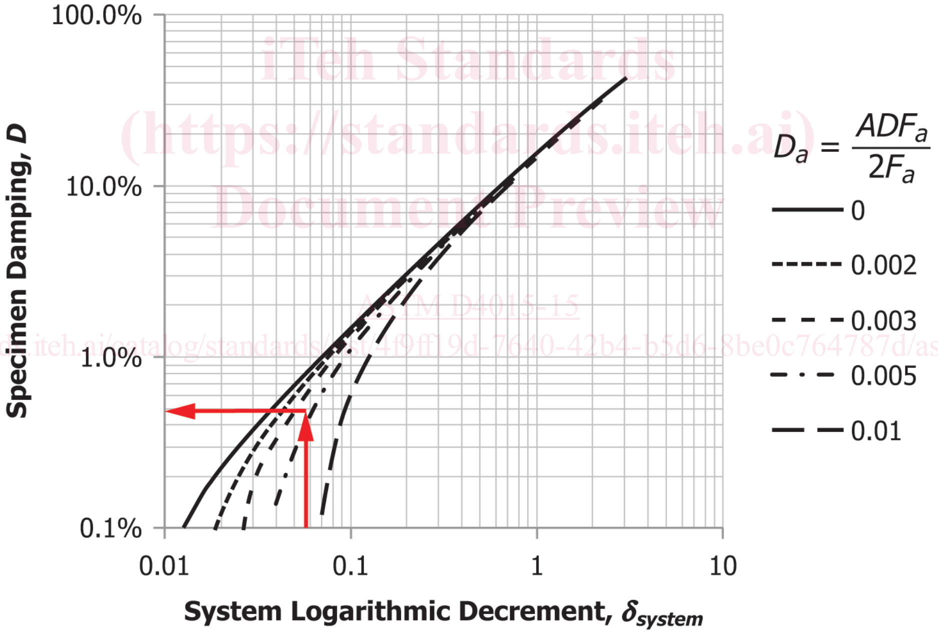
6.2.1 The active-end platen may have a portion of the excitation device, transducers, springs, and dashpots connected to it. The transducers and moving portions of the excitation device must be connected to the platen in such a fashion that they are to be considered part of the platen, be counterbalanced to maintain rotational symmetry and have the same motion as the platen for the full range of frequencies to be encountered when testing soils.

6.2.2 The theoretical model used for the resonant-column system represents the active-end platen, with all attachments, as a rigid mass that is attached to the specimen; this mass may also have massless springs and dashpots attached to it as shown in Fig. 1. If springs are used, the excitation device and active-end platen (without the specimen in place) form a one-degree-of-freedom system having an undamped natural frequency, f_a.



The computer program in Appendix X1 provides values of F_a and is recommended. For values of T_a below 1.0, the computer program must be used to get accurate values of F_a. For values of T_a above 10, $F_a \simeq T_a$ (The arrows show how the graph is used where T_a corresponds to the value in Appendix X1.)

FIG. 2 ~~(a)~~Dimensionless ~~Frequency Factors~~Modulus Factor F_a for Use in Eq 27 to Calculate Shear Modulus



The arrows show how the graph is used.

FIG. ~~2~~3 ~~(b)~~Specimen Damping Ratio *D* ~~Dimensionless Frequency Factors~~as a function of system logarithmic decrement δ_{system} and D_a ~~(continued)~~for Device Type 1.

6.2.3 The passive-end platen must be rigidly fixed. It may be assumed to be rigidly fixed when the inertia of it and the mass(es) attached to it are at least 500 times the inertia of the active-end platen (**1**)[3].

6.2.4 For Device Type 2, a torque transducer is placed between the passive end platen and the rigidly fixed base. The torque transducer even though relatively stiff in torsion (see 6.4), must allow for some small rotation of the passive end platen in order to register the transmitted torque. The inertia of the passive end platen system, J_p must include the inertia of the sensing head of

---

[3] ~~Enlarged and detailed copies of Figs. 2~~The boldface numbers in parentheses refer to~~a, 2b, 3a, 3b, 4a, and 4 b may be obtained at a nominal charge from ASTM Headquarters, 1916 Race St., Philadelphia, Pa. 19103. Request Adjunct No. 12-440150-00.~~ a list of references at the end of this standard.

the torque transducer which is rigidly fastened to it. With no specimen in place, the passive end platen system inertia, J_p, along with the stiffness, k_p, and damping coefficient, c_p, of the torque transducer constitute a single-degree-of-freedom system which are accounted for in Eq A1.3.

6.3 *Vibration Excitation ~~Device—~~Device (torque motor)—*This shall be ~~an electromagnetic~~ a device capable of applying a sinusoidal ~~longitudinal vibration or~~ torsional vibration ~~or both~~ to the active-end platen to which ~~it is~~ the moving parts of the device are rigidly coupled. The frequency of excitation shall be ~~adjustable and controlled to within 0.5 %. The~~ continuously variable and have a range that typically includes 10 Hz to 1 kHz. For Device Type 1 where the torque is measured at the active end, the excitation device shall have a means of measuring the ~~current~~torque applied to the ~~drive coils~~excitation device that has at least ~~a 5 % accuracy. The~~ 5 % accuracy of full scale output. If an electromagnetic excitation device is used, the voltage drop across a fixed, temperature-and-frequency-stable power resistor in series with the ~~drive coils may be used for this purpose.~~excitation device is proportional to applied torque (Note 2 ~~The force/current and torque/current factors for the vibration excitation devices must be linear within 5 % for the entire range of operating frequencies anticipated when~~). For Device Type 2, the torque is measured at the passive end with a torque transducer, see 6.4~~testing soils.~~.

NOTE 2—Calibrations at more than one frequency may be needed when testing frequencies vary over a wide range. Use of several calibration rods with differing torsional stiffness may be needed.

6.4 *Passive End Torque Transducer—*This torque transducer for Device Type 2 must be waterproof and insensitive to ambient pressure and temperature changes for the expected values. It may be a transducer that also measures axial force. The torque transducer must have a torque capacity of at least twice the maximum torque capability of the vibration excitation device, a linearity of ±0.5 % of full scale output, hysteresis less than ±0.1 % of full scale output, and repeatability better than ±0.5 % of full scale output. If the transducer is used to measure axial force, the specifications must be similar to those for torque. The transducer must be rigidly connected to the chamber base and the sensing head of the torque transducer shall be rigidly connected to the passive end platen.

6.5 *Sine Wave Generator—*The sine wave generator is an electric instrument capable of producing a sinusoidal current with a means of adjusting the frequency over the entire range of operating frequencies anticipated. This instrument shall provide sufficient power to produce the ~~required~~desired vibration amplitude, or its output may be electronically amplified to provide sufficient power. ~~The total distortion of the signal applied to the excitation device shall be less than 3 %.~~

6.6 *Vibration-Measuring Devices and Readout Instruments—*These devices and instruments shall be calibrated with an accuracy of 5 % and must be traceable to a government standards agency. The vibration-measuring devices shall be acceleration, velocity, or displacement transducers that can be attached to and become a part of the ~~active- and passive-end platens. On each platen, one transducer shall be mounted to produce a calibrated electrical output that is proportional to the longitudinal acceleration, velocity, or displacement of that platen (not required for torsion-only apparatus). The other~~ active-end platen. The transducer(s) shall be mounted to produce a calibrated electrical output that is proportional to the rotational acceleration, velocity, or ~~displacement (not required for longitudinal-only apparatus). The readout instrument and transducers shall have a sensitivity such that a displacement of 2.5 × 10~~displacement. The readout instruments must have a frequency resolution of at^{-6} ~~m (10~~least 0.1^{-4} ~~in.) and a rotation of 10~~Hz. $^{-5}$ ~~rad can be measured with 10 % accuracy for the entire range of frequency anticipated. It is also~~ It also is necessary to have an ~~x~~electronic device~~-y-time oscilloscope available for observing signal waveforms and~~ for establishing the phase difference between the applied and/or measured torque and resulting rotational motion for establishing the system resonant frequency. ~~This oscilloscope must have at least one amplifier (vertical or horizontal) with sufficient gain to observe the motion transducer output over the entire range of output voltages and frequencies anticipated. For measurement of damping by the free-vibration method, and for calibration of the apparatus damping, the readout instrument shall be capable of recording the decay of free vibration. Either a strip-chart recorder with appropriate response time and chart speed or an oscilloscope and camera or a digital oscilloscope may be used for this purpose.~~

6.6.1 For Device Type 1, an *x*-*y*-time oscilloscope may be used for this purpose. The electronic device must have amplifiers with sufficient gain to observe the torque motor input and motion transducer outputs over the entire range of frequencies anticipated. For measurement of damping by the free-vibration method, and for calibration of the apparatus damping, the readout instrument shall be capable of recording the decay of free vibration with appropriate response time. A digital *x*-*y*-time oscilloscope may be used for this purpose. For Device Type 2, a dual channel readout device or a spectrum analyzer must be used to measure the magnitude and phase (or real and imaginary) components of the measured θ_a/τ_{TT} at the resonant frequency.

6.7 *Support for Vibration Excitation Device—*~~For the special case where the passive end of the specimen is rigidly fixed and the vibration excitation device and active-end platen are placed on top of the specimen, it~~ It may be necessary to support all or a portion of the weight of the active-end platen and excitation device to prevent excessive axial stress or compressive failure of the specimen. This support may be provided by a spring, counterbalance weights, or pneumatic ~~cylinder~~device as long as the supporting system does not prevent axial movement of the active-end platen and as long as it does not alter the vibration characteristics of the excitation device.

6.8 *Temporary Platen Support Device—*~~The temporary support~~ Temporary support of the active-end platen may be any clamping device that can be used to support ~~one or both end platens~~ the platen during attachment of vibration excitation device to prevent specimen disturbance during apparatus assembly. This device is to be removed prior to the application of vibration.

6.9 *Specimen Dimension-Measuring Devices*—Dimension-measuring devices are needed to measure portions of the apparatus during calibration and specimen diameter and length. ~~All dimensions measurements should be accurate to 0.1 %.~~ Any suitable device may be used to make these measurements except that the device(s) used to measure the length and diameter of the specimen must not deform or otherwise affect the specimen. Specially designed perimeter tapes[4] that measure circumference but read out in diameter are preferred for measuring specimen diameters. Measurement accuracies are specified in 7.2.

6.10 *Balances*—Devices for determining the mass of the soil specimens as well as portions of the device during calibration. All measurements of mass should be accurate to 0.1 %. (Guide D4753)

6.11 *Specimen Preparation and Triaxial Equipment*—These methods cover specimen preparation and procedures related to the vibration of the specimen and do not cover the application and control of ~~ambient~~ static axial and lateral stresses. Any or all of the apparatus described in Test ~~Methods~~Method ~~D2166~~D2166/D2166M, D2850, or D4767 may be used for specimen preparation and application of ~~ambient~~ static axial and lateral stresses. Additional apparatus may be used for these purposes as ~~required.~~ needed.

6.12 *Miscellaneous Apparatus*—The miscellaneous apparatus consists of specimen trimming and carving tools, a membrane expander, remolding apparatus, and moisture content ~~cans, and data sheets~~ cans as required.

~~7. Test Specimen~~

~~7.1 *General*—These methods cover only the special specimen preparation procedures related to the vibration and resonant-column technique. Since the resonant-column test may be conducted in conjunction with controlled ambient stresses, the provisions for preparation of specimens in Test Methods D2166, D2850, or D4767 may be applicable or may be used as a guide in connection with other methods of application and control of ambient stresses.~~

~~7.2 *Specimen Size*—Specimens shall be of uniform circular cross section with ends perpendicular to the axis of the specimen. Specimens shall have a minimum diameter of 33 mm (1.3 in.). The largest particle contained within the test specimen shall be smaller than one tenth of the specimen diameter except that, for specimens having a diameter of 70 mm (2.8 in.) or larger, the largest particle size shall be smaller than one sixth of the specimen diameter. If, after completion of a test, it is found that larger particles than permitted are present, indicate this information in the report of test data under "Remarks." The length-to-diameter ratio shall be not less than 2 nor more than 7 except that, when an ambient axial stress greater than the ambient lateral stress is applied to the specimen, the ratio of length to diameter shall be between 2 and 3. Measure the length at the third points along the perimeter and average the values. Measure two diameters at each of three elevations and average the values. Determine the mass of the test specimen. For determination of moisture content (Test Method D2216), secure a representative specimen of the cuttings from intact specimens, or of the extra soil for remolded specimens, placing the specimen immediately in a covered container.~~

~~7.3 *End Coupling for Torsion*—For torsional motion, complete coupling of the ends of the specimen to the specimen cap and base must be assured. Complete coupling for torsion may be assumed if the mobilized coefficient of friction between the end platens and the specimen is less than 0.2 for all shear strain amplitudes. The coefficient of fraction is approximately given by:~~

~~Coefficient of friction = $\gamma G/\sigma'_a$ (5)~~

~~where:~~

~~γ = shear strain amplitude (see calculations section),~~
~~G = shear modulus (see calculations section), and~~
~~σ'_a = effective axial stress.~~

~~7.3.1 When this criterion is not met, other provisions such as the use of adhesives must be made in order to assure complete coupling. In such cases, the effectiveness of the coupling provisions shall be evaluated by testing two specimens of the same material but of different length. The lengths of these specimens shall differ by at least a factor of 1.5. The provisions for end coupling may be considered satisfactory if the values of the shear modulus for these two specimens of different length do not differ by more than 10 %.~~

7. Test Specimen

7.1 *General*—These methods are limited to the special specimen preparation procedures related to the vibration and resonant-column technique. Since the resonant-column test may be conducted in conjunction with controlled static axial and lateral stresses, the provisions for preparation of specimens in Test Method D2166/D2166M, D2850, or D4767 may be applicable or may be used as a guide in connection with other methods of application and control of static axial and lateral stresses.

7.2 *Specimen Size Limitations*—Specimens shall be of uniform circular cross section with ends perpendicular to the axis of the specimen. Specimens shall have a minimum diameter of 33 mm (1.3 in.). The largest particle contained within the test specimen

---

[4] The sole source of supply of the apparatus known to the committee at this time is **PI Tape**, Box 398, Lemon Grove, CA 92045 (http://www.pitape.com). If you are aware of alternative suppliers, please provide this information to ASTM International Headquarters. Your comments will receive careful consideration at a meeting of the responsible technical committee,[1] which you may attend.

shall be one sixth of the specimen diameter. If, after completion of a test, it is found that larger particles than permitted are present, indicate this information in the report of test data under "Remarks." The length-to-diameter ratio shall be not less than 2 or more than 7 except that, when a static axial stress greater than the lateral stress is applied to the specimen, the ratio of length to diameter shall be between 2 and 3. Take diameter measurements to the nearest 0.25 mm (0.01 in.), at the third points along the specimen length and average them. Take height measurements, to the nearest 0.25 mm (0.01 in.), at four quadrants and average them. For determination of moisture content (Test Method D2216), secure a representative specimen of the cuttings from intact specimens, or of the extra soil for remolded specimens, placing the specimen immediately in a covered container.

7.3 *End Coupling for Torsion*—For torsional motion, complete coupling of the ends of the specimen to the specimen cap and base must be assured. Coupling for torsion may be assumed if the mobilized coefficient of friction between the end platens and the specimen is less than 0.2 for all shear strain amplitudes. The coefficient of friction is approximately given by:

$$\text{Mobilized Coefficient of Friction} = \frac{\gamma G}{\sigma'_a} \tag{1}$$

where:

γ = shear strain amplitude (see Calculations section),
G = shear modulus (see Calculations section), and
σ'_a = effective axial stress.

NOTE 3—The shear strain is not in % for this calculation.

7.3.1 When this criterion is not met, other provisions such as the use of adhesives or other friction increasing measures must be made in order to assure complete coupling (**2**). In such cases, the effectiveness of the coupling provisions shall be evaluated by testing two specimens of the same material but of different length. The lengths of these specimens shall differ by at least a factor of 1.5. The provisions for end coupling may be considered satisfactory if the values of the shear modulus for these two specimens of different length do not differ by more than 10 %.

~~8. Calibration~~

~~8.1 *Motion Transducers*—Motion transducers shall be calibrated with each other and with an independent method to ensure calibration accuracy within 5 %. Linear motion transducers whose axes are located fixed distances from the axis of rotation may be used to measure rotational motion if the cross-axis sensitivities of the transducers are less than 5 %. For this case the distance between the axis of rotation and the transducer axes shall be known to within 5 %. The calibration factors for longitudinal motion shall be expressed in terms of peak-meters/peak-volt. The calibration factors for rotational motion shall be expressed in terms of peak-radians/peak-volt. This means that for velocity and acceleration transducers the vibration frequency shall be included as a term in the calibration factor. For velocity transducers, the displacement calibration factors are given by:~~

$$LCF \text{ or } RCF = \text{velocity calibration factor}/(2\pi f) \tag{6}$$

~~where f = frequency, Hz.~~

~~For acceleration transducers, the displacement calibration factors are given by:~~

$$LCF \text{ or } RCF = \text{acceleration calibration factor}/(2\pi f)^2 \tag{7}$$

~~8.1.1 Thus, for velocity and acceleration transducers, the displacement calibration factors will not be constants but will vary with measured frequency, f. Calibration factors for longitudinal motion are given by the symbol LCF with a subscript A or P denoting whether the transducer is located on the active-end platen or passive-end platen. Likewise, the calibration factors for rotational motion will be given by the symbol RCF and will have subscripts A or P depending on their location.~~

~~8.2 *Passive-End Platen Mass and Rotational Inertia*—The mass and rotational inertia of the passive-end platen shall be determined with all transducers and other rigid attachments securely in place. The mass, M_P, is determined by use of a balance. The rotational inertia of the concentric solid cylindrical components of the passive-end platen and its attachments is given by:~~

$$(J_P)_1 = \frac{1}{8}\sum_{i=1}^{n} M_i d_i^{\,2} \tag{8}$$

~~where:~~

~~M_i = mass of ith solid cylindrical component,~~
~~d_i = diameter of ith solid cylindrical component, and~~
~~n = number of solid cylindrical components.~~

~~Transducers and other masses attached to this platen can be accounted for by:~~

$$(J_P)_2 = \sum_{i=1}^{n} M_i r_i^{\,2} \tag{9}$$

~~where:~~

~~M_i = mass of ith component,~~

~~r_i = distance from the platen axis to center of mass for ith component, and~~
~~n = number of components attached to passive-end platen and not covered in determination of $(J_P)_1$.~~

~~The total rotational inertia for the passive end is given by:~~

$$J_P = (J_P)_1 + (J_P)_2 \tag{10}$$

~~8.3 *Active-End Platen Mass and Rotational Inertia*—The mass, M_A, and rotational inertia, J_A, of the active-end platen shall be determined with all transducers and rigid attachments, including attached portions of the vibration excitation device, securely in place. The equations just given may be used to obtain the mass and rotational inertia. For rotational inertia, if all components do not have simple geometry, an alternative procedure that involves a metal calibration rod of known torsional stiffness may be used. One end of the rod shall be rigidly fixed and the other end shall be rigidly fastened to the active-end platen. Since it may be very difficult to fasten the calibration rod to the platen without adding rotational inertia, it is recommended that the calibration rod be permanently fastened by welding, etc., to an auxiliary platen. If the auxiliary platen is not identical to the one to be used in testing, the difference between its rotational inertia and that of the platen for soil testing must be taken into account by use of aforementioned equations. (For example, suppose that the value of the active-end rotational inertia with the calibration rod was $J1$ and the rotational inertia of the calibration rod platen was $J2$. If the rotational inertia of the platen for testing soil is $J3$, then the value of J_A would be given by $J_A = J1 - J2 + J3$.) The torsional stiffness of the calibration rod should be chosen such that the system resonant frequency with the calibration rod in place is near the middle of the range of system resonant frequencies anticipated for soil testing. Several calibration rods may be necessary to account for different specimen sizes. With the calibration rod in place, determine the low-amplitude system resonant frequency for torsional vibration, $(f_{\text{rod}})_T$. The rotational inertia of the active end platen system is calculated from:~~

$$J_A = \frac{(K_{\text{rod}})_T}{(2\pi)^2\left[(f_{\text{rod}})_T^{\,2} - f_{oT}^{\,2}\right]} \tag{11}$$

~~where:~~

~~$(K_{\text{rod}})_T$ = torsional stiffness of calibration rod, $(I_pG)/L$,~~
~~I_p = polar moment of inertia of calibration rod, $(\pi d^4)/32$,~~
~~d = calibration rod diameter,~~
~~G = shear modulus for calibration rod material, and~~
~~f_{oT} = apparatus torsional resonant frequency as described in the following subsection.~~

~~8.3.1 The foregoing equations assume that the rotational inertia of the calibration rods is much less than the corresponding values for the active-end platen system. A second alternative procedure is to couple the metal calibration rod to the platens in place of the specimen and then use the procedures of the Calculations section to backfigure the active end inertias from the known moduli of the rod.~~

~~8.4 *Apparatus Resonant Frequencies, Spring Constants, and Damping Constants*—Apparatus resonant frequencies and spring constants are defined only for those apparatus that have springs attached to the active-end platen system. To determine the resonant frequencies, set up the apparatus complete with active-end platen and O-rings but no specimen. Vibrate at low amplitude and adjust the frequency of vibration until the input force is in phase with the velocity ofthe active-end platen system. For longitudinal vibration, this apparatus resonant frequency is f_{oL} and for torsional vibration it is f_{oT}. The longitudinal and torsional apparatus spring constants (K_{SL}, K_{ST}) may be calculated from:~~

$$K_{SL} = (2\pi f_{oL})^2 M_A \tag{12}$$

$$K_{ST} = (2\pi f_{oT})^2 J_A$$

~~where M_A and J_A are defined in the previous subsection.~~

~~8.4.1 To measure the damping constants for the apparatus, attach the same masses as used for the determination of apparatus resonant frequencies. For apparatus without springs attached to the active-end platen, insert the calibration rod described in the previous subsection. With the apparatus vibrating at the resonant frequency, cut off the power to the excitation device and record the decay curve for the vibration of the apparatus. From the decay curve, compute the logarithmic decrement, δ, as follows:~~

$$\delta = (1/n)\ln(A_1/A_{n+1}) \tag{13}$$

~~where:~~

~~A_1 = amplitude of vibration for first cycle after power is cut off, and~~
~~A_{n+1} = amplitude for (n + 1)th cycle.~~

~~The apparatus damping coefficient, ADC_L, from longitudinal vibration shall be given by:~~

$$ADC_{oL} = 2f_{oL}M_A\delta_L \tag{14}$$

~~where:~~

~~f_{oL} = longitudinal motion resonant frequency measured during apparatus damping determination,~~
~~M_A = active-end platen mass from previous subsection, and~~
~~δ_L = logarithmic decrement for longitudinal motion.~~

~~For torsional motion, the apparatus damping coefficient, ADC_{oT} is given by:~~

~~$ADC_{oT} = 2f_{oT}J_A\delta_T$ (15)~~

~~where:~~

~~f_{oT} = torsional motion resonant frequency measured during apparatus damping determination,~~
~~J_A = active-end rotational inertia from previous subsection, and~~
~~δ_T = logarithmic decrement for torsional motion.~~

~~8.5 *Force/Current and Torque/Current*—For apparatus without springs attached to the active-end platen, insert the calibration rod as described earlier. Determine the resonant frequency of this single-degree-of-freedom system consisting of the active-end platen and apparatus spring (or calibration rod) by use of the same procedure as described later in the procedures section. Then set the frequency to 0.707 times the resonant frequency and apply sufficient current to the vibration excitation device so that the vibration transducer output to the readout device has a signal of at least ten times the signal due to ambient vibrations and electrical noise when no power is applied to the excitation device. Read and record the output of both the vibration transducer and the current measuring instrument. Next, set the frequency to 1.414 times the system resonant frequency and obtain the vibration transducer and current instrument readings in a similar fashion to those at 0.707 times the resonant frequency. Calculate C_1 and C_2 from:~~

~~$C_1 = 0.5(VTCF)(TO1)/CR1$ (16)~~

~~$C_2 = (VTCF)(TO2)/CR2$~~

~~where:~~

~~*VTCF* = active-end vibration transducer displacement calibration factor (LCF or RCF) (Note 2) depending on whether vibration is longitudinal or torsional,~~
~~*TO1* = active-end transducer output at 0.707 times resonant frequency,~~
~~*CR1* = current instrument reading at 0.707 times resonant frequency (Note 3),~~
~~*TO2* = active-end transducer output at 1.414 times resonant frequency, and~~
~~*CR2* = current instrument reading at 1.414 times resonant frequency (Note 3).~~

~~NOTE 2—*LCF* and *RCF* will be functions of frequency for velocity and acceleration measuring transducers (see 8.1).~~

~~NOTE 3—If a current-measuring instrument is used, the units will be amperes. Alternatively, voltage drop across a fixed resistance may also be measured and the units will then be volts.~~

~~C_1 and C_2 should agree within 10 %. By use of C_1 and C_2 from longitudinal vibration, the force/current calibration factor, *FCF*, is obtained from:~~

~~$FCF = 0.5(C_1 + C_2)K$ (17)~~

~~where *K* = apparatus spring constant (or for apparatus without springs, the calibrating rod spring constant) for longitudinal motion.~~

~~By use of C_1 and C_2 from torsional vibration, the torque/current calibration factor, *TCF*, is obtained from:~~

~~$TCF = 0.5(C_1 + C_2)K$ (18)~~

~~where *K* = apparatus spring constant (or for apparatus without springs, the calibrating rod spring constant) for torsional motion.~~

8. Apparatus Properties (see Note 4)

NOTE 4—Practice D3740 provides information on calibration intervals, records, and quality assurance.

8.1 *Motion Transducers*—Motion transducers shall be calibrated with an independent method to ensure calibration accuracy within 5 % and must be traceable to a government standards agency.

8.1.1 *Rotational Motion Transducer*—The rotational motion at the free end of the soil specimen is normally measured using linear motion transducer(s) mounted at a radial distance r_t from the axis of rotation. Linear motion transducers that are sensitive to acceleration, velocity or displacement may be used. Rotational measuring transducers are acceptable as well. (See 6.6.)

8.1.1.1 The rotation transducer sensitivity S_θ in terms of millivolts/radian is computed as follows:

For an accelerometer transducer with sensitivity S_a [mV/g] the rotation transducer sensitivity at frequency f [Hz] is:

$$S_\theta = S_a r_t (2\pi f)^2 (1/9.81) \tag{2}$$

For a velocity transducer with sensitivity S_v [mV/(m/s)] the rotation transducer sensitivity is:

$$S_\theta = S_v r_t (2\pi f) \tag{3}$$

For a displacement transducer with sensitivity S_d [mV/m] the rotation transducer sensitivity is:

$$S_\theta = S_d r_t \tag{4}$$

Rotation of the top of the specimen is given by:

$$\theta[\text{rad}] = \frac{RTrdg[\text{mV}]}{S_\theta\left[\frac{\text{mV}}{\text{rad}}\right]} \tag{5}$$

where *RTrdg* is the output of the rotation transducer.

8.2 *Active-End Rotational Inertia (only needed for Device Type 1)*—The rotational inertia, J_a, of the active-end platen shall be determined with all transducers and rigid attachments, including attached portions of the vibration excitation device, securely in place. The rotational inertia of the concentric solid cylindrical components of the active-end platen and its attachments is computed from:

$$(J_a)_1 = \frac{1}{8}\sum_{i=1}^{n} M_i d_i^2 \tag{6}$$

where:

M_i = mass of i^{th} solid cylindrical component,
d_i = diameter of i^{th} solid cylindrical component, and
n = number of solid cylindrical components.

Transducers and other masses attached to this platen can be accounted for by:

$$(J_a)_2 = \sum_{i=1}^{n}\left(J_i + M_i\, r_i^2\right) \tag{7}$$

where:

J_i = rotational inertia of the i^{th} component,
M_i = mass of i^{th} component,
r_i = distance from the platen axis to center of mass for i^{th} component, and
n = number of components attached to active-end platen and not covered in determination of $(J_a)_1$.

The total rotational inertia for the active end is given by:

$$J_a = (J_a)_1 + (J_a)_2 \tag{8}$$

8.2.1 Acceptable alternate procedures for determining J_a are provided in A2.1.

8.3 *Apparatus Resonant Frequencies, Spring Constants, and Damping Constants (only needed for Device Type 1)*—(See Note 5) Apparatus resonant frequencies and spring constants are defined only for Device Type 1 that has springs attached to the active-end platen system. To determine the resonant frequencies, set up the apparatus complete with active-end platen and O-rings used to seal the membranes, but with no specimen. Vibrate at low amplitude and adjust the frequency of vibration until the input torque is in phase with the velocity of the active-end platen system. This apparatus resonant frequency is f_a. The apparatus spring constant, k_a, is calculated from:

$$k_a = (2\pi f_a)^2 J_a \tag{9}$$

where J_a is defined in the previous subsection.

NOTE 5—Device Type 2 apparatus may or may not have springs and dashpots attached to the active end platen but by Eq A1.3, these and the active end platen inertia do not affect the determination of shear modulus and damping of the soil.

8.3.1 Apparatus Damping Coefficient for Device Type 1 apparatus without springs attached to the active end platen. Device Type 1 without springs may still have a damping constant to account for back EMF, aerodynamic drag, vibration of wires attached to the platen, and eddy currents. To measure the damping constants for the apparatus, attach the same masses as used for the determination of apparatus resonant frequencies. For apparatus without springs attached to the active-end platen, insert the calibration rod described in the previous subsection. Vibrate the system at the resonant frequency and measure the torque and rotational motion. The apparatus damping coefficient is given by:

$$c_a = \frac{\tau_{appl}}{\theta\omega} = \frac{\tau_{appl}}{\frac{d\theta}{dt}} = \frac{\tau_{appl}\,\omega}{\frac{d^2\theta}{dt^2}} \tag{10}$$

where:

τ_{appl} = amplitude of applied torque,
θ = amplitude of rotation,