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Standard Terminology Relating to Design of Experiments[1]

This standard is issued under the fixed designation E1325; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This standard includes those statistical items related to the area of design of experiments for which standard definitions ~~appears~~appear desirable.

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E456 Terminology Relating to Quality and Statistics

3. Significance and Use

3.1 This standard is a subsidiary to Terminology E456.

3.2 It provides definitions, descriptions, discussion, and comparison of terms.

4. Terminology

aliases, *n—in a fractional factorial design*, two or more effects which are estimated by the same contrast and which, therefore, cannot be estimated separately.

[1] This terminology is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.10 on Sampling / Statistics. ~~The definitions in this standard were extracted from E456 – 89c.~~

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

DISCUSSION—

(*1*) The determination of which effects in a 2^n factorial are *aliased* can be made once the *defining contrast* (in the case of a half replicate) or *defining contrasts* (for a fraction smaller than ½) are stated. The *defining contrast* is that effect (or effects), usually thought to be of no consequence, about which all information may be sacrificed for the experiment. An identity, *I*, is equated to the *defining contrast* (or *defining contrasts*) and, using the conversion that $A^2 = B^2 = C^2 = I$, the multiplication of the letters on both sides of the equation shows the aliases. In the example under fractional factorial design, $I = ABCD$. So that: $A = A^2BCD = BCD$, and $AB = A^2B^2CD = CD$.

(*2*) With a large number of factors (and factorial treatment combinations) the size of the experiment can be reduced to ¼, ⅛, or in general to $½^k$ to form a 2^{n-k} fractional factorial.

(*3*) There exist generalizations of the above to factorials having more than 2 levels.

balanced incomplete block design (BIB), *n*—an incomplete block design in which each block contains the same number *k* of different versions from the *t* versions of a single principal factor arranged so that every pair of versions occurs together in the same number, λ, of blocks from the *b* blocks.

DISCUSSION—

The design implies that every version of the principal factor appears the same number of times *r* in the experiment and that the following relations hold true: $bk = tr$ and $r(k - 1) = \lambda(t - 1)$.

For randomization, arrange the blocks and versions within each block independently at random. Since each letter in the above equations represents an integer, it is clear that only a restricted set of combinations (t, k, b, r, λ) is possible for constructing balanced incomplete block designs. For example, $t = 7$, $k = 4$, $b = 7$, $\lambda = 2$. Versions of the principal factor:

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Block				
1	1	2	3	6
2	2	3	4	7
3	3	4	5	1
4	4	5	6	2
5	5	6	7	3
6	6	7	1	4
7	7	1	2	5

completely randomized design, *n*—a design in which the treatments are assigned at random to the full set of experimental units.

DISCUSSION—

No block factors are involved in a completely randomized design.

completely randomized factorial design, *n*—a factorial experiment (including all replications) run in a completely randomized design.

composite design, *n*—a design developed specifically for fitting second order response surfaces to study curvature, constructed by adding further selected treatments to those obtained from a 2^n factorial (or its fraction).

DISCUSSION—

If the coded levels of each factor are − 1 and + 1 in the 2^n factorial (see notation 2 under discussion for **factorial experiment**), the $(2n + 1)$ additional combinations for a *central composite design* are (0, 0, ..., 0), (±*a*, 0, 0, ..., 0) 0, ±*a*, 0, ..., 0) ..., (0, 0, ..., ± *a*). The minimum total number of treatments to be tested is $(2^n + 2n + 1)$ for a 2^n factorial. Frequently more than one center point will be run. For $n = 2$, 3 and 4 the experiment requires, 9, 15, and 25 units respectively, although additional replicate runs of the center point are usual, as compared with 9, 27, and 81 in the 3^n factorial. The reduction in experiment size results in confounding, and thereby sacrificing, all information about curvature interactions. The value of *a* can be chosen to make the coefficients in the quadratic polynomials as orthogonal as possible to one another or to minimize the bias that is created if the true form of response surface is not quadratic.

confounded factorial design, *n*—a factorial experiment in which only a fraction of the treatment combinations are run in each block and where the selection of the treatment combinations assigned to each block is arranged so that one or more prescribed effects is(are) confounded with the block effect(s), while the other effects remain free from confounding.

NOTE 1—All factor level combinations are included in the experiment.

DISCUSSION—

Example: In a 2^3 factorial with only room for 4 treatments per block, the *ABC* interaction (*ABC*: − (1) + *a* + *b* − *ab* + *c* − *ac* − *bc* + *abc*) can be sacrificed through confounding with blocks without loss of any other effect if the blocks include the ~~following.~~following:

	~~Block 1~~	~~Block 2~~
~~Treatment~~	~~(1)~~	~~a~~
~~Combination~~	~~ab~~	~~b~~
~~(Code identification shown in discus-~~	~~ac~~	~~c~~
~~sion under factorial experiment)~~	~~bc~~	~~abc~~

	Block 1	Block 2
Treatment	(1)	*a*
Combination	*ab*	*b*
(Code identification shown in discus-	*ac*	*c*
sion under factorial experiment)	*bc*	*abc*

~~The treatments to be assigned to each block can be determined once the effect(s) to be confounded is(are) defined. Where only one term is to be confounded with blocks, as in this example, those with a positive sign are assigned to one block and those with a negative sign to the other. There are generalized rules for more complex situations. A check on all of the other effects (A, B, AB, etc.) will show the balance of the plus and minus signs in each block, thus eliminating any confounding with blocks for them.~~

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confounding, *n*—combining indistinguishably the main effect of a factor or a differential effect between factors (interactions) with the effect of other factor(s), block factor(s) or interaction(s).

NOTE 2—Confounding is a useful technique that permits the effective use of specified blocks in some experiment designs. This is accomplished by deliberately preselecting certain effects or differential effects as being of little interest, and arranging the design so that they are confounded with block effects or other preselected principal factor or differential effects, while keeping the other more important effects free from such complications.

Sometimes, however, confounding results from inadvertent changes to a design during the running of an experiment or from incomplete planning of the design, and it serves to diminish, or even to invalidate, the effectiveness of an experiment.

contrast, *n*—a linear function of the observations for which the sum of the coefficients is zero.

NOTE 3—With observations Y_1, Y_2, ..., Y_n, the linear function $a_1Y_1 + a_2Y_2 + ... + a_1Y_n$ is a contrast if, and only if $\sum a_i = 0$, where the a_i values are called the contrast coefficients.

DISCUSSION—

Example 1: A factor is applied at three levels and the results are represented by A_1,A_2, A_3. If the levels are equally spaced, the first question it might be logical to ask is whether there is an overall linear trend. This could be done by comparing A_1 and A_3, the extremes of A in the experiment. A second question might be whether there is evidence that the response pattern shows curvature rather than a simple linear trend. Here the average of A_1 and A_3 could be compared to A_2. (If there is no curvature, A_2 should fall on the line connecting A_1 and A_3 or, in other words, be equal to the average.) The following example illustrates a regression type study of equally spaced continuous variables. It is frequently more convenient to use integers rather than fractions for contrast coefficients. In such a case, the coefficients for Contrast 2 would appear as (−1, + 2, − 1).

Response	A_1	A_2	A_3
Contrast coefficients for question 1	−1	0	+1
Contrast 1	$-A_1$	...	$+ A_3$
Contrast coefficients for question 2	−½	+1	−½
Contrast 2	$-½ A_1$	$+ A_2$	$-½A_3$

Example 2: Another example dealing with discrete versions of a factor might lead to a different pair of questions. Suppose there are three sources of supply, one of which, A_1, uses a new manufacturing technique while the other two, A_2 and A_3 use the customary one. First, does vendor A_1 with the new technique seem to differ from A_2 and A_3? Second, do the two suppliers using the customary technique differ? Contrast A_2 and A_3. The pattern of contrast coefficients is similar to that for the previous problem, though the interpretation of the results will differ.

Response	A_1	A_2	A_3
Contrast coefficients for question 1	−2	+1	+1
Contrast 1	$-2A_1$	$+A_2$	$+A_3$
Contrast coefficients for question 2	0	−1	+1
Contrast 2	...	$- A_2$	$+ A_3$

The coefficients for a contrast may be selected arbitrarily provided the $\sum a_i = 0$ condition is met. Questions of logical interest from an experiment may be expressed as contrasts with carefully selected coefficients. See the examples given in this discussion. As indicated in the examples, the response to each treatment combination will have a set of coefficients associated with it. The number of linearly independent contrasts in an experiment is equal to one less than the number of treatments. Sometimes the term *contrast* is used only to refer to the pattern of the coefficients, but the usual meaning of this term is the algebraic sum of the responses multiplied by the appropriate coefficients.

contrast analysis, *n*—a technique for estimating the parameters of a model and making hypothesis tests on preselected linear combinations of the treatments (contrasts). See Table 1 and Table 2.

NOTE 4—Contrast analysis involves a systematic tabulation and analysis format usable for both simple and complex designs. When any set of orthogonal contrasts is used, the procedure, as in the example, is straightforward. When terms are not orthogonal, the orthogonalization process to adjust for the common element in nonorthogonal contrast is also systematic and can be programmed.

DISCUSSION—

Example: Half-replicate of a 2^4 factorial experiment with factors *A, B* and *C* (X_1, X_2 and X_3 being quantitative, and factor *D* (X_4) qualitative. Defining contrast $I = + ABCD = X_1X_2X_3\ X_4$ (see **fractional factorial design** and **orthogonal ~~design~~contrasts** for derivation of the contrast *coefficients*).

~~**dependent variable,** *n*—see **response variable**.~~

design of experiments, *n*—the arrangement in which an experimental program is to be conducted, and the selection of the levels (versions) of one or more factors or factor combinations to be included in the experiment. Synonyms include experiment design and experimental design.

DISCUSSION—

TABLE 1 Contrast Coefficient

Source	Treatments	(1)	*ab*	*ac*	*bc*	*ad*	*bd*	*cd*	*abcd*	
Centre	X_0	+1	+1	+1	+1	+1	+1	+1	+1	See Note 1
A(+*BCD*): pH (8.0; 9.0)	X_1	−1	+1	+1	−1	+1	−1	−1	+1	
B(+ *ACD*): SO_4 (10 cm^3; 16 cm^3)	X_2	−1	+1	−1	+1	−1	+1	−1	+1	
C(+ *ABD*): Temperature (120°C; 150°C)	X_3	−1	−1	+1	+1	−1	−1	+1	+1	
D(+*ABC*): Factory (P; Q)	X_4	−1	−1	−1	−1	+1	+1	+1	+1	
AB + *CD*	$X_1X_2 = X_{12}$	+1	+1	−1	−1	−1	−1	+1	+1	
AC + *BD*	$X_1X_3 = X_{13}$	+1	−1	+1	−1	−1	+1	−1	+1	See Note 2
AD + *BC*	$X_1X_4 = X_{14}$	+1	−1	−1	+1	+1	−1	−1	+1	

NOTE 1—The center is not a constant ($\sum X_i \neq 0$) but is convenient in the contrast analysis calculations to treat it as one.

The purpose of designing an experiment is to provide the most efficient and economical methods of reaching valid and relevant conclusions from the experiment. The selection of an appropriate design for any experiment is a function of many considerations such as the type of questions to be answered, the degree of generality to be attached to the conclusions, the magnitude of the effect for which a high probability of detection (power) is desired, the homogeneity of the experimental units and the cost of performing the experiment. A properly designed experiment will permit relatively simple statistical interpretation of the results, which may not be possible otherwise. The *arrangement* includes the randomization procedure for allocating treatments to experimental units.

experimental design,*n*—see **design of experiments**.

experimental unit, *n*—a portion of the experiment space to which a treatment is applied or assigned in the experiment.

NOTE 5—The unit may be a patient in a hospital, a group of animals, a production batch, a section of a compartmented tray, etc.

experiment space, *n*—the materials, equipment, environmental conditions and so forth that are available for conducting an experiment.

DISCUSSION—

That portion of the experiment space restricted to the range of levels (versions) of the factors to be studied in the experiment is sometimes called the *factor space*. Some elements of the experiment space may be identified with blocks and be considered as block factors.

evolutionary operation (EVOP), *n*— a sequential form of experimentation conducted in production facilities during regular production.

NOTE 6—The principal theses of EVOP are that knowledge to improve the process should be obtained along with a product, and that designed experiments using relatively small shifts in factor levels (within production tolerances) can yield this knowledge at minimum cost. The range of variation of the factors for any one EVOP experiment is usually quite small in order to avoid making out-of-tolerance products, which may require considerable replication, in order to be able to clearly detect the effect of small changes.

2^n factorial experiment,*n*—a factorial experiment in which *n* factors are studied, each of them in two levels (versions).

DISCUSSION—

~~The 2^n factorial is a special case of the general factorial. (See **factorial experiment (general)**.) A popular code is to indicate a small letter when a factor is at its high level, and omit the letter when it is at its low level. When factors are at their low level the code is (*1*).~~

~~*Example (illustrating the discussion)*—A2^3 factorial with factors *A, B*, and *C*:~~

			~~Level~~					
~~Factor A~~	~~Low~~	~~High~~	~~Low~~	~~High~~	~~Low~~	~~High~~	~~Low~~	~~High~~
~~Factor B~~	~~Low~~	~~Low~~	~~High~~	~~High~~	~~Low~~	~~Low~~	~~High~~	~~High~~
~~Factor C~~	~~Low~~	~~Low~~	~~Low~~	~~Low~~	~~High~~	~~High~~	~~High~~	~~High~~
~~Code~~	~~(*1*)~~	~~*a*~~	~~*b*~~	~~*ab*~~	~~*c*~~	~~*ac*~~	~~*bc*~~	~~*abc*~~

~~This type of identification has advantages for defining blocks, confounding and aliasing. See~~ The ~~confounded factorial design and fractional factorial design.~~

~~Factorial experiments regardless of the form of analysis used, essentially involve contrasting the various levels (versions) of the factors.~~

~~Example (illustrating contrast)—Two-factor, two-level factorial~~ 2^2 ~~with factors~~ A^n ~~and~~factorial ~~B: A = [a − (1)] + [ab − b]. This is the contrast~~ is a special case of ~~A~~the ~~at the~~general factorial. ~~low~~(See ~~**level of**~~ **factorial experiment (general).** ~~B~~) ~~plus the contrast of~~ A ~~at the~~ popular code ~~high level of B. B = [b − (1)] + [ab − a]. This is the contrast of~~is to indicate a ~~B~~small ~~at the~~letter when a factor is at its ~~low~~high ~~level of~~level, and ~~A~~omit ~~plus the contrast of~~the letter when it ~~B~~is at ~~the~~its ~~high~~low ~~level of~~level. When ~~A: AB~~factors are ~~= [~~at~~ab − b] − [a − (1)] = [~~their low~~ab − a] − [~~level~~b −~~ the code is (*1*)~~]~~.).

~~This~~ ~~*is the contrast of the contrasts of*~~ *Example (illustrating the discussion*~~A~~)—A2^3 ~~at the~~factorial with ~~high~~factors ~~level~~ ~~*of*~~*A, B,* and ~~the *low*~~*C* ~~level of B or the contrast of the contrasts of B at the high level of A and at the low level of A.~~

~~Each contrast can be derived from the development of a symbolic product of two factors, these factors being of the form (a ± 1), (b ± 1), using − 1 when the capital letter (A, B) is included in the contrast and + 1 when it is not.~~

~~Example:~~

~~A~~:

			Level					
Factor A	Low	High	Low	High	Low	High	Low	High
Factor B	Low	Low	High	High	Low	Low	High	High
Factor C	Low	Low	Low	Low	High	High	High	High
Code	(*1*)	*a*	*b*	*ab*	*c*	*ac*	*bc*	*abc*

This type of identification has advantages for defining blocks, confounding and aliasing. See **confounded factorial design** and **fractional factorial design**.

Factorial experiments regardless of the form of analysis used, essentially involve contrasting the various levels (versions) of the factors.

NOTE 2—Once the contrast coefficients of the main effects (X_1, X_2, X_3 and X_4) are filled in, the coefficients for all interaction and other second or higher order effects can be derived as products ($X_{ij} = X_i\ X_i$) of the appropriate terms.

TABLE 2 Contrast Analysis

Source	Contrast $\sum_i X_{ij}Y_i{}^1$	Divisor $\sum_i X_{ij}^2$	Student's *t* ratio[2] $(\sum_i X_{ij}Y_i)/s\sqrt{\sum_i X_{ij}^2}$	Regression coefficient $B_j = (\sum_i X_{ij}Y_I)\ /\ \sum_i X_{ij}^2$
X_0: Centre	$\sum X_0Y$	$\sum X_0^2$	$(\sum X_0Y)/s\sqrt{\sum X_0^2}$	$B_0=(\sum X_0Y)/\sum X_0^2$
X_1: $A + BCD$	$\sum X_1Y$	$\sum X_1^2$	$(\sum X_1Y)/s\sqrt{\sum X_1^2}$	$B_1=(\sum X_1Y)/\sum X_1^2$
X_2: $B + ACD$	$\sum X_2Y$	$\sum X_2^2$	$(\sum X_2Y)/s\sqrt{\sum X_2^2}$	$B_2=(\sum X_2Y)/\sum X_2^2$
X_3: $C + ABD$	$\sum X_3Y$	$\sum X_3^2$	$(\sum X_3Y)/s\sqrt{\sum X_3^2}$	$B_3=(\sum X_3Y)/\sum X_3^2$
X_4: $D + ABC$	$\sum X_4Y$	$\sum X_4^2$	$(\sum X_4Y)/s\sqrt{\sum X_4^2}$	$B_4=(\sum X_4Y)/\sum X_4^2$
X_{12}: $AB + CD$	$\sum X_{12}Y$	$\sum X_{12}^2$	$(\sum X_{12}Y)/s\sqrt{\sum X_{12}^2}$	$B_{12}=(\sum X_{12}Y)/\sum X_{12}^2$
X_{13}: $AC + BD$	$\sum X_{13}Y$	$\sum X_{13}^2$	$(\sum X_{13}Y)/s\sqrt{\sum X_{13}^2}$	$B_{13}=(\sum X_{13}Y)/\sum X_{13}^2$
X_{14}: $AD + BC$	$\sum X_{14}Y$	$\sum X_{14}^2$	$(\sum X_{14}Y)/s\sqrt{\sum X_{14}^2}$	$B_{14}=(\sum X_{14}Y)/\sum X_{14}^2$

NOTE 1—The notation for contrast analysis usually uses Y to indicate the response variable and X the predictor variables.

NOTE 2—The measure of experimental error, s, can be obtained in various ways. If the experiment is replicated, s is the square root of the pooled variances of the pairs for each treatment combination. (Each row of X values would be expanded to account for the additional observations in the contrast analysis computations). If some effects were felt to be pseudo-replicates (example, no interactions were logical) multiplying the contrast by the regression coefficient of these terms forms a sum of squares (as in analysis of variance) and these would be summed and divided by the number of terms involved to give s^2. Also, in many experiments, past experience may already provide an estimate of this error. Assumed model: $Y = B_0 + B_1X_{1i} + B_2X_{3i} + B_4X_{4i} + e$). In a simple 2-level experiment such as this, the regression coefficient measures the half-effect of shifting a factor, say pH, between its low and high level, or the effect of shifting from a center level to the high level. In general, substitution of the appropriate contrast coefficients for the X terms in the model will permit any desired comparisons. The difference between quantitative and qualitative factors lies in the interpretation. Since a unit of X_1 represents a pH shift of 0.5, there is a meaningful translation into physical units. On the other hand, the units of the qualitative variable (factories) have no significance other than for identification and in the substitution process to obtain estimates of the average response values.

Example (illustrating contrast)—Two-factor, two-level factorial 2^2 with factors *A* and *B*: $A = [a - (1)] + [ab - b]$. This is the contrast of *A* at the *low* level of *B* plus the contrast of *A* at the *high* level of *B*. $B = [b - (1)] + [ab - a]$. This is the contrast of *B* at the *low* level of *A* plus the contrast of *B* at the *high* level of *A*: $AB = [ab - b] - [a - (1)] = [ab - a] - [b - (1)]$. This is the contrast of the contrasts of *A* at the *high* level of *B* and the *low* level of *B* or the contrast of the contrasts of *B* at the *high* level of *A* and at the *low* level of *A*.

Each contrast can be derived from the development of a *symbolic product* of two factors, these factors being of the form ($a \pm 1$), ($b \pm 1$), using – 1 when the capital letter (*A, B*) is included in the contrast and + 1 when it is not.

Example:

A: $(a - 1)(b + 1)$
B: $(a + 1)(b - 1)$
AB: $(a - 1)(b - 1)$

These expressions are usually written in a standard order, in this case:

A: $-(1) + a - b + ab$
B: $-(1) - a + b + ab$
AB: $(1) - a - b + ab$

Note that the coefficient of each treatment combination in AB (+ 1 or – 1) is the product of the corresponding coefficients in A and B. This property is general in 2^n factorial experiments. After grouping, the A term 2^n represents the effect of A averaged over the two levels of B, that is, a main effect or average effect. Similarly, B represents the average effect of B over both levels of A. The AB term contrasts the effect of A at the high and the low levels of B (or the effect of B at the high and low levels of A), that is an interaction or differential effect.

This example is, of course, the simplest case, but it illustrates the basic principles. The contrasts may appear more complex as additional factors are introduced.

~~(a – 1)(b + 1)~~
~~B: (a + 1)(b – 1)~~
~~AB: (a – 1)(b – 1)~~
~~These expressions are usually written in a standard order, in this case:~~
~~A: –(1) + a – b + ab~~
~~B: –(1) – a + b + ab~~
~~AB: (1) – a – b + ab~~

~~Note that the coefficient of each treatment combination in AB (+ 1 or – 1) is the product of the corresponding coefficients in A and B. This property is general in 2^n factorial experiments. After grouping, the A term 2^n represents the effect of A averaged over the two levels of B, that is, a main effect or average effect. Similarly, B represents the average effect of B over both levels of A. The AB term contrasts the effect of A at the high and the low levels of B (or the effect of B at the high and low levels of A), that is an interaction or differential effect.~~

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factorial experiment (general), *n*—in general, an experiment in which all possible treatments formed from two or more factors, each being studied at two or more levels (versions), are examined so that interactions (differential effects) as well as main effects can be estimated.

DISCUSSION—

The term is descriptive of the combining of the various factors in all possible combinations, but in itself does not describe the experimental design in which these combinations, or a subset of these combinations, will be studied.

The most commonly used designs for the selected arrangement of the factorial treatment combinations are the completely randomized design, the randomized block design and the balanced incomplete block design, but others also are used.

A factorial experiment is usually described symbolically as the product of the number of levels (versions) of each factor. For example, an experiment based on 3 levels of factor *A*, 2 versions of factor *B* and 4 levels of factor *C* would be referred to as a 3 × 2 × 4 factorial. The product of these numbers indicates the number of factorial treatments.

When a factorial experiment includes factors all having the same number of levels (versions), the description is usually given in terms of the number of levels raised to the power equal to the number of factors, *n*. Thus, an experiment with three factors all run at two levels would be referred to as a 2^3 factorial (*n* being equal to 3) and has 8 factorial treatment combinations. Some commonly used notations for describing the treatment combinations for a factorial experiment are as follows:

(1) Use a letter to indicate the factor and a numerical subscript the level (version) of the factor, for example, three factors *A*, *B*, and *C* in a 2 × 3 × 2 factorial. The 12 combinations would be:

$$A_1B_1C_1,\ A_2B_1C_1,\ A_1B_2C_1,\ A_2B_2C_1,\ A_1B_3C_1,\ A_2B_3C_1,$$
$$A_1B_1C_2,\ A_2B_1C_2,\ A_1B_2C_2,\ A_2B_2C_2,\ A_1B_3C_2,\ A_2B_3C_2.$$

Sometimes only the subscripts, listed in the same order as the factors are used, such as: 111, 211, 121, 221, 131, 231, 112, 212, 122, 222, 132, 232. A variation which permits the use of modulo 2 and modulo 3 arithmetic for the purpose of listing the treatment combinations in blocked and fractional designs is: 000, 100, 010, 110, 020, 120, 001, 101, 011, 111, 021, 121.

(*2*) Describe the levels in terms of the number of unit deviations from the center level, including sign. In the case of an even number of levels where there is no actual treatment at the center level, the coefficients describing the levels are usually given in terms of half-unit deviations. For example, with two levels, if a unit of deviation between these levels is 4 mm, the −1 coefficient might be assigned to 3 mm and the + 1 to 7 mm with 0 being assigned to the non-included 5 mm level. In this example the code would appear as follows.

(−1, −1, −1); (+1, −1, −1); (−1, 0, −1); (+1, 0, −1);
(−1, +1, −1); (+1, +1, −1); (−1, −1, +1); (+1, −1, +1);
(−1, 0, +1); (+1, 0, +1); (−1, +1, +1); (+1, +1, +1)

This descriptive coding has many advantages, particularly in analyzing contrasts when levels are equally spaced. Unequal spacing of the levels or weighted emphasis for the various versions can also be reflected in the coefficients.

fractional factorial design, *n*—a factorial experiment in which only an adequately chosen fraction of the treatments required for the complete factorial experiment is selected to be run.

NOTE 7—This procedure is sometimes called fractional replication.

DISCUSSION—

The fraction selected is obtained by choosing one or several *defining contrasts* which are considered of minor importance, or negligible, generally interaction(s) of high order. These *defining contrasts* cannot be estimated and thus are sacrificed. By *adequately chosen* is meant selection according to specified rules which include consideration of effects to be confounded and aliased (see **confounding** and **aliases**). It is possible to use tables of orthogonal arrays, algorithms or a listing of designs to obtain the factorial treatment combinations for the fractional replicate without actually specifying the defining contrasts, but this entails a loss of information.

Fractional factorial designs are often used very effectively in screening tests to determine which factor or factors are large contributors to variability, or as part of a sequential series of tests, but there are risks of getting biased estimates of main effects or of misjudging the relative importance of various factors. When there is a large number of factor level combinations resulting from a large number of factors to be tested, it is often impracticable to test all the combinations with one experiment. In such cases resort may be made to a fractional, that is, partial, replication. The usefulness of these designs stems from the fact that, in general, higher order interactions are not likely to occur. When this assumption is not valid, biased estimates will result.

Example—Two half-replicates of a 2^4 factorial (refer to the discussion under factorial experiment for the code interpretation). Defining contrast, *ABCD*:

+	−
abcd	*abc*
ab	*abd*
ac	*acd*
ad	*bcd*
bc	*a*
bd	*b*
cd	*c*
(1)	*d*

Either of these half-replicates can be used as a *fractional replicate*.

In the example, the factorial combinations in the first column are those with *a* + (plus) sign in the development of symbolic product of the *ABCD defining contrast*, as illustrated in the example o f 2^n factorial experiment. $ABCD = (a - 1)(b - 1)(c - 1)(d - 1)$. Those factorial combinations in the second column are those with *a* – (minus) sign.

Because only those elements of the *ABCD* interaction having the same sign are run, no *ABCD* contrast measure is obtainable, so that the *ABCD* interaction is completely confounded and unestimable. In addition, it will be found that because only half of the full factorial experiment is run, each contrast represents two effects.

From the + sign fractional replicate in this example, we should compute the factorial effects as follows: