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An American National Standard

Standard Practice for Use of Control Charts in Statistical Process Control[1]

This standard is issued under the fixed designation E2587; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice provides guidance for the use of control charts in statistical process control programs, which improve process quality through reducing variation by identifying and eliminating the effect of special causes of variation.

1.2 Control charts are used to continually monitor product or process characteristics to determine whether or not a process is in a state of statistical control. When this state is attained, the process characteristic will, at least approximately, vary within certain limits at a given probability.

1.3 This practice applies to variables data (characteristics measured on a continuous numerical scale) and to attributes data (characteristics measured as percentages, fractions, or counts of occurrences in a defined interval of time or space).

1.4 The system of units for this practice is not specified. Dimensional quantities in the practice are presented only as illustrations of calculation methods. The examples are not binding on products or test methods treated.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.6 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods
E456 Terminology Relating to Quality and Statistics
E1994 Practice for Use of Process Oriented AOQL and LTPD Sampling Plans
E2234 Practice for Sampling a Stream of Product by Attributes Indexed by AQL
E2281 Practice for Process Capability and Performance Measurement
E2762 Practice for Sampling a Stream of Product by Variables Indexed by AQL

3. Terminology

3.1 *Definitions:*

3.1.1 See Terminology E456 for a more extensive listing of statistical terms.

3.1.2 *assignable cause, n*—factor that contributes to variation in a process or product output that is feasible to detect and identify (see *special cause*).

3.1.2.1 *Discussion*—Many factors will contribute to variation, but it may not be feasible (economically or otherwise) to identify some of them.

3.1.3 *accepted reference value, ARV, n*—value that serves as an agreed-upon reference for comparison and is derived as: (*1*) a theoretical or established value based on scientific principles, (*2*) an assigned or certified value based on experimental work of some national or international organization, or (*3*) a consensus or certified value based on collaborative experimental work under the auspices of a scientific or engineering group. **E177**

3.1.4 *attributes data, n*—observed values or test results that indicate the presence or absence of specific characteristics or counts of occurrences of events in time or space.

3.1.5 *average run length (ARL), n*—the average number of times that a process will have been sampled and evaluated before a shift in process level is signaled.

3.1.5.1 *Discussion*—A long ARL is desirable for a process located at its specified level (so as to minimize calling for unneeded investigation or corrective action) and a short ARL is desirable for a process shifted to some undesirable level (so that corrective action will be called for promptly). ARL curves are used to describe the relative quickness in detecting level shifts of various control chart systems (see 5.1.4). The average

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[1] This practice is under the jurisdiction of ASTM Committee E11 on Quality and Statistics and is the direct responsibility of Subcommittee E11.30 on Statistical Quality Control.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

number of units that will have been produced before a shift in level is signaled may also be of interest from an economic standpoint.

3.1.6 *c chart, n*—control chart that monitors the count of occurrences of an event in a defined increment of time or space.

3.1.7 *center line, n*—line on a control chart depicting the average level of the statistic being monitored.

3.1.8 *chance cause, n*—source of inherent random variation in a process which is predictable within statistical limits (see *common cause*).

3.1.8.1 *Discussion*—Chance causes may be unidentifiable, or may have known origins that are not easily controllable or cost effective to eliminate.

3.1.9 *common cause, n*—(see *chance cause*).

3.1.10 *control chart, n*—chart on which are plotted a statistical measure of a subgroup versus time of sampling along with limits based on the statistical distribution of that measure so as to indicate how much common, or chance, cause variation is inherent in the process or product.

3.1.11 *control chart factor, n*—a tabulated constant, depending on sample size, used to convert specified statistics or parameters into a central line value or control limit appropriate to the control chart.

3.1.12 *control limits, n*—limits on a control chart that are used as criteria for signaling the need for action or judging whether a set of data does or does not indicate a state of statistical control based on a prescribed degree of risk.

3.1.12.1 *Discussion*—For example, typical three-sigma limits carry a risk of 0.135 % of being out of control (on one side of the center line) when the process is actually in control and the statistic has a normal distribution.

3.1.13 *EWMA chart, n*—control chart that monitors the exponentially weighted moving averages of consecutive subgroups.

3.1.14 *EWMV chart, n*—control chart that monitors the exponentially weighted moving variance.

3.1.15 *exponentially weighted moving average (EWMA), n*—weighted average of time ordered data where the weights of past observations decrease geometrically with age.

3.1.15.1 *Discussion*—Data used for the EWMA may consist of individual observations, averages, fractions, numbers defective, or counts.

3.1.16 *exponentially weighted moving variance (EWMV), n*—weighted average of squared deviations of observations from their current estimate of the process average for time ordered observations, where the weights of past squared deviations decrease geometrically with age.

3.1.16.1 *Discussion*—The estimate of the process average used for the current deviation comes from a coupled EWMA chart monitoring the same process characteristic. This estimate is the EWMA from the previous time period, which is the forecast of the process average for the current time period.

3.1.17 *I chart, n*—control chart that monitors the individual subgroup observations.

3.1.18 *lower control limit (LCL), n*—minimum value of the control chart statistic that indicates statistical control.

3.1.19 *MR chart, n*—control chart that monitors the moving range of consecutive individual subgroup observations.

3.1.20 *p chart, n*—control chart that monitors the fraction of occurrences of an event.

3.1.21 *R chart, n*—control chart that monitors the range of observations within a subgroup.

3.1.22 *rational subgroup, n*—subgroup chosen to minimize the variability within subgroups and maximize the variability between subgroups (see *subgroup*).

3.1.22.1 *Discussion*—Variation within the subgroup is assumed to be due only to common, or chance, cause variation, that is, the variation is believed to be homogeneous. If using a range or standard deviation chart, this chart should be in statistical control. This implies that any assignable, or special, cause variation will show up as differences between the subgroups on a corresponding $\bar{X}$ chart.

3.1.23 *s chart, n*—control chart that monitors the standard deviations of subgroup observations.

3.1.24 *special cause, n*—(see *assignable cause*).

3.1.25 *standardized chart, n*—control chart that monitors a standardized statistic.

3.1.25.1 *Discussion*—A standardized statistic is equal to the statistic minus its mean and divided by its standard error.

3.1.26 *state of statistical control, n*—process condition when only common causes are operating on the process.

3.1.26.1 *Discussion*—In the strict sense, a process being in a state of statistical control implies that successive values of the characteristic have the statistical character of a sequence of observations drawn independently from a common distribution.

3.1.27 *statistical process control (SPC), n*—set of techniques for improving the quality of process output by reducing variability through the use of one or more control charts and a corrective action strategy used to bring the process back into a state of statistical control.

3.1.28 *subgroup, n*—set of observations on outputs sampled from a process at a particular time.

3.1.29 *u chart, n*—control chart that monitors the count of occurrences of an event in variable intervals of time or space, or another continuum.

3.1.30 *upper control limit (UCL), n*—maximum value of the control chart statistic that indicates statistical control.

3.1.31 *variables data, n*—observations or test results defined on a continuous scale.

3.1.32 *warning limits, n*—limits on a control chart that are two standard errors below and above the centerline.

3.1.33 *X-bar chart, n*—control chart that monitors the average of observations within a subgroup.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *allowance value, K, n*—amount of process shift to be detected.

3.2.2 *allowance multiplier, k, n*—multiplier of standard deviation that defines the allowance value, K.

3.2.3 *average count* ($\bar{c}$), *n*—arithmetic average of subgroup counts.

3.2.4 *average moving range* $\left(\overline{MR}\right)$, *n*—arithmetic average of subgroup moving ranges.

3.2.5 *average proportion* ($\bar{p}$), *n*—arithmetic average of subgroup proportions.

3.2.6 *average range* $\left(\bar{R}\right)$, *n*—arithmetic average of subgroup ranges.

3.2.7 *average standard deviation* ($\bar{s}$), *n*—arithmetic average of subgroup sample standard deviations.

3.2.8 *cumulative sum, CUSUM, n*—cumulative sum of deviations from the target value for time-ordered data.

3.2.8.1 *Discussion*—Data used for the CUSUM may consist of individual observations, subgroup averages, fractions defective, numbers defective, or counts.

3.2.9 *CUSUM chart, n*—control chart that monitors the cumulative sum of consecutive subgroups.

3.2.10 *decision interval, H, n*—the distance between the center line and the control limits.

3.2.11 *decision interval multiplier, h, n*—multiplier of standard deviation that defines the decision interval, H.

3.2.12 *grand average* ($\bar{\bar{X}}$), *n*—average of subgroup averages.

3.2.13 *inspection interval, n*—a subgroup size for counts of events in a defined interval of time space or another continuum.

3.2.13.1 *Discussion*—Examples are 10 000 metres of wire inspected for insulation defects, 100 square feet of material surface inspected for blemishes, the number of minor injuries per month, or scratches on bearing race surfaces.

3.2.14 *moving range (MR), n*—absolute difference between two adjacent subgroup observations in an *I* chart.

3.2.15 *observation, n*—a single value of a process output for charting purposes.

3.2.15.1 *Discussion*—This term has a different meaning than the term defined in Terminology E456, which refers there to a component of a test result.

3.2.16 *overall proportion, n*—average subgroup proportion calculated by dividing the total number of events by the total number of objects inspected (see average proportion).

3.2.16.1 *Discussion*—This calculation may be used for fixed or variable sample sizes.

3.2.17 *process, n*—set of interrelated or interacting activities that convert input into outputs.

3.2.18 *process target value, T, n*—target value for the observed process mean.

3.2.19 *relative size of process shift, δ, n*—size of process shift to detect in standard deviation units.

3.2.20 *subgroup average* ($\bar{X}_i$), *n*—average for the *i*th subgroup in an *X*-bar chart.

3.2.21 *subgroup count* (c_i), *n*—count for the *i*th subgroup in a *c* chart.

3.2.22 *subgroup EWMA* (Z_i), *n*—value of the EWMA for the *i*th subgroup in an EWMA chart.

3.2.23 *subgroup EWMV* (V_i), *n*—value of the EWMV for the *i*th subgroup in an EWMV chart.

3.2.24 *subgroup individual observation* ($\bar{X}_i$), *n*—value of the single observation for the *i*th subgroup in an *I* chart.

3.2.25 *subgroup moving range* (MR_i), *n*—moving range for the *i*th subgroup in an *MR* chart.

3.2.25.1 *Discussion*—If there are *k* subgroups, there will be *k*-1 moving ranges.

3.2.26 *subgroup proportion* (p_i), *n*—proportion for the *i*th subgroup in a *p* chart.

3.2.27 *subgroup range* (R_i), *n*—range of the observations for the *i*th subgroup in an *R* chart.

3.2.28 *subgroup size* (n_i), *n*—the number of observations, objects inspected, or the inspection interval in the *i*th subgroup.

3.2.28.1 *Discussion*—For fixed sample sizes the symbol *n* is used.

3.2.29 *subgroup standard deviation* (s_i), *n*—sample standard deviation of the observations for the *i*th subgroup in an *s* chart.

3.3 *Symbols:*

A_2 = factor for converting the average range to three standard errors for the *X*-bar chart (Table 1)

A_3 = factor for converting the average standard deviation to three standard errors of the average for the *X*-bar chart (Table 1)

TABLE 1 Control Chart Factors

	for *X*-Bar and *R* Charts				for *X*-Bar and *S* Charts			
n	A_2	D_3	D_4	d_2	A_3	B_3	B_4	c_4
2	1.880	0	3.267	1.128	2.659	0	3.267	0.7979
3	1.023	0	2.575	1.693	1.954	0	2.568	0.8862
4	0.729	0	2.282	2.059	1.628	0	2.266	0.9213
5	0.577	0	2.114	2.326	1.427	0	2.089	0.9400
6	0.483	0	2.004	2.534	1.287	0.030	1.970	0.9515
7	0.419	0.076	1.924	2.704	1.182	0.118	1.882	0.9594
8	0.373	0.136	1.864	2.847	1.099	0.185	1.815	0.9650
9	0.337	0.184	1.816	2.970	1.032	0.239	1.761	0.9693
10	0.308	0.223	1.777	3.078	0.975	0.284	1.716	0.9727

Note: for larger numbers of *n*, see Ref. (**1**).[A]

[A] The boldface numbers in parentheses refer to a list of references at the end of this standard.

B_3, B_4 = factors for converting the average standard deviation to three-sigma limits for the s chart (Table 1)
B_5^*, B_6^* = factors for converting the initial estimate of the variance to three-sigma limits for the EWMV chart (Table 11)
C_0 = cumulative sum (CUSUM) at time zero (12.2.2)
c_4 = factor for converting the average standard deviation to an unbiased estimate of sigma (*see* σ) (Table 1)
c_i = counts of the observed occurrences of events in the ith subgroup (10.2.1)
C_i = cumulative sum (CUSUM) at time, i (12.1)
$\bar{c}$ = average of the k subgroup counts (10.2.1)
$\bar{d}_2$ = factor for converting the average range to an estimate of sigma (see σ) (Table 1)
D_3, D_4 = factors for converting the average range to three-sigma limits for the R chart (Table 1)
D_i^2 = the squared deviation of the observation at time i minus its forecast average (13.1)
h = decision interval multiplier for calculation of the decision interval, H (12.1.5)
H = decision interval for calculation of CUSUM control limits (12.1.5)
k = number of subgroups used in calculation of control limits (6.2.1)
k = allowance multiplier for calculation of K (12.1.5.1)
K = amount of process shift to detect with a CUSUM chart (12.1.5)
MR_i = absolute value of the difference of the observations in the (i-1)th and the ith subgroups in a MR chart (8.2.1)
$(\overline{MR})$ = average of the subgroup moving ranges (8.2.2.1)
n = subgroup size, number of observations in a subgroup (5.1.3)
n_i = subgroup size, number of observations (objects inspected) in the ith subgroup (9.1.2)
p_i = proportion of the observed occurrences of events in the ith subgroup (9.2.1)
$\bar{p}$ = average of the k subgroup proportions (9.2.1)
R_i = range of the observations in the ith subgroup for the R chart (6.2.1.2)
$\bar{R}$ = average of the k subgroup ranges (6.2.2)
s_i = Sample standard deviation of the observations in the ith subgroup for the s chart (7.2.1)
s_z = standard error of the EWMA statistic (11.2.1.2)
$\bar{s}$ = average of the k subgroup standard deviations (7.2.2)
T = process target value for process mean (12.1.1)
u_i = counts of the observed occurrences of events in the inspection interval divided by the size of the inspection interval for the ith subgroup (10.4.2)
V_0 = exponentially-weighted moving variance at time zero (13.2.1)
V_i = exponentially-weighted moving variance statistic at time i (13.1)
X_i = single observation in the ith subgroup for the I chart (8.2.1)
X_{ij} = the jth observation in the ith subgroup for the X-bar chart (6.2.1)
$\bar{X}$ = average of the individual observations over k subgroups for the I chart (8.2.2)
$\bar{X}_i$ = average of the ith subgroup observations for the X-bar chart (6.2.1)
$\bar{\bar{X}}$ = average of the k subgroup averages for the X-bar chart (6.2.2)
Y_i = value of the statistic being monitored by an EWMA chart at time i (11.2.1)
z_i = the standardized statistic for the ith subgroup (9.4.1.3)
Z_0 = exponentially-weighted moving average at time zero (11.2.1.1)
Z_i = exponentially-weighted average (EWMA) statistic at time i (11.2.1)
δ = relative process shift for calculation of the allowance multiplier, k (12.1.5.1)
λ = factor ($0 < \lambda < 1$) which determines the weighing of data in the EWMA statistic (11.2.1)
$\hat{\sigma}$ = estimated common cause standard deviation of the process (6.2.4)
$\hat{\sigma}_c$ = standard error of c, the number of observed counts (10.2.1.2)
$\hat{\sigma}_p$ = standard error of p, the proportion of observed occurrences (9.2.2.4)
v = effective degrees of freedom for the EWMV (13.1.2)
ω = factor ($0 < \omega < 1$) which determines the weighting of squared deviations in the EWMV statistic (13.1)

4. Significance and Use

4.1 This practice describes the use of control charts as a tool for use in statistical process control (SPC). Control charts were developed by Shewhart (**2**)[3] in the 1920s and are still in wide use today. SPC is a branch of statistical quality control (**3, 4**), which also encompasses process capability analysis and acceptance sampling inspection. Process capability analysis, as described in Practice E2281, requires the use of SPC in some of its procedures. Acceptance sampling inspection, described in Practices E1994, E2234, and E2762, requires the use of SPC so as to minimize rejection of product.

4.2 *Principles of SPC*—A process may be defined as a set of interrelated activities that convert inputs into outputs. SPC uses various statistical methodologies to improve the quality of a process by reducing the variability of one or more of its outputs, for example, a quality characteristic of a product or service.

4.2.1 A certain amount of variability will exist in all process outputs regardless of how well the process is designed or maintained. A process operating with only this inherent variability is said to be in a state of statistical control, with its output variability subject only to chance, or common, causes.

4.2.2 Process upsets, said to be due to assignable, or special causes, are manifested by changes in the output level, such as a spike, shift, trend, or by changes in the variability of an

[3] The boldface numbers in parentheses refer to a list of references at the end of this standard.

output. The control chart is the basic analytical tool in SPC and is used to detect the occurrence of special causes operating on the process.

4.2.3 When the control chart signals the presence of a special cause, other SPC tools, such as flow charts, brainstorming, cause-and-effect diagrams, or Pareto analysis, described in various references (**4-8**), are used to identify the special cause. Special causes, when identified, are either eliminated or controlled. When special cause variation is eliminated, process variability is reduced to its inherent variability, and control charts then function as a process monitor. Further reduction in variation would require modification of the process itself.

4.3 The use of control charts to adjust one or more process inputs is not recommended, although a control chart may signal the need to do so. Process adjustment schemes are outside the scope of this practice and are discussed by Box and Luceño (**9**).

4.4 The role of a control chart changes as the SPC program evolves. An SPC program can be organized into three stages (**10**).

4.4.1 *Stage A, Process Evaluation*—Historical data from the process are plotted on control charts to assess the current state of the process, and control limits from this data are calculated for further use. See Ref. (**1**) for a more complete discussion on the use of control charts for data analysis. Ideally, it is recommended that 100 or more numeric data points be collected for this stage. For single observations per subgroup at least 30 data points should be collected (**6, 7**). For attributes, a total of 20 to 25 subgroups of data are recommended. At this stage, it will be difficult to find special causes, but it would be useful to compile a list of possible sources for these for use in the next stage.

4.4.2 *Stage B, Process Improvement*—Process data are collected in real time and control charts, using limits calculated in Stage A, are used to detect special causes for identification and resolution. A team approach is vital for finding the sources of special cause variation, and process understanding will be increased. This stage is completed when further use of the control chart indicates that a state of statistical control exists.

4.4.3 *Stage C, Process Monitoring*—The control chart is used to monitor the process to confirm continually the state of statistical control and to react to new special causes entering the system or the reoccurrence of previous special causes. In the latter case, an out-of-control action plan (OCAP) can be developed to deal with this situation (**7, 11**). Update the control limits periodically or if process changes have occurred.

NOTE 1—Some practitioners combine Stages A and B into a Phase I and denote Stage C as Phase II (**10**).

5. Control Chart Principles and Usage

5.1 One or more observations of an output characteristic are periodically sampled from a process at a defined frequency. A control chart is basically a time plot summarizing these observations using a sample statistic, which is a function of the observations. The observations sampled at a particular time point constitute a subgroup. Control limits are plotted on the chart based on the sampling distribution of the sample statistic being evaluated (see 5.2 for further discussion).

NOTE 2—Subgroup statistics commonly used are the average, range, standard deviation, variance, percentage or fraction of an occurrence of an event among multiple opportunities, or the number of occurrences during a defined time period or in a defined space.

5.1.1 The subgroup sampling frequency is determined by practical considerations, such as time and cost of an observation, the process dynamics (how quickly the output responds to upsets), and consequences of not reacting promptly to a process upset.

NOTE 3—Sampling at too high of a frequency may introduce correlation between successive subgroups. This is referred to as autocorrelation. Control charts that can handle this type of correlation are outside the scope of this practice.

NOTE 4—Rules for nonrandomness (see 5.2.2) assume that the plotted points on the chart are independent of one another. This shall be kept in mind when determining the sampling frequency for the control charts discussed in this practice.

5.1.2 The sampling plan for collecting subgroup observations should be designed to minimize the variation of observations within a subgroup and to maximize variation between subgroups. This is termed *rational subgrouping*. This gives the best chance for the within-subgroup variation to estimate only the inherent, or common-cause, process variation.

NOTE 5—For example, to obtain hourly rational subgroups of size four in a product-filling operation, four bottles should be sampled within a short time span, rather than sampling one bottle every 15 min. Sampling over 1 h allows the admission of special cause variation as a component of within-subgroup variation.

5.1.3 The subgroup size, n, is the number of observations per subgroup. For ease of interpretation of the control chart, the subgroup size should be fixed (symbol n), and this is the usual case for variables data (see 5.3.1). In some situations, often involving retrospective data, variable subgroup sizes may be unavoidable, which is often the situation for attributes data (see 5.3.2).

5.1.4 *Subgroup Size and Average Run Length*—The average run length (ARL) is a measure of how quickly the control chart signals a sustained process shift of a given magnitude in the output characteristic being monitored. It is defined as the average number of subgroups needed to respond to a process shift of h sigma units, where sigma is the intrinsic standard deviation estimated by σ (see 6.2.4). The theoretical background for this relationship is developed in Montgomery (**4**), and Fig. 1 gives the curves relating ARL to the process shift for selected subgroup sizes in an X-bar chart. An ARL = 1 means that the next subgroup will have a very high probability of detecting the shift.

5.2 The control chart is a plot of the subgroup statistic in time order. The chart also features a center line, representing the time-averaged value of the statistic, and the lower and upper control limits, that are located at ±three standard errors of the statistic around the center line. The center line and control limits are calculated from the process data and are not based in any way on specification limits. The presence of a special cause is indicated by a subgroup statistic falling outside the control limits.

5.2.1 The use of three standard errors for control limits (so-called "three-sigma limits") was chosen by Shewhart (**2**), and therefore are also known as Shewhart Limits. Shewhart

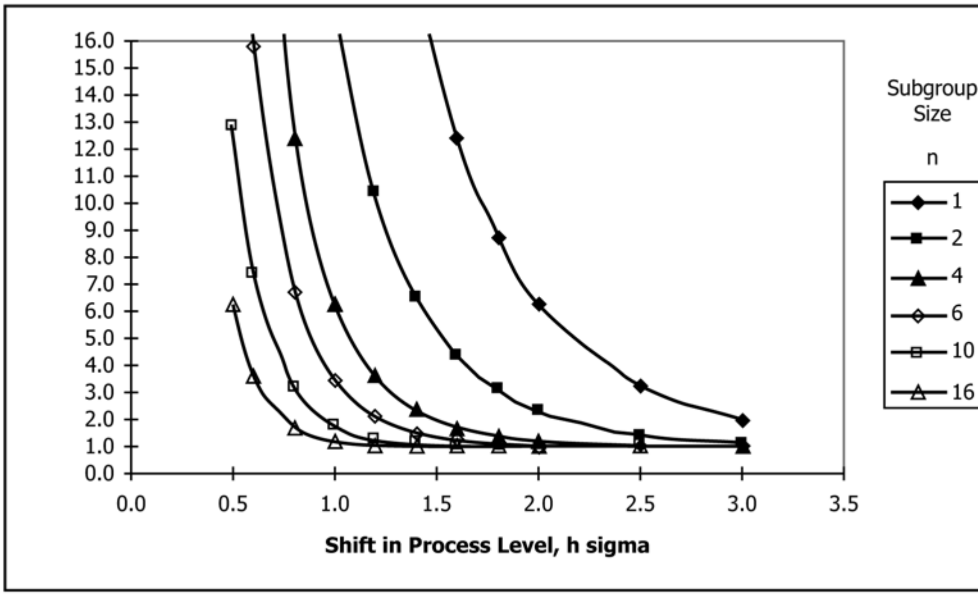

FIG. 1 ARL for the *X*-Bar Chart to Detect an *h*-Sigma Process Shift by Subgroup Size, *n*

chose these limits to balance the two risks of: (*1*) failing to signal the presence of a special cause when one occurs, and (*2*) occurrence of an out-of-control signal when the process is actually in a state of statistical control (a false alarm).

5.2.2 Special cause variation may also be indicated by certain nonrandom patterns of the plotted subgroup statistic, as detected by using the so-called Western Electric Rules (**3**). To implement these rules, additional limits are shown on the chart at ±two standard errors (warning limits) and at ±one standard error (see 7.3 for example).

5.2.2.1 *Western Electric Rules*—A shift in the process level is indicated if:

(1) One value falls outside either control limit,

(2) Two out of three consecutive values fall outside the warning limits on the same side,

(3) Four out of five consecutive values fall outside the ±one-sigma limits on the same side, and

(4) Eight consecutive values either fall above or fall below the center line.

5.2.2.2 Other Western Electric rules indicate less common situations of nonrandom behavior:

(1) Six consecutive values in a row are steadily increasing or decreasing (trend),

(2) Fifteen consecutive values are all within the ±one-sigma limits on either side of the center line,

(3) Fourteen consecutive values are alternating up and down, and

(4) Eight consecutive values are outside the ±one-sigma limits.

5.2.2.3 These rules should be used judiciously since they will increase the risk of a false alarm, in which the control chart indicates lack of statistical control when only common causes are operating. The effect of using each of the rules, and groups of these rules, on false alarm incidence is discussed by Champ and Woodall (**12**).

5.3 This practice describes the use of control charts for variables and attributes data.

5.3.1 Variables data represent observations obtained by observing and recording the magnitude of an output characteristic measured on a continuous numerical scale. Control charts are described for monitoring process variability and process level, and these two types of charts are used as a unit for process monitoring.

5.3.1.1 For multiple observations per subgroup, the subgroup average is the statistic for monitoring process level (*X*-bar chart) and either the subgroup range (*R* chart), or the subgroup standard deviation (*s* chart) is used for monitoring process variability. The range is easier to calculate and is nearly as efficient as the standard deviation for small subgroup sizes. The *X*-bar, *R* chart combination is discussed in Section 6. The *X*-bar, *s* chart combination is discussed in Section 7.

NOTE 6—For processes producing discrete items, a subgroup usually consists of multiple observations. The subgroup size is often five or less, but larger subgroup sizes may be used if measurement ease and cost are low. The larger the subgroup size, the more sensitive the control chart is to smaller shifts in the process level (see 5.1.4).

5.3.1.2 For single observations per subgroup, the subgroup individual observation is the statistic for monitoring process level (*I* chart) and the subgroup moving range is used for monitoring process variability (*MR* chart). The *I*, *MR* chart combination is discussed in Section 8.

NOTE 7—For batch or continuous processes producing bulk material, often only a single observation is taken per subgroup, as multiple observations would only reflect measurement variation.

5.3.2 Attributes data consist of two types: (*1*) observations representing the frequency of occurrence of an event in the subgroup, for example, the number or percentage of defective units in a subgroup of inspected units, or (*2*) observations representing the count of occurrences of an event in a defined interval of time or unit of space, for example, numbers of auto accidents per month in a given region. For attributes data, the standard error of the mean is a function of the process average, so that only a single control chart is needed.

NOTE 8—The subgroup size for attributes data, because of their lower cost and quicker measurement, is usually much greater than for numeric observations. Another reason is that variables data contain more information than attributes data, thus requiring a smaller subgroup size.

5.3.2.1 For monitoring the frequency of occurrences of an event with fixed subgroup size, the statistic is the proportion or fraction of objects having the attribute (*p* chart). An alternate statistic is the number of occurrences for a given subgroup size (*np* chart) and these charts are described in Section 9. For monitoring with variable subgroup sizes, a modified *p* chart with variable control limits or a standardized control chart is used, and these charts are also described in Section 9.

5.3.2.2 For monitoring the count of occurrences over a defined time or space interval, termed the *inspection interval*, the statistic depends on whether or not the inspection interval is fixed or variable over subgroups. For a fixed inspection interval for all subgroups the statistic is the count (*c* chart); for variable inspection units the statistic is the count per inspection interval (*u* chart). Both charts are described in Section 10.

5.3.3 The EWMA chart plots the exponentially weighted moving average statistic which is described by Hunter (**13**). The EWMA may be calculated for individual observations and averages of multiple observations of variables data, and for percent defective, or counts of occurrences over time or space for attributes data. The calculations for the EWMA chart are defined and discussed in Section 11.

5.3.3.1 The EWMA chart is also a useful supplementary control chart to the previously discussed charts in SPC, and is a particularly good companion chart to the *I* chart for individual observations. The EWMA reacts more quickly to smaller shifts in the process characteristic, on the order of 1.5 standard errors or less, whereas the Shewhart-based charts are more sensitive to larger shifts. Examples of the EWMA chart as a supplementary chart are given in 11.4 and Appendix X1.

5.3.3.2 The EMWA chart is also used in process adjustment schemes where the EWMA statistic is used to locate the local mean of a non-stationary process and as a forecast of the next observation from the process. This usage is beyond the scope of this practice but is discussed by Box and Paniagua-Quiñones (**14**) and by Lucas and Saccucci (**15**).

5.3.4 The CUSUM chart plots the accumulated total value of differences between the measured values or monitored statistics and the predefined target or reference value as described in Montgomery (**4**). The CUSUM may be calculated for variables data using individual observations or subgroup averages and attributes data using percent defectives or counts of occurrences over time or space. The calculations for the CUSUM chart are defined and discussed in Section 12.

5.3.4.1 The CUSUM chart is used when smaller process shifts (1 to 1.5 sigma) are of interest. The CUSUM chart effectively detects a sustained small shift in the process mean or a slow process drift or trend. The CUSUM chart can also be used to evaluate the direction and the magnitude of the drift from the process target or reference value.

5.3.5 The CUSUM chart is not very effective in detecting large process shifts. Therefore, it is often used as a supplementary chart to I-chart or *X*-bar chart. In this case, either the I-chart or *X*-bar chart detects larger process shifts. The CUSUM chart detects smaller shifts (1 to 1.5 sigma) in process.

5.4 The EWMV chart is useful for monitoring the variance of a process characteristic from a continuous process where single measurements have been taken at each time point (see 5.3.1.2), and the EWMV chart may be considered as an alternative or companion to the Moving Range chart. The EWMV chart is based on the squared deviation of the current process observation from an estimate of the current process average, which is obtained from a companion EWMA chart. The calculations for the EWMV chart are defined and discussed in Section 13.

6. Control Charts for Multiple Numerical Measurements per Subgroup (*X*-Bar, *R* Charts)

6.1 *Control Chart Usage*—These control charts are used for subgroups consisting of multiple numerical measurements. The *X*-bar chart is used for monitoring the process level, and the *R* chart is used for monitoring the short-term variability. The two charts use the same subgroup data and are used as a unit for SPC purposes.

6.2 *Control Chart Setup and Calculations:*

6.2.1 Denote an observation X_{ij}, as the *j*th observation, $j = 1, \ldots, n$, in the *i*th subgroup $i = 1, \ldots, k$. For each of the *k* subgroups, calculate the *i*th subgroup average,

$$\bar{X}_i = \sum_{j=l}^{n} X_{ij}/n = (X_{i1} + X_{i2} \ldots + X_{in})/n \quad (1)$$

6.2.1.1 Averages may be rounded to one more significant figure than the data.

6.2.1.2 For each of the *k* subgroups, calculate the *i*th subgroup range, the difference between the largest and the smallest observation in the subgroup.

$$R_i = Max(X_{i1}, \ldots, X_{in}) - Min(X_{i1}, \ldots, X_{in}) \quad (2)$$

6.2.1.3 The averages and ranges are plotted as dots on the *X*-bar chart and the *R* chart, respectively. The dots may be connected by lines, if desired.

6.2.2 Calculate the grand average and the average range over all *k* subgroups:

$$\bar{\bar{X}} = \sum_{i=l}^{k} \bar{X}_i/k = \left(\bar{X}_1 + \bar{X}_2 + \ldots + \bar{X}_k\right)/k \quad (3)$$

$$\bar{R} = \sum_{i=l}^{k} R_i/k = (R_1 + R_2 + \ldots + R_k)/k \quad (4)$$

6.2.2.1 These values are used for the center lines on the control chart, which are usually depicted as solid lines on the control chart, and may be rounded to one more significant figure than the data.

6.2.3 Using the control chart factors in Table 1, calculate the lower control limits (*LCL*) and upper control limits (*UCL*) for the two charts.

6.2.3.1 *For the X-Bar Chart:*

$$LCL = \bar{\bar{X}} - A_2\bar{R} \quad (5)$$

$$UCL = \bar{\bar{X}} + A_2\bar{R} \tag{6}$$

6.2.3.2 *For the R Chart:*

$$LCL = D_3\bar{R} \tag{7}$$

$$UCL = D_4\bar{R} \tag{8}$$

6.2.3.3 The control limits are usually depicted as dashed lines on the control chart.

6.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \bar{R}/d_2 \tag{9}$$

6.2.4.1 This estimate is useful in process capability studies (see Practice E2281).

6.2.5 Subgroup statistics falling outside the control limits on the *X*-bar chart or the *R* chart indicate the presence of a special cause. The Western Electric Rules may also be applied to the *X*-bar and *R* chart (see 5.2.2).

6.3 *Example—Liquid Product Filling into Bottles*—At a frequency of 30 min, four consecutive bottles are pulled from the filling line and weighed. The observations, subgroup averages, and subgroup ranges are listed in Table 2, and the grand average and average range are calculated at the bottom of the table.

6.3.1 The control limits are calculated as follows:

6.3.1.1 *X-Bar Chart:*

$$LCL = 246.44 - (0.729)(5.92) = 242.12$$

$$UCL = 246.44 + (0.729)(5.92) = 250.76$$

6.3.1.2 *R Chart:*

$$LCL = (0)(5.92) = 0$$

$$UCL = (2.282)(5.92) = 13.51$$

6.3.1.3 Estimate of inherent standard deviation:

$$\sigma = 5.92/2.059 = 2.87$$

6.3.1.4 The control charts are shown in Fig. 2 and Fig. 3 Both charts indicate that the filling weights are in statistical control.

7. Control Charts for Multiple Numerical Measurements per Subgroup (*X*-Bar, *s* Charts)

7.1 *Control Chart Usage*—These control charts are used for subgroups consisting of multiple numerical measurements, the *X*-bar chart for monitoring the process level, and the *s* chart for monitoring the short-term variability. The two charts use the same subgroup data and are used as a unit for SPC purposes.

7.2 *Control Chart Setup and Calculations:*

7.2.1 Denote an observation X_{ij}, as the *j*th observation, $j = 1, \ldots n$, in the *i*th subgroup, $i = 1, \ldots, k$. For each of the *k* subgroups, calculate the *i*th subgroup average and the *i*th subgroup standard deviation:

$$\bar{X}_i = \sum_{j=1}^{n} X_{ij}/n = (X_{i1} + X_{i2} + \ldots + X_{in})/n \tag{10}$$

$$s_i = \sqrt{\sum_{j=1}^{n} \left(X_{ij} - \bar{X}_i\right)^2/(n-1)} \tag{11}$$

7.2.1.1 Averages may be rounded to one more significant figure than the data.

7.2.1.2 Sample standard deviations may be rounded to two or three significant figures.

7.2.1.3 The averages and standard deviations are plotted as dots on the *X*-bar chart and the *s* chart, respectively. The dots may be connected by lines, if desired.

7.2.2 Calculate the grand average and the average standard deviation over all *k* subgroups:

TABLE 2 Example of *X*-Bar, *R* Chart for Bottle-Filling Operation

Subgroup	Bottle 1	Bottle 2	Bottle 3	Bottle 4	Average	Range
1	246.5	250.7	246.1	250.2	248.38	4.6
2	246.5	243.7	241.7	248.0	244.98	6.3
3	246.5	243.3	250.1	243.5	245.85	6.8
4	246.5	248.5	250.5	242.0	246.88	8.5
5	246.5	242.9	248.0	249.4	246.70	6.5
6	246.7	250.6	246.0	246.1	247.35	4.6
7	246.6	247.3	251.6	248.8	248.58	5.0
8	246.5	249.6	246.6	243.6	246.58	6.0
9	246.4	251.1	247.7	245.5	247.68	5.6
10	246.4	245.7	245.8	247.0	246.23	1.3
11	246.5	242.6	241.5	248.3	244.73	6.8
12	246.4	247.3	244.1	243.3	245.28	4.0
13	246.4	250.1	249.0	245.3	247.70	4.8
14	246.3	247.8	239.4	245.7	244.80	8.4
15	246.6	242.7	244.1	249.7	245.78	7.0
16	246.6	248.4	246.8	251.0	248.20	4.4
17	246.4	246.0	250.3	246.2	247.23	4.3
18	246.5	250.2	243.2	246.9	246.70	7.0
19	246.4	247.5	246.6	244.8	246.33	2.7
20	246.3	248.4	244.6	244.9	246.05	3.8
21	246.5	244.7	243.0	248.0	245.55	5.0
22	246.6	249.2	250.5	242.6	247.23	7.9
23	246.5	249.7	240.7	246.7	245.90	9.0
24	246.6	244.0	238.5	243.0	243.03	8.1
25	246.5	251.5	248.9	242.0	247.23	9.5
			Grand average	246.44		
			Average range			5.92

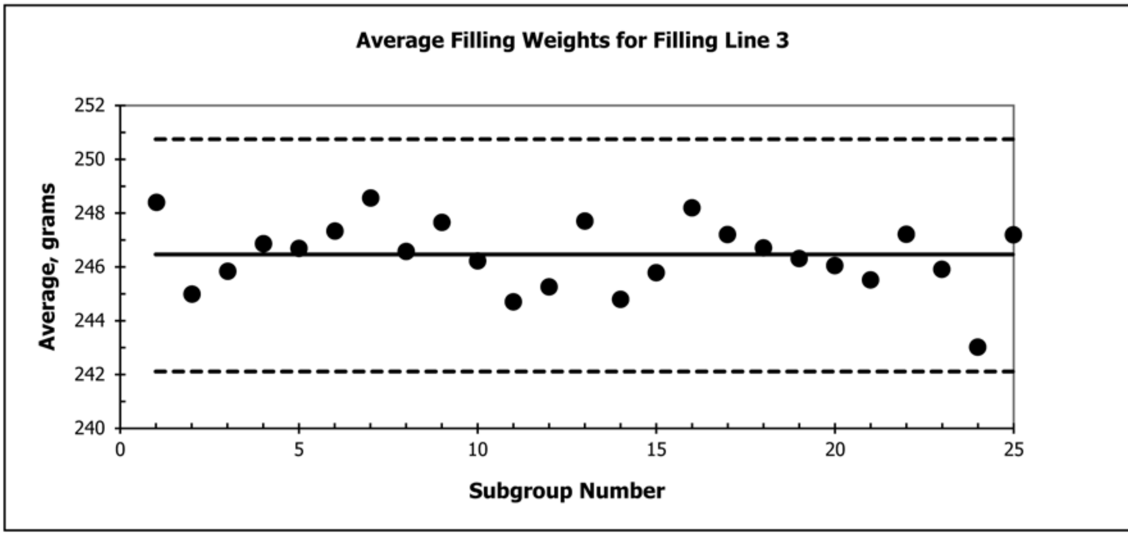

FIG. 2 X-Bar Chart for Filling Line 3

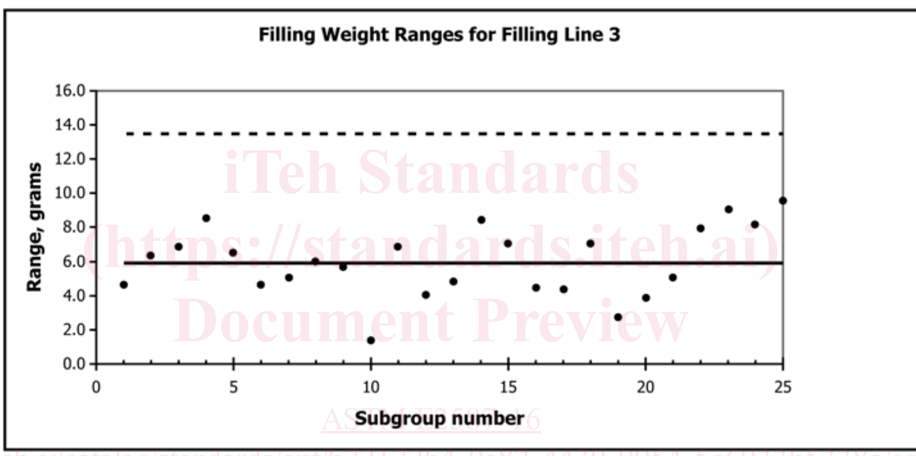

FIG. 3 Range Chart for Filling Line 3

$$\bar{\bar{X}} = \sum_{i=1}^{k} \bar{X}_i/k = \left(\bar{X}_1+\bar{X}_2+\ldots+\bar{X}_k\right)/k \tag{12}$$

$$\bar{s} = \sum_{i=1}^{k} s_i/k = \left(s_1+s_2+\ldots+s_k\right)/k \tag{13}$$

7.2.2.1 These values are used for the center lines on the control chart, usually depicted as solid lines, and may be rounded to the same number of significant figures as the subgroup statistics.

7.2.3 Using the control chart factors in Table 1, calculate the *LCL* and *UCL* for the two charts.

7.2.3.1 *For the X-Bar Chart:*

$$LCL = \bar{\bar{X}} - A_3\bar{s} \tag{14}$$

$$UCL = \bar{\bar{X}} + A_3\bar{s} \tag{15}$$

7.2.3.2 *For the s Chart:*

$$LCL = B_3\bar{s} \tag{16}$$

$$UCL = B_4\bar{s} \tag{17}$$

7.2.3.3 The control limits are usually depicted by dashed lines on the control charts.

7.2.4 An estimate of the inherent (common cause) standard deviation may be calculated as follows:

$$\hat{\sigma} = \bar{s}/c_4 \tag{18}$$

7.2.5 Subgroup statistics falling outside the control limits on the *X*-bar chart or the *s* chart indicate the presence of a special cause.

7.3 *Example*—Vitamin tablets are compressed from blended granulated powder and tablet hardness is measured on ten tablets each hour. The observations, subgroup averages, and subgroup standard deviations are listed in Table 3, and the grand average and average range are calculated at the bottom of the table.

7.3.1 The control limits are calculated as follows:

7.3.1.1 *X-Bar Chart:*

$$LCL = 24.141 - (0.975)(1.352) = 22.823$$

$$UCL = 24.141 + (0.975)(1.352) = 25.459$$

7.3.1.2 *The s Chart:*

$$LCL = (0.284)(1.352) = 0.384$$

$$UCL = (1.716)(1.352) = 2.320$$

TABLE 3 Example of *X*-Bar, *S* Chart for Tablet Hardness

Subgroup	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	Avg	Std
1	21.3	19.5	21.3	23.1	22.4	24.6	23.4	22.4	21.4	22.9	22.23	1.419
2	21.4	22.2	22.1	23.3	23.9	22.9	21.6	24.6	25.7	24.1	23.18	1.399
3	23.9	24.2	22.8	22.9	25.9	21.4	23.1	20.5	23.6	23.8	23.21	1.494
4	23.4	26.3	24.4	25.3	22.0	25.8	22.7	26.5	21.6	25.0	24.30	1.781
5	25.6	22.9	24.6	23.8	23.6	22.7	27.1	25.3	25.1	25.5	24.62	1.372
6	26.8	25.7	25.6	24.8	25.7	26.0	23.6	22.8	22.1	24.7	24.78	1.507
7	26.6	24.0	25.0	26.3	25.3	25.2	25.0	24.1	22.6	26.0	25.01	1.199
8	26.7	25.9	22.6	23.8	27.3	26.7	25.7	24.1	25.2	25.2	25.32	1.470
9	25.0	24.8	22.4	22.9	24.9	24.4	22.5	22.7	23.8	24.0	23.74	1.037
10	26.1	25.3	25.7	25.0	24.7	26.2	23.6	24.2	25.1	24.3	25.02	0.844
							Center line				24.141	1.352
							LCL				22.823	0.384
							UCL				25.459	2.320
							Lower warning limit				23.262	
							Upper warning limit				25.020	
							Lower one-sigma limit				23.702	
							Upper one-sigma limit				24.580	

7.3.2 The two-sigma warning limits and the one-sigma limits are also calculated for the *X*-bar chart to illustrate the use of the Western Electric Rules.

7.3.3 The warning limits and one-sigma limits for the *X*-bar chart were calculated as follows.

7.3.3.1 *Warning Limits:*

$$LCL = 24.141 - 2(0.975)(1.352)/3 = 23.262$$

$$UCL = 24.141 + 2(0.975)(1.352)/3 = 25.020$$

7.3.3.2 *One-Sigma Limits:*

$$LCL = 24.141 - (0.975)(1.352)/3 = 23.702$$

$$UCL = 24.141 + (0.975)(1.352)/3 = 24.580$$

7.3.3.3 *Estimate of Inherent Standard Deviation:*

$$\hat{\sigma} = 1.352/0.9727 = 1.39$$

7.3.3.4 The control charts are shown in Fig. 4 and Fig. 5. The *s* chart indicates statistical control in the process variation.

7.3.4 *The X-Bar Chart Gives Several Out-of-Control Signals:*

7.3.4.1 *Subgroup 1*—Below the *LCL*.

7.3.4.2 *Subgroups 2 and 3*—Two points outside the warning limit on the same side.

7.3.4.3 *Subgroups 6, 7, and 8*—End points of six points in a row steadily increasing.

7.3.4.4 *Subgroup 10*—Four out of five points on the same side of the upper one-sigma limits.

7.3.5 It appears that the process level has been steadily increasing during the run. Some possible special causes are particle segregation in the feed hopper or a drift in the press settings.

8. Control Charts for Single Numerical Measurements per Subgroup (*I*, *MR* Charts)

8.1 *Control Chart Usage*—These control charts are used for subgroups consisting of a single numerical measurement. The *I* chart is used for monitoring the process level and the *MR* chart is used for monitoring the short-term variability. The two charts are used as a unit for SPC purposes, although some practitioners state that the *MR* chart does not add value and recommend against its use for other than calculating the control limits for the *I* chart (**16**).

8.2 *Control Chart Setup and Calculations:*

8.2.1 Denote the observation, X_i, as the individual observation in the *i*th subgroup, *i* = 1, 2,…, *k*.

8.2.1.1 Note that the first subgroup will not have a moving range. For the *k–1* subgroups, *i* = 2, …, *k* calculate the moving range, the absolute value of the difference between two successive values:

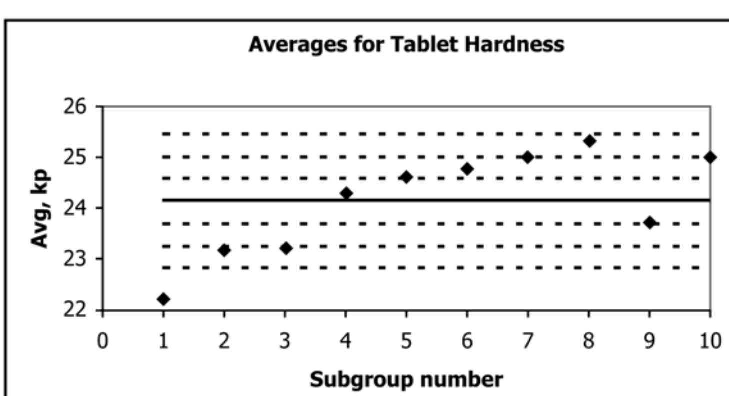

FIG. 4 *X*-Bar Chart for Tablet Hardness