



Designation: D2244 – 16

# Standard Practice for Calculation of Color Tolerances and Color Differences from Instrumentally Measured Color Coordinates<sup>1</sup>

This standard is issued under the fixed designation D2244; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.

*This standard has been approved for use by agencies of the U.S. Department of Defense.*

## INTRODUCTION

This practice originally resulted from the consolidation of a number of separately published methods for the instrumental evaluation of color differences. As revised in 1979, it included four color spaces in which color-scale values could be measured by instruments, many of which were obsolete, and the color differences calculated by ten equations for different color scales. The sections on apparatus, calibration standards and methods, and measurement procedures served little purpose in the light of modern color-measurement technology. The revision published in 1993 omitted these sections, and limited the color spaces and color-difference equations considered, to the three most widely used in the paint and related coatings industry. A previous revision added two new color tolerance equations and put two of the color difference equations from the 1993 version in an informative appendix for historical purposes.

### 1. Scope\*

1.1 This practice covers the calculation, from instrumentally measured color coordinates based on daylight illumination, of color tolerances and small color differences between opaque specimens such as painted panels, plastic plaques, or textile swatches. Where it is suspected that the specimens may be metameric, that is, possess different spectral curves though visually alike in color, Practice D4086 should be used to verify instrumental results. The tolerances and differences determined by these procedures are expressed in terms of approximately uniform visual color perception in CIE 1976 CIELAB opponent-color space (1),<sup>2</sup> CMC tolerance units (2), CIE94 tolerance units (3), the DIN99 color difference formula given in DIN 6176(4), or the new CIEDE2000 color difference units (5).

1.2 For product specification, the purchaser and the seller shall agree upon the permissible color tolerance between test specimen and reference and the procedure for calculating the color tolerance. Each material and condition of use may require specific color tolerances because other appearance factors, (for

example, specimen proximity, gloss, and texture), may affect the correlation between the magnitude of a measured color difference and its commercial acceptability.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory requirements prior to use.*

### 2. Referenced Documents

2.1 *ASTM Standards:*<sup>3</sup>

- D1729 Practice for Visual Appraisal of Colors and Color Differences of Diffusely-Illuminated Opaque Materials
- D4086 Practice for Visual Evaluation of Metamerism
- E284 Terminology of Appearance
- E308 Practice for Computing the Colors of Objects by Using the CIE System
- E805 Practice for Identification of Instrumental Methods of Color or Color-Difference Measurement of Materials
- E1164 Practice for Obtaining Spectrometric Data for Object-Color Evaluation

<sup>1</sup> This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

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<sup>2</sup> The boldface numbers in parentheses refer to the list of references at the end of this standard.

<sup>3</sup> For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

\*A Summary of Changes section appears at the end of this standard

## 2.2 Other Standards:

**DIN 6176** *Farbmetrische, Bestimmung von Farbabständen bei Körperfarben nach der DIN99-Formel*<sup>4</sup>

## 3. Terminology

3.1 Terms and definitions in Terminology **E284** are applicable to this practice.

### 3.2 Definitions of Terms Specific to This Standard:

3.2.1 *colorimetric spectrometer, n*—spectrometer, one component of which is a dispersive element (such as a prism, grating or interference filter or wedge or tunable or discrete series of monochromatic sources), that is normally capable of producing as output colorimetric data (such as tristimulus values and derived color coordinates or indices of appearance attributes). Additionally, the colorimetric spectrometer may also be able to report the underlying spectral data from which the colorimetric data were derived.

3.2.1.1 *Discussion*—At one time, UV-VIS analytical spectrophotometers were used for colorimetric measurements. Today, while instruments intended for use in color measurements share many common components, UV-VIS analytical spectrophotometers are designed to optimize their use in chemometric quantitative analysis, which requires very precise spectral position and very narrow bandpass and moderate baseline stability. Colorimetric spectrometers are designed to optimize their use as digital simulations of the visual colorimeter or as the source of spectral and colorimetric information for computer-assisted color matching systems. Digital colorimetry allows more tolerance on the spectral scale and spectral bandwidth but demand much more stability in the radiometric scale.

3.2.2 *color tolerance equation, n*—a mathematical expression, derived from acceptability judgments, which distorts the metric of color space based on the coordinates in that color space, of a reference color, for the purpose of single number shade passing.

3.2.2.1 *Discussion*—The color tolerance equation computes a pass/fail value based on which of the pair of specimens is assigned the designation “standard.” Thus, inter-changing the reference and test specimens will result in a change in the predicted level of acceptance between the specimens while the perceived difference is unchanged. A color difference equation quantifies distance in a color space using the metric of that space. Inter-changing the reference and test specimens does not change either the perceived or predicted color differences.

## 4. Summary of Practice

4.1 The differences in color between a reference and a test specimen are determined from measurements made by use of a spectral based or filter based colorimeter. Reflectance readings from spectral instruments are converted by computations to color-scale values in accordance with Practice **E308**, or these color-scale values may be read directly from instruments that automatically make the computations. Color-difference units

are computed, from these color-scale values, and approximate the perceived color differences between the reference and the test specimen.

## 5. Significance and Use

5.1 The original CIE color scales based on tristimulus values  $X, Y, Z$  and chromaticity coordinates  $x, y$  are not uniform visually. Each subsequent color scale based on CIE values has had weighting factors applied to provide some degree of uniformity so that color differences in various regions of color space will be more nearly comparable. On the other hand, color differences obtained for the same specimens evaluated in different color-scale systems are not likely to be identical. To avoid confusion, color differences among specimens or the associated tolerances should be compared only when they are obtained for the same color-scale system. There is no simple factor that can be used to convert accurately color differences or color tolerances in one system to difference or tolerance units in another system for all colors of specimens.

5.2 Color differences calculated in  $\Delta E_{CMC}$  or  $\Delta E_{00}$  units are highly recommended for use with color-differences in the range of 0.0 to 5.0  $\Delta E_{ab}^*$  units. Both are appropriate for and widely used in industrial and commercial applications including, but not limited to, automobiles, coatings, cosmetics, inks, packaging, paints, plastics, printing, security, and textiles. The Hunter color difference components  $\Delta L_H, \Delta a_H, \Delta b_H$ , and their color difference unit  $\Delta E_H$ , are used by the coil coating and aluminum extrusion coating industries, as well as the customers of these users. They are, therefore, included in **Appendix X1** for historical purposes and use.

5.3 Users of color tolerance equations have found that, in each system, summation of three, vector color-difference components into a single scalar value is very useful for determining whether a specimen color is within a specified tolerance from a standard. However, for control of color in production, it may be necessary to know not only the magnitude of the departure from standard but also the direction of this departure. It is possible to include information on the direction of a small color difference by listing the three instrumentally determined components of the color difference.

5.4 Selection of color tolerances based on instrumental values should be carefully correlated with a visual appraisal of the acceptability of differences in hue, lightness, and saturation obtained by using Practice **D1729**. The three tolerance equations given here have been tested extensively against such data for textiles and plastics and have been shown to agree with the visual evaluations to within the experimental uncertainty of the visual judgments. That implies that the equations themselves misclassify a color difference with a frequency no greater than that of the most experienced visual color matcher.

5.5 While color difference equations and color tolerance equations are routinely applied to a wide range of illuminants, they have been derived or optimized, or both, for use under daylight illumination. Good correlation with the visual judgments may not be obtained when the calculations are made with other illuminants. Use of a tolerance equation for other than daylight conditions will require visual confirmation of the level of metamerism in accordance with Practice **D4086**.

<sup>4</sup> Available from Beuth Verlag GmbH, 10772, Berlin, Germany, <http://www.beuth.de/>.

## 6. Description of Color-Difference and Color-Tolerance Equations

6.1 *CIE 1931 and 1964 Color Spaces*—The daylight colors of opaque specimens are represented by points in a space formed by three rectangular axes representing the lightness scale  $Y$  and chromaticity scales  $x$  and  $y$ , where:

$$x = \frac{X}{X+Y+Z} \quad (1)$$

$$y = \frac{Y}{X+Y+Z} \quad (2)$$

where  $X$ ,  $Y$ , and  $Z$  are tristimulus values for either the 1931 CIE standard observer (2° observer) or the 1964 CIE standard observer (10° observer) and standard illuminant  $D_{65}$ , or other phase of daylight. These scales do not provide a perceptually uniform color space. Consequently, color differences are seldom if ever computed directly from differences in  $x$ ,  $y$ , and  $Y$ .

6.2 *CIE 1976  $L^* a^* b^*$  Uniform Color Space and Color-Difference Equation (1, 6)*—This is an approximately uniform color space based on nonlinear expansion of the tristimulus values and taking differences to produce three opponent axes that approximate the percepts of lightness-darkness, redness-greenness and yellowness-blueness. It is produced by plotting in rectangular coordinates the quantities  $L^*$ ,  $a^*$ ,  $b^*$ , calculated as follows:

$$L^* = 116f(Q_Y) - 16 \quad (3)$$

$$a^* = 500[f(Q_X) - f(Q_Y)] \quad (4)$$

$$b^* = 200[f(Q_Y) - f(Q_Z)] \quad (5)$$

where

$$Q_X = (X/X_n); Q_Y = (Y/Y_n); Q_Z = (Z/Z_n)$$

and

$$f(Q_i) = Q_i^{1/3} \quad \text{if } Q_i > (6/29)^3$$

$$f(Q_i) = (841/108)Q_i + 4/29 \quad \text{if } Q_i \leq (6/29)^3.$$

Here,  $i$  varies as  $X$ ,  $Y$ , and  $Z$ .

The tristimulus values  $X_n$ ,  $Y_n$ ,  $Z_n$  define the color of the nominally white object-color stimulus. Usually, the white object-color stimulus is given by the spectral radiant power of one of the CIE standard illuminants, for example,  $C$ ,  $D_{65}$  or another phase of daylight, reflected into the observer's eye by the perfect reflecting diffuser. Under these conditions,  $X_n$ ,  $Y_n$ ,  $Z_n$  are the tristimulus values of the standard illuminant with  $Y_n$  equal to 100.

6.2.1 The total color-difference  $\Delta E_{ab}^*$  between two colors each given in terms of  $L^*$ ,  $a^*$ ,  $b^*$  is calculated as follows:

$$\Delta E_{ab}^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \quad (6)$$

NOTE 1—The color space defined above is called the CIE 1976  $L^* a^* b^*$  space and the color-difference equation is the CIE 1976  $L^* a^* b^*$  color-difference formula. The abbreviation CIELAB (with all letters capitalized) is recommended.

6.2.2 The magnitude,  $\Delta E_{ab}^*$ , gives no indication of the character of the difference since it does not indicate the relative quantity and direction of hue, chroma, and lightness differences.

6.2.3 The direction of the color difference is described by the magnitude and algebraic signs of the components  $\Delta L^*$ ,  $\Delta a^*$ , and  $\Delta b^*$ :

$$\Delta L^* = L_{B}^* - L_{S}^* \quad (7)$$

$$\Delta a^* = a_{B}^* - a_{S}^* \quad (8)$$

$$\Delta b^* = b_{B}^* - b_{S}^* \quad (9)$$

where  $L_{S}^*$ ,  $a_{S}^*$ , and  $b_{S}^*$  refer to the reference or standard, and  $L_{B}^*$ ,  $a_{B}^*$ , and  $b_{B}^*$  refer to the test specimen or batch. The signs of the components  $\Delta L^*$ ,  $\Delta a^*$ , and  $\Delta b^*$  have the following approximate meanings (7):

$$+\Delta L^* = \text{lighter} \quad (10)$$

$$-\Delta L^* = \text{darker} \quad (11)$$

$$+\Delta a^* = \text{redder (less green)} \quad (12)$$

$$-\Delta a^* = \text{greener (less red)} \quad (13)$$

$$+\Delta b^* = \text{yellow (less blue)} \quad (14)$$

$$-\Delta b^* = \text{bluer (less yellow)} \quad (15)$$

6.2.4 For judging the direction of the color difference between two colors, it is useful to calculate hue angles  $h_{ab}$  and CIE 1976 metric chroma  $C_{ab}^*$  according to the following pseudocode:

if  $b^* = 0$  then (16)

$$h_{ab} = 90 \operatorname{sign}(a^*) [\operatorname{sign}(a^*) - 1]$$

else

$$h_{ab} = 180 - (180/\pi) \arctan(a^*/b^*) - 90 \operatorname{sign}(b^*)$$

end if.

Here  $\operatorname{sign}$  is a function that returns the sign of the argument, and  $\arctan$  is the inverse tangent function returning angles in units of radians. The units of  $h_{ab}$  calculated by the above are degrees counter-clockwise from the positive  $a^*$  axis. The function  $\operatorname{sign}$  is expected to return a minus one for negative values of the argument, a zero when the argument is zero, and a positive one for positive values of the argument.

$$C_{ab}^* = \sqrt{(a^*)^2 + (b^*)^2} \quad (17)$$

Differences in hue angle  $h_{ab}$  between the test specimen and reference can be correlated with differences in their visually perceived hue, except for very dark colors (8). Differences in chroma  $\Delta C_{ab}^* = ([C_{ab}^*]_{\text{batch}} - [C_{ab}^*]_{\text{standard}})$  can similarly be correlated with differences in visually perceived chroma.

6.2.5 For judging the relative contributions of lightness differences, chroma differences, and hue differences between two colors, it is useful to calculate the CIE 1976 Metric Hue Difference  $\Delta H_{ab}^*$  between the colors as follows:

$$\Delta H_{ab}^* = s [2(C_{ab,B}^* C_{ab,S}^* - a_{B}^* a_{S}^* - b_{B}^* b_{S}^*)]^{0.5} \quad (18)$$

where

$$\text{if } a_{S}^* b_{B}^* > a_{B}^* b_{S}^* \text{ then} \quad (19)$$

$$s = 1$$

else

$$s = -1$$

end if.

When  $\Delta E^*_{ab}$  is calculated as in 6.2.1 and  $\Delta C^*_{ab}$  is calculated as in 6.2.4, then

$$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta C^*)^2 + (\Delta H^*)^2]^{0.5} \quad (20)$$

contains terms showing the relative contributions of lightness differences  $\Delta L^*_{ab}$ , chroma differences  $\Delta C^*_{ab}$ , and hue differences  $\Delta H^*_{ab}$ .

**6.3 CMC Color Tolerance Equation**—The Colour Measurement Committee of the Society of Dyers and Colourists undertook a task to improve upon the results of the JPC79 tolerance equation (2) developed at J & P Coats thread company in the United Kingdom. It was a combination of the CIELAB equation and local optimization based on the position of the standard used to derive the FMC-2 equation. It was based on the more intuitive perceptual variables of lightness, chroma and hue instead of the lightness, redness/greenness and yellowness/blueness of the older equation. It is intended to be used as a single-number shade-passing equation. There should not be a need to break the equation down into perceptual components—the CIELAB components of the model do that already. Fig. 1(9) shows the CIELAB chromaticness plane ( $a^*$ ,  $b^*$ ) with a large number of CMC ellipsoids plotted on that plane. The figure clearly shows the change in area of the ellipses with increases in CIELAB metric chroma  $C^*_{ab}$  and with respect to changes in CIELAB metric hue angle  $h^*_{ab}$ . The CMC components and single number tolerances are computed as follows:

$$\Delta E_{CMC}(l:c) = \sqrt{\left(\frac{\Delta L^*}{l \cdot S_L}\right)^2 + \left(\frac{\Delta C^*}{c \cdot S_C}\right)^2 + \left(\frac{\Delta H^*}{S_H}\right)^2} \quad (21)$$

The most common values for the lightness to chroma ratio  $l:c$  is (2:1) for textiles and plastics that are molded to simulate a woven material, implying that lightness differences carry half the importance of chroma and hue differences (10). The values (1:1), often assumed to represent a just perceptible difference, should be applied to materials that require very critical

tolerances or have glossy surfaces. For specimens that are matte, randomly rough, or mildly textured, values intermediate between (1:1) and (2:1) can be used, with the value (1.3:1) being reported most frequently.

The color dependent functions are defined as:

$$S_L = \frac{0.040975 \cdot L^*}{(1 + 0.01765 \cdot L^*)} \quad \text{for } L^* \geq 16 \quad (22)$$

$$S_L = 0.511, \quad \text{for } L^* < 16$$

$$S_C = \frac{0.0638 \cdot C^*}{(1 + 0.0131 \cdot C^*)} + 0.638$$

$$S_H = S_C(T \cdot f + 1 - f)$$

where

$$f = \left\{ \frac{(C^*)^4}{(C^*)^4 + 1900} \right\}^{\frac{1}{2}}$$

$$T = 0.56 + |0.2 \cos(h + 168^\circ)|, \quad \text{if } 164^\circ < h < 345^\circ$$

else

$$T = 0.36 + |0.4 \cos(h + 35^\circ)|$$

All angles are given in degrees but will generally need to be converted to radians for processing on a digital computer. In Eq 22, the values of  $L^*$ ,  $C^*$ , and  $h$  are taken to be those of the standard specimen.

The use of a commercial factor  $cf$  is no longer recommended.

**6.4 CIE94 Color Tolerance Equation (3)**—The development of this color tolerance equation was prompted by the success of the CMC tolerance equation. It was derived primarily from visual observations of automotive paints on steel panels. Like the CMC equation, it is based on the CIELAB color metric and uses the position of the standard in CIELAB color space to derive a set of analytical functions that modify the spacing of the CIELAB space in the region around the standard. Its weighting functions are much simpler than those of the CMC equation. CIE94 tolerances are computed as follows:

$$\Delta E^*_{94} = \left[ \left( \frac{\Delta L^*}{k_L S_L} \right)^2 + \left( \frac{\Delta C^*}{k_C S_C} \right)^2 + \left( \frac{\Delta H^*}{k_H S_H} \right)^2 \right]^{0.5}$$

Unlike many previous color difference equations, CIE94 comes with a well defined set of conditions under which the equation will provide optimum results and departures from this set of conditions will cause the agreement between the visually evaluated color-difference and the computed color-difference to be significantly poorer. Those conditions are given in Table 1. The parameters  $k_L$ ,  $k_C$ ,  $k_H$  are the parametric factors that can be used to compensate for texture and other specimen presentation effects. These should not be used to introduce a commercial factor into the equation. For more information on the use of commercial factors in color tolerance equations, see Appendix X3. All the  $k$  values default to 1 in the absence of specific information or agreement between parties. The parameters  $S_L$ ,  $S_C$ ,  $S_H$  are used to perform the local distortion of

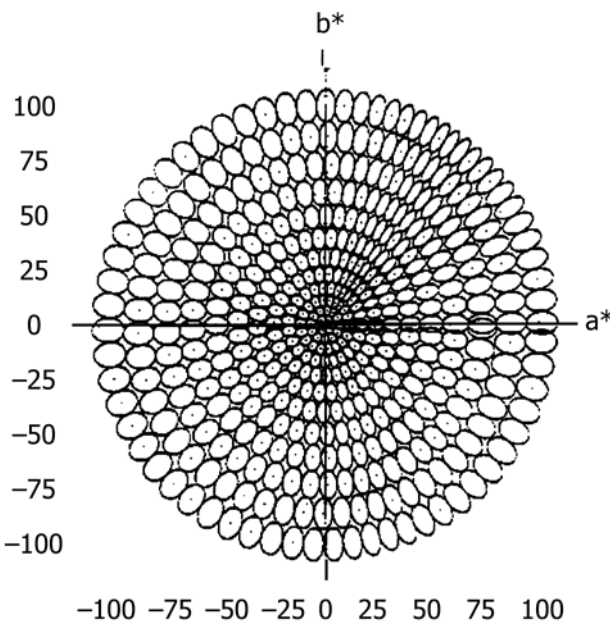


FIG. 1 CMC Ellipse Distribution in the CIELAB ( $a^*$ ,  $b^*$ ) Plane



**TABLE 1 Basis Conditions for CIE94 Tolerance Equation**

Attribute	Requirement
Illumination	D65 source
Specimen Illuminance	1000 lx
Observer	Normal color vision
Background	Uniform neutral gray $L^* = 50$
Viewing Mode	Object
Sample Size	>4° subtended visual angle
Sample Separation	Minimum possible
Size of Color Differences	0 to 5 CIELAB units
Sample Structure	Visually homogenous

CIELAB color space, again based on the position of the standard specimen in that space. They are computed using the following equations:

$$S_L = 1 \quad (24)$$

$$S_C = 1 + 0.045 \cdot C^*$$

$$S_H = 1 + 0.015 \cdot C^*$$

In Eq 24, the value of  $C^*$  is taken to be that of the standard specimen.

**6.5 DIN99 Color Difference Equation**—The publication in 1996 of the paper by Rohner and Rich (4) prompted the German standards institute (DIN) to further develop and standardize a modified version as a new color difference formula that globally models color space using logarithms of the CIELAB coordinates rather than the linear and hyperbolic functions of CMC and CIE94. The equations derived and documented in standard DIN 6176 provides an axes rotation and the logarithmic expansion of the new axes to match that of the spacing of the CIE94 color tolerance formula without the need to make the specimen identified as standard the source of the distortion of distances in the CIELAB color space. Also, as neither the tristimulus values XYZ nor the CIELAB axes  $a^*$ ,  $b^*$  are perceptual variables while the axes  $L^*$ ,  $C^*$  and  $h^*_{ab}$  are correlates of the perceptions of lightness, chroma and hue, it seemed appropriate to scale the differences or distances in color space following the Weber-Fechner law of perception. This resulted in a formula which is easy to use and has equivalent performance to CMC or CIE94. It also eliminates the annoying reference-color based distortion of CIELAB. Thus computed color differences are based only on the Euclidean distance in the DIN99 space. The procedures for computing the DIN99 formula are:

Step 1

$$\text{Redness } e = \cos(16^\circ) a^* + \sin(16^\circ) b^* \quad (25)$$

$$\text{Yellowness } f = 0.7(-\sin(16^\circ) a^* + \cos(16^\circ) b^*)$$

$$\text{Chroma } G = (e^2 + f^2)^{0.5}$$

$$\text{Hue angle } h_{ef} = \arctan\left(\frac{f}{e}\right)$$

Step 2

$$\text{Chroma } C_{99} = \frac{(\log_e(1 + 0.045 G))}{(0.045 k_{CH} k_E)} \quad (26)$$

$$\text{Hue angle } h_{99} = h_{ef} \frac{180}{\pi}$$

$$\text{Redness } a_{99} = C_{99} \cos(h_{ef})$$

$$\text{Yellowness } b_{99} = C_{99} \sin(h_{ef})$$

$$\text{Lightness } L_{99} = 105.509 [\log_e(1 + 0.0158 L^*)] k_E$$

Step 3

$$\Delta E_{99} = \sqrt{(\Delta L_{99})^2 + (\Delta a_{99})^2 + (\Delta b_{99})^2} \quad (27)$$

or

$$\Delta E_{99} = \sqrt{(\Delta L_{99})^2 + (\Delta C_{99})^2 + (\Delta H_{99})^2}$$

with

$$\Delta C_{99} = C_{99,B} - C_{99,S}$$

$$\Delta H_{99} = \frac{(a_{99,S} \cdot b_{99,B} - a_{99,B} \cdot b_{99,S})}{\sqrt{0.5 \cdot (C_{99,B} \cdot C_{99,S} + a_{99,B} \cdot a_{99,S} + b_{99,B} \cdot b_{99,S})}}$$

Where subscripts  $S$  refers to the product standard and subscript  $B$  refers to the current product batch or test sample.

Default parameters are:  $k_E = k_{CH} = 1$ ,  $k_E (1 : k_{CH})$ .

For textiles the following equivalence relations holds: To obtain an equivalent computed difference to a  $CMC(l=2, c=1)$  difference, use the parameters: 2 (1 : 0.5), which indicate that  $k_E = 2$  and,  $k_{CH} = 0.5$ .

**6.6 CIEDE2000 Color Difference Equation (5)**—The development of this color difference equation grew out of the research being performed to try to determine which of the two color tolerance equations, CMC or CIE94, was the better formula. In the process, the researchers came to the conclusion that neither formula was truly optimum. Therefore the CIE set up a new technical committee, TC 1-47, Hue & Lightness Dependant Correction to Industrial Colour Difference Equations, to recommend a new equation that addresses the short-comings in both color tolerance equations. One of the major weaknesses of the color tolerance equations was using the position of the reference color in CIELAB color space for computing the local distortion of CIELAB color space. When the identifications of the two specimens are reversed (calling the original test specimen the reference and the original reference now the test specimen) the computation results in a different computed color difference. This is contrary to what is observed. Visually, there is no change in the magnitude of the difference between the specimens simply by switching roles. By using the position of the arithmetic average color between the two specimens to compute the local distortions to CIELAB color space, the roles of the two specimens may be switched without changing the magnitude of the computed color-difference, in full agreement with the visual assessments. The report from CIE TC 1-47 has shown that CIEDE2000 outperforms both CMC and CIE94 across a wide array of specimens. The CIEDE2000 color differences are computed from the following equations:

$$L' = L^* \quad a' = (1 + G) \cdot a^* \quad b' = b^* \quad (28)$$

$$C' = \sqrt{a'^2 + b'^2}$$

if  $b' = 0$  then

$$h' = 90 \operatorname{sign}(a') [\operatorname{sign}(a') - 1]$$

else

$$h' = 180 - (180/\pi) \arctan(a'/b') - 90 \operatorname{sign}(b')$$

end if.

Here  $\operatorname{sign}$  and  $\arctan$  are functions that are defined in and are expected to return values as stated in 6.2.4.

$$G = 0.5 \cdot \left( 1 - \sqrt{\frac{\overline{C}^{*7}}{\overline{C}^{*7} + 25^7}} \right)$$

where  $\overline{C}^*$  is the arithmetic mean of the CIELAB  $C^*$  values for the pair of specimens (standard and batch).

$$\Delta L' = L'_B - L'_S$$

$$\Delta C' = C'_B - C'_S$$

$$\Delta H' = s [2 (C'_B C'_S - a'_B a'_S - b'_B b'_S)]^{0.5}$$

where

$$s = 1 \text{ if } a'_S b'_B > a'_B b'_S, \text{ else } s = -1.$$

$$\Delta E_{00}^2 = \left( \frac{\Delta L'}{k_L \cdot S_L} \right)^2 + \left( \frac{\Delta C'}{k_C \cdot S_C} \right)^2 + \left( \frac{\Delta H'}{k_H \cdot S_H} \right)^2 + R_T \cdot \left( \frac{\Delta C' \cdot \Delta H'}{k_C \cdot S_C \cdot k_H \cdot S_H} \right)$$

$$\Delta E_{00} = \sqrt{\Delta E_{00}^2}$$

The specimen or industry dependent parameters are  $k_L$ ,  $k_C$ ,  $k_H$  (all defaulting to unity in the absence of specific information or agreement between parties),  $S_L$ ,  $S_C$ ,  $S_H$  and  $R_T$ . The three  $S$  terms operate on the, assumed orthogonal, CIELAB coordinates and the  $R_T$  term computes a rotation of the color difference volume in the blue and purple-blue regions of the CIELAB diagram. The four color space terms are computed as follows:

$$S_L = 1 + \frac{0.015 \cdot (\overline{L} - 50)^2}{\sqrt{20 + (\overline{L} - 50)^2}}$$

$$S_C = 1 + 0.045 \cdot \overline{C}$$

$$S_H = 1 + 0.015 \cdot \overline{C} \cdot T$$

$$R_T = -\sin(2 \cdot \Delta\theta) \cdot R_C$$

$$R_C = 2 \cdot \sqrt{\frac{\overline{C}^7}{\overline{C}^7 + 25^7}}$$

$$\Delta\theta = 30 \cdot \exp\left(-\left[\frac{(\overline{h} - 275^\circ)}{25}\right]^2\right)$$

$$T = 1 - 0.17 \cdot \cos(\overline{h} - 30^\circ) + 0.24 \cdot \cos(2\overline{h}) + 0.32 \cdot \cos(3\overline{h} + 6^\circ) - 0.20 \cdot \cos(4\overline{h} - 63^\circ)$$

The following pseudocode (see 11) will calculate  $\overline{h}$  for substitution in the above equation:

$$p = (h'_S + h'_B) / 2$$

$$q = \operatorname{Abs}(h'_S - h'_B)$$

if  $C'_S C'_B = 0$  then

$$\overline{h} = 2p$$

else if  $q > 180$  then

if  $p < 180$  then

$$\overline{h} = p + 180$$

else

$$\overline{h} = p - 180$$

end if

else

$$\overline{h} = p$$

end if

Here  $\operatorname{Abs}$  means the absolute value of the argument.

While not obvious from this listing, all displayed angles are assumed to be given in degrees, including  $\Delta\theta$  and thus must generally be converted into radians for trigonometric analysis on digital computers.

6.6.1 Using the arithmetic average of the CIELAB color coordinates of the reference and test specimens to compute the local distortion of CIELAB color space introduces a new problem. Current color tolerance difference equations which base the distortion of CIELAB space on the position of the standard allows a user to predefine the acceptance volume. This is convenient for certain textile sorting applications and for graphical quality control charting. Such a predetermination is not possible with CIEDE2000. Nor is it possible or reasonable to plot groups of colors in terms of the modified space coordinates,  $L^*, a', b^*$  since the meaning of  $a'$  is determined uniquely for each pair of colors. Thus the equation is highly optimized for pairwise comparison of a product standard to a production test specimen but not for statistical process control.

## 7. Test Specimens

7.1 This practice does not cover specimen preparation techniques. Unless otherwise specified or agreed, prepare specimens in accordance with appropriate test methods and practices.

## 8. Procedure

8.1 Select appropriate geometric conditions for color measurement in accordance with Practice E805.

8.2 Operate the instrument in accordance with the manufacturer's instructions and the procedures given in Practice E1164.

8.3 When a colorimetric spectrometer is used, obtain the reflectance values of the reference specimen and test specimens, in turn, at a sufficient number of wavelength intervals to permit accurate calculation of CIE tristimulus values. See Practice E308.

8.4 Measure at least three portions of each specimen surface to obtain an indication of uniformity. Record the location where these measurements were made on the specimen.