International Standard ISO 3951

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Sampling procedures and charts for inspection by variables for percent defective

Règles et tables d'échantillonnage pour les contrôles par mesures des pourcentages de défectueux

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Foreword

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Sampling procedures and charts for inspection by variables for percent defective

Section one : General

1 Scope and field of application

1.1 Scope

1.1.1 This International Standard establishes sampling plans and procedures for inspection by variables. It is complementary to ISO 2859. When specified by the responsible authority, both this International Standard and ISO 2859 may be referenced in a product or process specification, contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The "responsible authority" shall be designated in one of the above documents.

1.1.2 The object of the methods laid down in this International Standard is to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of rejection of inferior lots is as high as possible.

1.1.3 In common with ISO 2859, the percentage of non-conforming products, or the **percent defective**, in the lots is used to define the quality of these lots and of the production process in question.

1.2 Field of application

This International Standard is primarily designed for use under the following conditions :

a) where the inspection procedure is to be applied to a **continuous series of lots** of discrete products all supplied by one producer using one production process. If there are different producers, this International Standard shall be applied to each one separately;

b) where only a **single quality characteristic** x of these products is taken into consideration, which must be **measurable on a continuous scale.** If several such characteristics are of importance, this International Standard shall be applied to each separately;

c) where production is stable (under statistical control) and the quality characteristic x is distributed according to a normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines an **upper specification limit** U, a **lower specification limit** L, or both; a product is qualified as non-conforming, or **defective**, when its measured quality characteristic x satisfies one of the following inequalities :

$$x > U \quad \ldots(1)$$

$$x < L \quad \ldots(2)$$

$$\text{either } x > U \text{ or } x < L \quad \ldots(3)$$

Inequalities (1) and (2) are called cases with a **single specification limit**, and (3) a case with **double specification limits.** In this last situation a further distinction is made between separate or combined double limits according to whether the AQL is applied to each limit separately or to both limits combined (see clause 4).

2 References

ISO 2854, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

ISO 2859, *Sampling procedures and tables for inspection by attributes.*

ISO 3534, *Statistics — Vocabulary and symbols.*

ISO 5725, *Precision of test methods — Determination of repeatability and reproducibility by inter-laboratory tests.*

3 Definitions and symbols

3.1 Definitions

For the purpose of this International Standard, the definitions given in ISO 3534 and ISO 2859 apply. The following additional terms also apply.

3.1.1 inspection by variables (contrôle par mesures) : A method which consists in measuring a quantitative characteristic for each item of a population or a sample taken from this population.

3.1.2 acceptance sampling by variables (échantillonnage par mesures en vue d'acceptation) : An acceptance procedure wherein a specified characteristic is measured to establish statistically the acceptability of a lot from the result obtained from the items in a sample.

3.1.3 acceptable quality level (AQL) [niveau de qualité acceptable (NQA)] : The maximum percent defective that, for purposes of sampling inspection, can be considered satisfactory as a process average. (See clause 4.)

3.1.4 limiting quality (qualité limite) : In a sampling plan, a quality level which corresponds to a specified and relatively low probability of acceptance (in this International Standard : 10 %). (See 12.1.)

3.1.5 defective (défectueux) : Any item that does not conform to the specification.

3.1.6 "*s*" method (méthode «*s*») : A method of assessing the acceptability of a lot by using the estimate of the standard deviation of the lot based on measurements of all the items in a sample. (See clause 14.)

3.1.7 "*σ*" method (méthode «*σ*») : A method of assessing the acceptability of a lot using previous knowledge of its standard deviation. (See clause 15.)

3.1.8 "*R*" method (méthode «*R*») : A method of assessing the acceptability of a lot by using an estimate of the standard deviation of the lot based on the average range of the measurements of the items in sub-groups of a sample. (See annex C.)

3.1.9 specification limit (limite de spécification) : The specified maximum or minimum acceptable value of the characteristic.

3.1.10 lower specification limit (*L*) [limite inférieure de spécification (L_i)] : The specified minimum acceptable value of the characteristic.

3.1.11 upper specification limit (*U*) [limite supérieure de spécification (L_s)] : The specified maximum acceptable value of the characteristic.

3.1.12 single specification limit (limite unique de spécification) : The term used when one limit only is specified.

3.1.13 separate double specification limits (limites de spécifications doubles séparées) : The term used when both upper and lower limits are specified and separate AQLs are applied to each limit individually. (See 4.3.)

3.1.14 combined double specification limit (limite de spécification double combinée) : The term used when both upper and lower limits are specified and an AQL is given which applies to the combined percent defective at both the limits. (See 4.3.)

3.1.15 acceptability constant (*K*) (constante d'acceptabilité) : A constant dependent on the specified value of the acceptable quality level and the sample size. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.16 quality parameter (*q*) (paramètre de qualité) : A function of the specification limit, the mean and the standard deviation of the lot. (See clause B.3 in annex B.)

3.1.17 quality statistic (*Q*) (statistique de qualité) : A function of the specification limit, the sample mean, and the estimate of the standard deviation of the lot. The lot is sentenced on the result of comparing *Q* with the acceptability constant *K*. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.18 lower quality statistic (Q_L) [statistique de qualité correspondant à la limite inférieure (Q_i)] : A function of the lower specification limit, the sample mean, and the estimate of the standard deviation of the lot. The lot is sentenced on the result of comparing Q_L with the acceptability constant *K*. (See 14.2.)

3.1.19 upper quality statistic (Q_U) [statistique de qualité correspondant à la limite supérieure (Q_s)] : A function of the upper specification limit, the sample mean, and the estimate of the standard deviation of the lot. The lot is sentenced on the result of comparing Q_U with the acceptability constant *K*. (See 14.2.)

3.1.20 maximum standard deviation (MSD) [écart-type maximal (ETM)] : Under given conditions, the largest acceptable standard deviation. (See 14.4 and sub-clause B.5.2 in annex B.)

3.1.21 switching rules (règles de modification du contrôle) : Rules that govern the decision to increase or decrease the severity of inspection. (See clause 19.)

3.2 Symbols

The symbols used are as follows :

f A factor, given in table IV, that relates the maximum standard deviation to the difference between *U* and *L*.

K The (general) acceptability constant.

k The acceptability constant when using the "*s*" method, "*σ*" method or "*R*" method.

L Lower specification limit. (As a suffix to a variable, denotes its value at *L*.)

U Upper specification limit. (As a suffix to a variable, denotes its value at *U*.)

n Number of units in a sample.

N Number of units in a lot.

P The cumulative probability function (distribution function).

p The estimate of the total lot percent or fraction defective

$$p = p_U + p_L$$

p_L Estimate of fraction defective below *L*.

p_U Estimate of fraction defective above *U*.

q The quality parameter : equal to z_U or $-z_L$.

Q The quality statistic.

Q_L Lower quality statistic.

Q_U Upper quality statistic.

s Estimate from the sample of the standard deviation of the lot

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}$$

(See also annex A.)

x Measured value of a characteristic in the sample.

$\overline{x}$ Mean value of *x* for the sample of *n* items.

z Value of the standardized variate

$$z = \frac{x - \mu}{\sigma}$$

z_L Value of *z* at the lower specification limit

$$z_L = \frac{L - \mu}{\sigma}$$

z_U Value of *z* at the upper specification limit

$$z_U = \frac{U - \mu}{\sigma}$$

μ Mean of the lot.

σ Standard deviation of the process (or of a lot) (σ^2 = variance).

Σ "The sum of" (for example, $\Sigma x =$ the sum of the *x* values).

$\sum_{i=1}^{n} x_i$ The sum of all the *x* values when *i* takes integral values from 1 to *n*.

$>$ "Greater than" (for example, $a > b$ means *a* is greater than *b*).

$\geqslant$ "Greater than or equal to" (for example, $a \geqslant b$ means *a* is greater than or equal to *b*).

$<$ "Less than" (for example, $a < b$ means *a* is less than *b*).

$\leqslant$ "Less than or equal to" (for example, $a \leqslant b$ means *a* is less than or equal to *b*).

3.3 Bibliography

A bibliography of documents utilized in the development of this International Standard is given in annex D.

4 Acceptable Quality Level (AQL)

4.1 Definition

The maximum percent defective that, for the purposes of sampling inspection, can be considered satisfactory as a process average.

4.2 Use

The AQL, together with the code letter, is used to index the sampling plans provided herein.

4.3 Specifying AQLs

The AQL to be used will be designated in the product specification contract or by the responsible authority. Where both upper and lower specifications limits are given, separate AQLs may be given to the individual limits, which are then known as "separate double specification limits". Alternatively, an overall AQL may be given which applies to the combined percent defective at both the upper and lower limits; this is then known as a "combined double specification limit".

4.4 Preferred AQLs

The values of AQLs given in this International Standard are preferred AQLs. If, for any product, an AQL is designated other than a preferred AQL, then this International Standard is not applicable. (See 12.2.)

4.5 Caution

From the above definition of the AQL, it follows that desired protection can only be obtained when a continuous series of lots is provided for inspection.

4.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any defective unit of product.

5 Switching rules for normal, tightened and reduced inspection

5.1 In order to discourage the process average exceeding the AQL, this International Standard prescribes a switch to tightened inspection when inspection results indicate that the process average exceeds the AQL, and stopping sampling inspection altogether when tightened inspection does not in time stimulate the producer to improve his production process.

5.2 Hence, tightened inspection and the stopping rule are integral, and therefore obligatory, procedures of this International Standard if the protection implied by the AQL is to be maintained.

5.3 This International Standard also provides the possibility of switching to reduced inspection when inspection results indicate that the process average is stable and reliable at a level below the AQL. This practice is, however, optional (at the discretion of the responsible authority).

5.4 When there is sufficient evidence from the control charts (see 18.1) that the variability is in statistical control, consideration should be given to switching to the "σ" method. If this appears advantageous, the consistent value of s shall be taken as σ.

5.5 When it has been necessary to discontinue sampling inspection, tightened inspection may not be resumed until action is taken by the producer to improve the quality of the submitted product.

5.6 Details of the operation of the switching rules are given in clause 19.

6 Relation to ISO 2859

6.1 Similarities

a) This International Standard is a complement to ISO 2859; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans and the preferred values used in this document are identical with those given in ISO 2859 for the same range of values (i.e. from 0,1 % to 10 %).

c) In the two documents, lot size and inspection level (inspection level II being preferred in default of other instructions) determine a code letter. Then general tables give the sample size to be taken and the acceptability criterion in terms of the code letter and the AQL according to the method chosen ("s", "σ" or, contingently, "R"). Separate tables are given for normal, tightened and reduced inspection.

d) The switching rules are almost identical.

e) The definitions of critical, major and minor defects remain unchanged, but the concept is less useful as characteristics cannot be grouped together for sentencing in inspection by variables. It does assist, however, in specifying the AQL for the various characteristics.

6.2 Differences

a) **Determination of acceptability.** The acceptability of an attributes sampling plan, taken from ISO 2859, is determined by the number of defectives found in the sample; the acceptability criterion in inspection by variables is based on estimates of the location and variability of the distributed measurements of the lot, in relation to the specification limits, that is in terms of the mean and standard deviation. In this International Standard two ways of estimating the standard deviation are considered : the "s" method and the "σ" method (a third method, "R" method, is given in annex C). In the case of a single specification limit or of two separate limits, the acceptability may be calculated from a formula (see 14.2 and 15.2), but is more easily established by a graphical method (see 14.3). In the case of a combined double limit, this International Standard provides for a graphical method (see 14.4 and 15.3).

b) **Normality.** In ISO 2859 there is no requirement relating to the distribution of the characteristics, but in this International Standard it is necessary to the efficient operation of a plan that the measurements should be distributed acording to a normal distribution or a close approximation of the normal distribution.

c) **Operating Characteristics curves** (OC curves). While an individual variables plan may be devised whose OC curve corresponds closely to that of a given attributes plan, it would not be possible to make all the OC curves in this International Standard identical with the corresponding OC curves in ISO 2859 (which are indexed with the same code letter and AQL), as the sample size for a given code letter would then have to increase with the AQL; this would be undesirable for the practical application of this sampling scheme.

d) **Probability of acceptance at the AQL.** The probability that a lot, whose quality is precisely at the AQL, will be accepted increases with the sample size and follows a similar, but not identical, scale to that used in ISO 2859.

e) **Sample sizes.** The variables sample sizes corresponding to given code letters are usually smaller than the attributes sample sizes for the same letters.

f) **Double sampling plans.** No double sampling plans are given in this International Standard.

g) **Average Outgoing Quality Limit (AOQL).** Since it is anticipated that variables plans will mainly be used with destructive testing, where 100 % inspection and rectification of rejected lots is not possible, the AOQL concept can not be applied and therefore these tables have not been included.

7 Non-continuous production

7.1 The sampling scheme contained in this International Standard was not designed to be applied under circumstances different from those specified in 1.2, for example, to an isolated lot or limited number of lots, where tightened inspection and the stopping rules cannot be applied.

7.2 Under such conditions, the concept of AQL as a maximum for the process average of accepted lots no longer holds, and the AQL can no longer be interpreted as a measure for the degree of protection of the consumer against receiving lots of poor average quality. The AQL then only indicates the percent defective that has a high probability of acceptance and acts as an index to a sampling plan.

7.3 The degree of protection of the consumer provided by the individual sampling plans of this International Standard can, however, be judged from their Operating Characteristic (OC) curves as given in charts V-B to V-P and tables V-B-1 to V-P-1, and these should be consulted in choosing a sampling plan.

The OC curves calculated for the "s" method are applicable to the "σ" method. They are identified by code letters and AQL values.

NOTE — The OC curves given in charts V-B to V-P are in terms of normal inspection. Separate curves are not given for tightened or reduced inspection.

Diagram A will assist in finding a suitable OC curve. On this diagram, the intersection of a vertical line through the chosen value for the limiting quality and a horizontal line through the acceptable quality with a 95 % probability of acceptance (approximately equal to AQL) will lie on, or under, a sloping line indexed with the code letter of a standard plan which meets the specified requirements approximately.

Section two : Choice of sampling plan

8 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge of both statistics and the product to be inspected. This section of this International Standard is intended to suggest to those responsible for specifying sampling plans the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

9 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account :

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more expensive in time and money per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the more precise information obtained indicates how good the product is; earlier warning will be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any defectives. (See the example in 14.4.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859) and the standard plans in this International Standard, is given in table I-B. It will be seen that the smallest samples are required by the "σ" method, used when the standard deviation of the lot is known.

e) Inspection by variables is appropriate particularly in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

g) A variables scheme becomes less suitable as the number of measurements to be taken on one item increases, as each characteristic has to be considered separately. It may be advantageous to apply "attributes" to the majority of the characteristics and "variables" to one or two of the more important requirements, for example, proof load tests, safety and reliability requirements.

h) The use of this International Standard is only applicable when there is reason to believe that the distribution of measurements is normal. In case of any doubt, the responsible authority should be consulted.

NOTES

1 Tests for departure from normality are dealt with in section two of ISO 2854, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

2 More comprehensive documentation on this subject is currently in course of preparation within ISO/TC 69/SC 2 which will additionally give guidance on various numerical tests which might also be utilized. Subsequently, it is envisaged that tests for departure from normality will form the subject of a separate International Standard.

10 Choice of method

If it is desired to apply inspection by variables, the next question is which method should be used, the "s" method or the "σ" method (or the "R" method).

The "σ" method is the most economical in sample size, but before this method may be employed, the value of σ has to be established.

In terms of sample size, the "s" method has a slight advantage over the "R" method, but the calculation of s does involve more computation; the extent and difficulty of this is more apparent than real, especially if an electronic calculator is available. Methods of calculating s are given in annex A.

The "R" method (given in annex C) is simple to calculate, but requires a somewhat larger sample size for the same AQL.

Initially, it will be necessary to begin with the "s" (or the "R" method), but if the quality is satisfactory, the standard switching rules will permit the responsible authority to commence reduced inspection and use a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the "σ" method? It should be noted that the size of the sample will not necessarily be smaller if the AQL is large, but the acceptability criteria become simpler. (See clauses 15.2 and 15.3.) On the other hand, it will still be necessary to calculate s for record purposes and to keep the control charts up to date. (See clause 18.)

11 Choice of inspection level and AQL

In standard sampling plans, the inspection level in conjunction with the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve given in one of the tables V-B to V-P shows the extent of the risk that is involved in such a plan.

The choice of the inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of defective items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

12 Choice of a sampling plan

12.1 Standard plans

The standard procedure can be used only when the production of lots is continuous.

The standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the "*s*" method, has been found in practice to produce workable sampling plans; but it assumes that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see clause 19), which quickly increase the severity of inspection and finally terminate it, if the quality of the process is worse than the AQL.

NOTE — It should also be remembered that the limiting quality is the quality which if offered for inspection would have a 10 % probability of acceptance. The actual risk taken by the consumer therefore also depends on the probability of goods of this low quality being offered for inspection.

However, if, in certain circumstances, the limiting quality has a higher priority than the sample size (for example, when only a limited number of lots are being produced), a suitable plan in this International Standard may be selected by using diagram A. The intersection of a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (approximately equal to AQL) will lie on, or under, a sloping line indexed with the code letter of a standard plan which meets the specified requirements. This should be verified by inspecting the OC curve given in table V for this code letter and AQL.

If the lines intersect at a point above the line marked P (see diagram A), this implies that a sample of over 200 would be necessary and the specification cannot be met by the plans in this International Standard.

12.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. The choice is then to decide which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent, for, when any two have been chosen, the third follows.

NOTE — This choice is not completely unfettered : the "non-central *t*" tables referred to in annex D have only been computed for a limited, though generally adequate, range of variables and the fact that the size of the sample is necessarily a whole number imposes some constraints. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

Section three : Operation of a variables sampling plan

13 Preliminary operations

Before starting inspection by variables, check

a) that the distribution can be considered to be normal and that production is considered to be continuous;

b) whether the "s" (or "R") method is to be used initially or whether the standard deviation is stable and known and the "σ" method is to be used;

c) that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) that the AQL has been designated and that it is one of the preferred AQLs for use with this International Standard. If it is not, the tables are not applicable;

e) if a double specification limit has to be met, whether the limits are separate or combined and, if the limits are separate, whether AQLs are determined for each limit.

14 Standard procedure for "s" method

14.1 Obtaining a plan

The procedure for obtaining a plan is as follows :

a) With the inspection level given (normally this will be II) and with the lot size, obtain the code letter using table I-A.

b) With this code letter and the AQL, enter table II-A and obtain the sample size n and acceptability constant k.

c) Taking a random sample of this size, measure the characteristic x in each item and then calculate $\bar{x}$, the sample mean, and s, the estimated standard deviation (see annex A). If $\bar{x}$ is outside the specification limit, the lot can be rejected without calculating s. It may, however, be necessary to calculate s for record purposes.

14.2 Acceptability criteria for single or separate specification limits

If single or separate specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s}$$

and/or

$$Q_L = \frac{\bar{x} - L}{s}$$

as appropriate,

then compare the quality statistic (Q_U and/or Q_L) with the acceptability constant k obtained from table II-A for normal inspection. If the appropriate quality statistic is greater than or equal to the acceptability constant, accept the lot; if less, reject.

Thus, if only the upper specification limit U is given,

accept if $Q_U \geqslant k$

reject if $Q_U < k$

Or, if only the lower specificatioin limit L is given,

accept if $Q_L \geqslant k$

reject if $Q_L < k$

When both U and L are given (k values are different if the AQL are different for the upper limit and the lower limit),

accept if **both** $Q_L \geqslant k_L$ and $Q_U \geqslant k_U$

reject if **either** $Q_L < k_L$ or $Q_U < k_U$

Example

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From table I-A, the letter code is F; from table II-A it is seen that a sample size of 10 is required and that the acceptability constant k is 1,41. Suppose the measurements are as follows : 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 55 °C; 50 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size : n	10
Sample mean $\bar{x}$: $\Sigma\, x/n$	54,9
Standard deviation s : $\sqrt{\Sigma\,(x - \bar{x})^2/(n - 1)}$	3,414
Specification limit (upper) : U	60
$Q_U = (U - \bar{x})/s$	1,494
Acceptability constant : k (see table II-A)	1,41
Acceptability criterion : compare Q_U with k	1,494 > 1,41

The lot meets the acceptability criterion, since Q_U is greater than k.

14.3 Graphical method for single or separate specification limits

Write the acceptance conditions

$$\frac{U - \overline{x}}{s} \geqslant k \quad \text{and/or} \quad \frac{\overline{x} - L}{s} \geqslant k$$

as $x \leqslant U - ks$ and/or $\overline{x} \geqslant L + ks$.

It can be seen on a graph with s values as the horizontal axis and $\overline{x}$ values as the vertical axis, that the line of general equation $\overline{x} = U - ks$ [straight line through the point ($s = 0$, $\overline{x} = U$) with a slope $-k$] and/or line $\overline{x} = L + ks$ [straight line through the point ($s = 0$, $\overline{x} = L$) with a slope k] divides the plan in two zones : accept zone and reject zone according to figure 1.

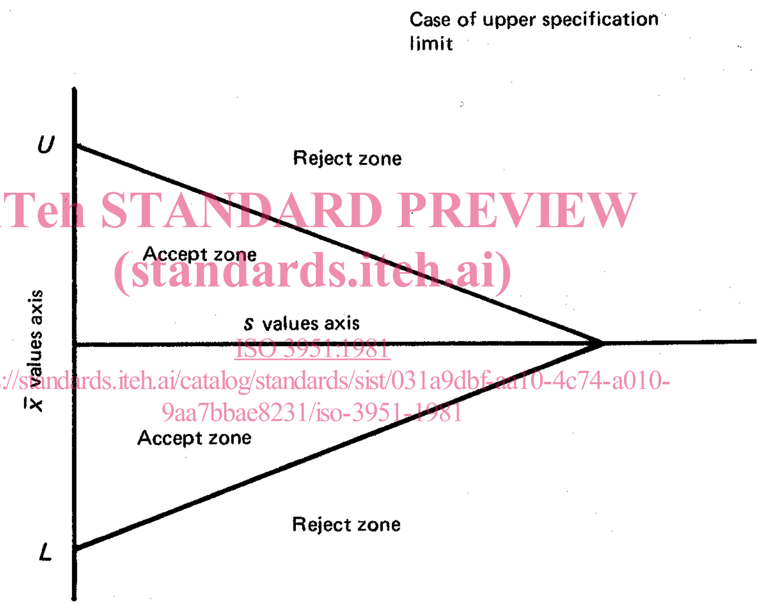

Figure 1

In case of double separate specification limits, the accept zone is the inside area limited by the two lines. Figure 1 assumes that a single value of AQL has been determined for each limit (single k value). The lines intersect at the point whose abscissa is

$$\frac{U - L}{2k}$$

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot plot the point (s, $\bar{x}$) and decide to accept or reject the lot.

Example

Using the figures given in the example in 14.2, mark the point U = 60 on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $-k$ [as k = 1,41, this means the line passes through points (s = 1, $\bar{x}$ = 58,59), (s = 2, $\bar{x}$ = 57,18), (s = 3, $\bar{x}$ = 55,77), etc.]. Select a suitable point and draw a straight line through it and (s = 0, $\bar{x}$ = 60), i.e. U. The accept zone is then the area under this line. The calculated values of s and $\bar{x}$ are 3,414 and 54,9. Plotting the point (s, $\bar{x}$), it will be seen from figure 2 that it lies just inside the accept zone and the lot is acceptable.

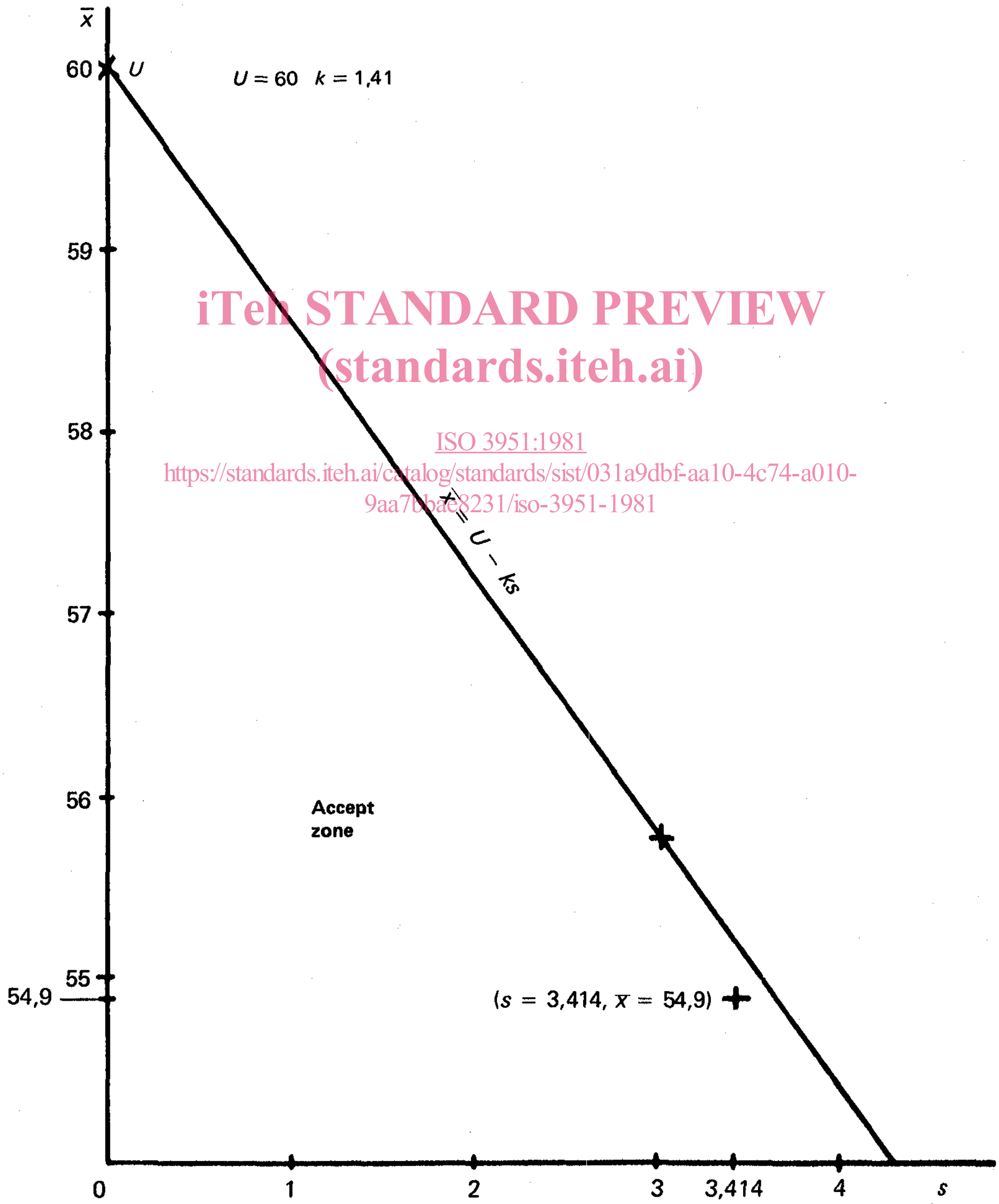

Figure 2

14.4 Acceptability criterion for a combined double specification limit

If a combined upper and lower specification limit has been given, it will be necessary to use a graphical method, unless s is greater than the value of the MSD obtained from table IV, when the lot has to be rejected at once.

Consult the chart in the "s" series which is labelled with the appropriate code letter and select the acceptance curve with the AQL specified for the two limits.

Then calculate the values of

$$\frac{s}{U - L} \text{ and } \frac{\bar{x} - L}{U - L}$$

and plot a point representing these values on the graph (or a copy of it). If the point lies outside the accept zone defined by the curve, reject the lot, otherwise accept.

For greater convenience, it is recommended that before the inspection operations begin the acceptance curve be copied (or traced) onto graph paper and that the scales be adjusted so that s *and* $\bar{x}$ can be plotted directly (the upper limit is given instead of 1,0 and the lower limit instead of 0 on the $\bar{x}$ scale).

Then plot on the chart the values of s and $\bar{x}$ found from the sample and, if the point lies outside the accept zone, reject the lot, otherwise accept.

NOTES

1 For code letters B and C (i.e. sample sizes 3 and 4), the drawing of the accept zone is bounded by 4 straight lines : the $\bar{x}$ axis, the line $\bar{x} = U - ks$, a line parallel to the $\bar{x}$ axis through the MSD (see table IV) and the line $\bar{x} = L + ks$. The value of k is obtained from the table II-A, II-B or II-C.

2 In order to provide a reasonable scale for the most frequently used values, the AQLs 6,5 % and 10 % have had to be omitted in certain places (for example, diagram s-D). However, the procedure for their construction can be found in annex B.

Example

The minimum temperature of operation for a certain device is specified as 60,0 °C and the maximum temperature as 70,0 °C. Production is in inspection lots of 96 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From table I-A, the code letter is F; from the table I-B it is seen that a sample size of 10 is required and from table IV that the value of f for the MSD is 0,276. Suppose the measurements obtained are as follows : 63,5 °C; 62,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Value obtained**
Sample size : n	10
Sample mean : $\bar{x}$: $\Sigma x/n$	64,57
Standard deviation s : $\sqrt{\Sigma (x - \bar{x})^2/(n - 1)}$ (See clause A.2, annex A.)	3,01
Standardized mean : $(\bar{x} - L)/(U - L)$	0,457
Standardized standard deviation : $s/(U - L)$	0,301
Value of f for MSD (table IV)	0,276
MSD = $f(U - L)$	2,76

The appropriate acceptance curve is taken from diagram s-F.

If, as on figure 3, the scales have been adjusted to the real measurements, plot the point (s = 3,01, $\bar{x}$ = 64,57). This lies outside the acceptance curve and the lot is rejected. It could have been rejected as soon as it was seen that s was greater than the MSD.

If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required :

Standardized mean : $(\bar{x} - L)/(U - L) = (64,57 - 60)/(70 - 60) = 0,457$

Standardized s : $s/(U - L) = 3,01/(70 - 60) = 0,301$

The point (0,301, 0,457) is plotted on figure 3.

As it lies outside the acceptance curve for AQL = 1,5 %, the lot is rejected.

NOTE — This lot is rejected even though all tested items **in the sample** are within specification limits.