INTERNATIONAL STANDARD

**ISO
3951**

Sampling procedures and charts for inspection by variables for percent nonconforming

Règles et tables d'échantillonnage pour les contrôles par mesures des pourcentages de non conformes

INTERNATIONAL

ISO

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council. They are approved in accordance with ISO procedures requiring at least 75 % approval by the member bodies voting.

International Standard ISO 3951 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods.*

This second edition cancels and replaces the first edition (ISO 3951 : 1981), of which it constitutes a technical revision.

The principal changes to the first edition are as follows:

a) a distinction has been drawn between maximum process standard deviation (MPSD) under the "σ" method and maximum sample standard deviation (MSSD) under the "s" method;

b) all the "σ" method acceptance curves have been truncated at the appropriate maximum process standard deviations;

c) the terminology has been aligned on that of ISO 2859 and ISO 3534.

Users should note that all International Standards undergo revision from time to time and that any reference made herein to any other International Standard implies its latest edition, unless otherwise stated.

Contents

Sampling procedures and charts for inspection by variables for percent nonconforming

Section one : General

1 Scope and field of application

1.1 Scope

1.1.1 This International Standard establishes sampling plans and procedures for inspection by variables. It is complementary to ISO 2859. When specified by the responsible authority, both this International Standard and ISO 2859 may be referenced in a product or process specification, contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The "responsible authority" shall be designated in one of the above documents.

1.1.2 The object of the methods laid down in this International Standard is to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as possible.

1.1.3 In common with ISO 2859, the percentage of nonconforming products in the lots is used to define the quality of these lots and of the production process in question.

1.2 Field of application

This International Standard is primarily designed for use under the following conditions :

a) where the inspection procedure is to be applied to a **continuous series of lots** of discrete products all supplied by one producer using one production process. If there are different producers, this International Standard shall be applied to each one separately;

b) where only a **single quality characteristic** x of these products is taken into consideration, which must be **measurable on a continuous scale**. If several such characteristics are of importance, this International Standard shall be applied to each separately;

c) where production is stable (under statistical control) and the quality characteristic x is distributed according to a normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines an **upper specification limit** U, a **lower specification limit** L, or both; a product is qualified as nonconforming when its measured quality characteristic x satisfies one of the following inequalities :

$$x > U \qquad \qquad \dots(1)$$

$$x < L \qquad \qquad \dots(2)$$

$$\text{either } x > U \text{ or } x < L \qquad \dots(3)$$

Inequalities (1) and (2) are called cases with a **single specification limit**, and (3) a case with **double specification limits**. In this last situation a further distinction is made between separate or combined double limits according to whether the AQL is applied to each limit separately or to both limits combined (see clause 4).

2 References

ISO 2854, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

ISO 2859, *Sampling procedures and tables for inspection by attributes.*

ISO 3534, *Statistics — Vocabulary and symbols.*

ISO 5479, *Normality tests.* [1]

ISO 5725, *Precision of test methods — Determination of repeatability and reproducibility for a standard test method by inter-laboratory tests.*

3 Definitions and symbols

3.1 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534 and ISO 2859, together with the following, apply.

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1) At present at the stage of draft.

3.1.1 inspection by variables (contrôle par mesures) : A method which consists in measuring a quantitative characteristic for each item of a population or a sample taken from this population.

3.1.2 acceptance sampling by variables (échantillonnage par mesures en vue d'acceptation) : An acceptance procedure wherein a specified characteristic is measured to establish statistically the acceptability of a lot from the result obtained from the items in a sample.

3.1.3 acceptable quality level (AQL) [niveau de qualité acceptable (NQA)] : When a continuous series of lots is considered, a quality level which for the purposes of sampling inspection is the limit of a satisfactory process average percent nonconforming. (See clause 4.)

3.1.4 limiting quality (qualité limite) : When a lot is considered in isolation, a quality level which for the purposes of sampling inspection is limited to a low probability of acceptance (in this International Standard : 10 %). (See 12.1.)

3.1.5 nonconformity (non-conformité) : Failure to fulfil a specified requirement by a quality characteristic of an item or service, the assessment of which does not depend essentially on the passage of time.

Nonconformities will generally be classified by their degree of seriousness, such as :

Class A. Those nonconformities of a type considered to be of the highest concern for the product or service. In acceptance sampling, such types of nonconformity will be assigned very small AQL values.

Class B. Those nonconformities of a type considered to have the next lower degree of concern; therefore these can be assigned a larger AQL value than those in class A and smaller than in class C, if a third class exists, and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

3.1.6 nonconforming unit (unité non conforme) : A unit of product or service containing at least one nonconformity.

3.1.7 "s" method (méthode «s») : A method of assessing the acceptability of a lot by using the sample standard deviation. (See clause 14.)

3.1.8 "σ" method (méthode «σ») : A method of assessing the acceptability of a lot by using knowledge of the process standard deviation. (See clause 15.)

3.1.9 "R" method (méthode «R») : A method of assessing the acceptability of a lot by indirectly using an estimate of the standard deviation of the process based on the average range of the measurements of the items in sub-groups of the sample. (See annex C.)

3.1.10 specification limit (limite de spécification) : The limiting value (lower or upper) specified for a quantitative characteristic.

3.1.11 lower specification limit (L) [limite inférieure de spécification (L_i)] : A specification limit that defines the lower conformance boundary for an individual unit of a manufacturing or service operation.

3.1.12 upper specification limit (U) [limite supérieure de spécification (L_s)] : A specification limit that defines the upper conformance boundary for an individual unit of a manufacturing or service operation.

3.1.13 single specification limit (limite unique de spécification) : The term used when one limit only is specified.

3.1.14 separate double specification limits (limites de spécifications doubles séparées) : The term used when both upper and lower limits are specified and separate AQLs are applied to each limit individually. (See 4.3.)

3.1.15 combined double specification limit (limite de spécification double combinée) : The term used when both upper and lower limits are specified and an AQL is given which applies to the combined percent nonconforming at the two limits. (See 4.3.)

3.1.16 acceptability constant (k) [constante d'acceptabilité (k)] : A constant dependent on the specified value of the acceptable quality level and the sample size. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.17 quality statistic (Q) [statistique de qualité (Q)] : A function of the specification limit, the sample mean, and the standard deviation. The lot is sentenced on the result of comparing Q with the acceptability constant k. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.18 lower quality statistic (Q_L) [statistique de qualité correspondant à la limite inférieure (Q_i)] : A function of the lower specification limit, the sample mean, and the standard deviation. The lot is sentenced on the result of comparing Q_L with the acceptability constant k. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.19 upper quality statistic (Q_U) [statistique de qualité correspondant à la limite supérieure (Q_s)] : A function of the upper specification limit, the sample mean, and the standard deviation. The lot is sentenced on the result of comparing Q_U with the acceptability constant k. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.20 maximum sample standard deviation (MSSD) [écart-type maximal de l'échantillon (ETME)] : Under given conditions, the largest acceptable sample standard deviation. (See 14.6 and B.8.3 in annex B.)

3.1.21 maximum process standard deviation (MPSD) [écart-type maximal du procédé (ETMP)] : Under given conditions, the largest acceptable process standard deviation. (See 15.3 and B.5.2 in annex B).

3.1.22 switching rules (règles de modification du contrôle): Instructions within a sampling scheme for shifting from one sampling plan to another based on demonstrated quality history. (See clause 19.)

3.2 Symbols

The symbols used are as follows:

f_s A factor, given in table IV-s, that relates the maximum sample standard deviation to the difference between U and L.

f_σ A factor, given in table IV-σ, that relates the maximum process standard deviation to the difference between U and L.

k The acceptability constant when using the "s" method, "σ" method or "R" method.

K The acceptability constant when both μ and σ are known.

L Lower specification limit. (As a suffix to a variable, denotes its value at L.)

U Upper specification limit. (As a suffix to a variable, denotes its value at U.)

n Sample size (number of units in a sample).

N Lot size (number of units in a lot).

P_a The probability of acceptance.

Q The quality statistic.

Q_L Lower quality statistic.

Q_U Upper quality statistic.

s Standard deviation of a sample (estimate of the standard deviation of the process).

$$s = \sqrt{\frac{\sum\limits_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}$$

(See also annex A.)

x Measured value of a characteristic in the sample.

$\overline{x}$ Mean value of x for the sample of n items.

$\overline{x}_L$ Lower acceptance value.

$\overline{x}_U$ Upper acceptance value.

μ Mean of the process.

σ Standard deviation of the process. (σ^2, the square of the standard deviation, is known as the variance.)

Σ "The sum of" (for example, Σx = the sum of the x values).

$\sum\limits_{i=1}^{n} x_i$ The sum of all the x values when i takes integral values from 1 to n.

$>$ "Greater than" (for example, $a > b$ means a is greater than b).

$\geqslant$ "Greater than or equal to" (for example, $a \geqslant b$ means a is greater than or equal to b).

$<$ "Less than" (for example, $a < b$ means a is less than b).

$\leqslant$ "Less than or equal to" (for example, $a \leqslant b$ means a is less than or equal to b).

3.3 Bibliography

A bibliography of documents used in the development of this International Standard is given in annex D.

4 Acceptable Quality Level (AQL)

4.1 Definition

When a continuous series of lots is considered, a quality level which for the purposes of sampling inspection is the limit of a satisfactory process average percent nonconforming.

4.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this International Standard.

4.3 Specifying AQLs

The AQL to be used will be designated in the product specification contract or by the responsible authority. Where both upper and lower specifications limits are given, separate AQLs may be given to the individual limits, which are then known as "separate double specification limits". Alternatively, an overall AQL may be given which applies to the combined percent nonconforming at both the upper and lower limits; this is then known as a "combined double specification limit".

4.4 Preferred AQLs

Eleven AQLs given in this International Standard, ranging in value from 0,10 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this International Standard is not applicable. (See 12.2.)

Two further AQLs, 0,065 % and 15 %, are given to complete the range of plans needed for the operation of the switching rules. (See clauses 19 and 21.) Plans and curves designated by an AQL of 0,065 % or 15 % will only be used when the AQL under normal inspection is 0,10 % or 10 % respectively and the switching rules are invoked.

4.5 Caution

From the above definition of the AQL, it follows that desired protection can only be obtained when a continuous series of lots is provided for inspection.

4.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming unit of product.

5 Switching rules for normal, tightened and reduced inspection

5.1 In order to discourage the producer from operating at a process average percent nonconforming that exceeds the AQL, this International Standard prescribes a switch to tightened inspection when inspection results indicate that the process average does exceed the AQL, and discontinuation of sampling inspection altogether when tightened inspection does not in time stimulate the producer to improve his production process.

5.2 Hence, tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this International Standard if the protection implied by the AQL is to be maintained.

5.3 This International Standard also provides the possibility of switching to reduced inspection when inspection results indicate that the process average percent nonconforming is stable and reliable at a level below the AQL. This practice is, however, optional (at the discretion of the responsible authority).

5.4 When there is sufficient evidence from the control charts (see 18.1) that the variability is in statistical control, consideration should be given to switching to the "σ" method. If this appears advantageous, the consistent value of s shall be taken as σ.

5.5 When it has been necessary to discontinue sampling inspection, inspection may not be resumed until action is taken by the producer to improve the quality of the submitted product.

5.6 Details of the operation of the switching rules are given in clauses 19 and 20.

6 Relation to ISO 2859

6.1 Similarities

a) This International Standard is a complement to ISO 2859; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans and the preferred values used in this document are identical with those given in ISO 2859 for the same range of values (i.e. from 0,1 % to 10 %).

c) In the two International Standards, lot size and inspection level (inspection level II being preferred in default of other instructions) determine a code letter. Then general tables give the sample size to be taken and the acceptability criterion in terms of the code letter and the AQL according

to the method chosen ("s", "σ" or, contingently, "R"). Separate tables are given for normal, tightened and reduced inspection.

d) The switching rules are essentially the same.

e) The classification of nonconformities by degree of seriousness into class A, class B, etc. remains unchanged.

6.2 Differences

a) **Determination of acceptability.** The acceptability of an attributes sampling plan, taken from ISO 2859, is determined by the number of nonconforming units found in the sample; the acceptability criterion in inspection by variables is based on estimates of the location and variability of the distributed measurements of the lot, in relation to the specification limits, that is in terms of the mean and standard deviation. In this International Standard two methods are considered: the "s" method for use when the process standard deviation σ is unknown and the "σ" method for use when σ is considered to be known. A third method, called the "R" method, is given in annex C. In the case of a single specification limit or of two separate limits, the acceptability may be calculated from a formula (see 14.2 and 15.2), but is more easily established by a graphical method (see 14.3). In the case of a combined double limit, this International Standard provides for a graphical method (see 14.6 and 15.3).

b) **Normality.** In ISO 2859 there is no requirement relating to the distribution of the characteristics, but in this International Standard it is necessary to the efficient operation of a plan that the measurements should be distributed according to a normal distribution or a close approximation to the normal distribution.

c) **Operating characteristic curves** (OC curves). While an individual variables plan may be devised the OC curve of which corresponds closely to that of a given attributes plan, it would not be possible to make all the OC curves in this International Standard identical with the corresponding OC curves in ISO 2859 (which are indexed with the same code letter and AQL), without the sample size increasing with the AQL for a given sample size code letter. For the "s" method the sample size has been kept fixed for a given lot size across the full range of AQLs; for the "σ" method this constraint has been removed in order to match the "s" and "σ" method OC curves as closely as possible, both at the AQL and at the limiting quality.

d) **Probability of acceptance at the AQL.** The probability that a lot, whose quality is precisely at the AQL, will be accepted increases with the sample size and follows a similar, but not identical, scale to that used in ISO 2859.

e) **Sample sizes.** The variables sample sizes corresponding to given code letters are usually smaller than the attributes sample sizes for the same letters.

f) **Double sampling plans.** No double sampling plans are given in this International Standard.

g) **Average Outgoing Quality Limit (AOQL).** Under destructive or expensive testing, where 100 % inspection and rectification of non-accepted lots is not feasible, the AOQL concept cannot be applied. As variables plans will generally be used under these circumstances, no AOQL tables have been included in this International Standard.

7 Non-continuous production and operating characteristic curves

7.1 Non-continuous production

a) The sampling schemes contained in this International Standard were not designed to be applied under circumstances different from those specified in 1.2, for example to an isolated lot or limited number of lots, where tightened inspection and the discontinuation rules cannot be applied.

b) Under such conditions, the concept of an AQL becomes irrelevant, as the consumer's concern narrows to the quality of the limited number of submitted lots, and he is no longer involved with exerting controls on the quality of the production process. The AQL will still indicate a submitted quality which has a high probability of acceptance, and can therefore still be used as an index to a sampling plan. (See clause 12.)

7.2 Operating characteristic curves

a) The degree of protection of the consumer provided by the individual sampling plans of this International Standard can, however, be judged from their operating characteristic (OC) curves as given in charts V-B to V-P and tables V-B-1 to V-P-1, and these should be consulted when choosing a sampling plan.

b) These curves are for normal inspection using the "s" method with a single specification limit, but they provide a good approximation to the case of a combined double specification limit.

c) These curves also provide a good approximation to the OC curves for the "σ" method (and the "R" method) sampling plans indexed by the same code letters and AQL, unless the sample size is small.

d) Separate OC curves are not given for tightened or reduced inspection, but the curves may be found from among those given for normal inspection (see tables VI-A and VI-B).

Section two : Choice of sampling plan

8 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge of both statistics and the product to be inspected. This section of this International Standard is intended to suggest to those responsible for specifying sampling plans the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

9 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account :

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more expensive in time and money per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the more precise information obtained indicates how good the product is; earlier warning will be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the example in 14.6.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859) and the standard plans in this International Standard is given in table I-B. It will be seen that the smallest samples are required by the "σ" method, used when the standard deviation of the process is known.

e) Inspection by variables is appropriate particularly in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.

g) A variables scheme becomes less suitable as the number of measurements to be taken on one item increases, as each characteristic has to be considered separately. It may be advantageous to apply "attributes" to the majority of the characteristics and "variables" to one or two of the more important requirements, for example proof load tests, safety and reliability requirements.

h) The use of this International Standard is only applicable when there is reason to believe that the distribution of measurements is normal. In case of any doubt, the responsible authority should be consulted.

NOTES

1 Tests for departure from normality are dealt with in section two of ISO 2854, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

2 ISO 5479 gives a more detailed study on the subject of normality tests.

10 Choice of method

If it is desired to apply inspection by variables, the next question is which method should be used, the "s" method or the "σ" method (or the "R" method).

The "σ" method is the most economical in sample size, but before this method may be employed, the value of σ has to be established.

In terms of sample size, the "s" method has a slight advantage over the "R" method, but the calculation of s involves more computation; the extent and difficulty of this is more apparent than real, especially if an electronic calculator is available. Methods of calculating s are given in annex A.

The "R" method (given in annex C) is simple to calculate, but requires a somewhat larger sample size for the same AQL. Also it has the undesirable property that for samples of size 10 or more the acceptability of a lot can depend upon how the sample is divided into sub-groups.

Initially, it will be necessary to begin with the "s" (or the "R" method), but if the quality is satisfactory, the standard switching rules will permit the responsible authority to commence reduced inspection and use a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the "σ" method ?

The size of the sample will generally be smaller and the acceptability criteria become simpler. (See clauses 15.2 and 15.3.) On the other hand, it will still be necessary to calculate s for record purposes and to keep the control charts up to date. (See clause 18.)

11 Choice of inspection level and AQL

In standard sampling plans, the inspection level in conjunction with the size of the lots and the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve given in one of the tables V-B to V-P shows the extent of the risk that is involved in such a plan.

The choice of the inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

12 Choice of sampling plan

12.1 Standard plans

The standard procedure can be used only when the production of lots is continuous.

The standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the "s" method, has been found in practice to produce workable sampling plans; but it assumes that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see clause 19), which quickly increase the severity of inspection and finally terminate it, if the quality of the process is worse than the AQL.

NOTE — It should also be remembered that the limiting quality is the quality which if offered for inspection would have a 10 % probability of acceptance. The actual risk taken by the consumer therefore also depends on the probability of goods of this low quality being offered for inspection.

However, if, in certain circumstances, the limiting quality has a higher priority than the sample size (for example, when only a limited number of lots are being produced), a suitable plan in this International Standard may be selected by using diagram A. The intersection of a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (approximately equal to AQL) will lie on, or under, a sloping line indexed with the sample size code letter of a standard plan which meets the specified requirements. This should be verified by inspecting the OC curve given in table V for this code letter and AQL.

If the lines intersect at a point above the line marked P (see diagram A), this implies that, for example, a sample of over 200 would be necessary for the s method and the specification cannot be met by the plans in this International Standard.

12.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. The choice is then to decide which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent, for, when any two have been chosen, the third follows.

NOTE — This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some constraints. If a special scheme is necessary it should be devised only with the assistance of a statistician experienced in quality control.

Section three : Operation of a variables sampling plan

13 Preliminary operations

Before starting inspection by variables, check

a) that the distribution can be considered to be normal and that production is considered to be continuous;

b) whether the "s" (or "R") method is to be used initially or whether the standard deviation is stable and known, in which case the "σ" method should be used;

c) that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) that the AQL has been designated and that it is one of the preferred AQLs for use with this International Standard. If it is not, the tables are not applicable;

e) if a double specification limit has to be met, whether the limits are separate or combined and, if the limits are separate, whether AQLs are determined for each limit.

14 Standard procedure for "s" method

14.1 Obtaining a plan

The procedure for obtaining a plan is as follows :

a) With the inspection level given (normally this will be II) and with the lot size, obtain the sample size code letter using table I-A.

b) With this code letter and the AQL, enter table II-A and obtain the sample size n and acceptability constant k.

c) Taking a random sample of this size, measure the characteristic x in each item and then calculate $\overline{x}$, the sample mean, and s, the estimated standard deviation (see annex A). If $\overline{x}$ is outside the specification limit, the lot can be judged unacceptable without calculating s. It may, however, be necessary to calculate s for record purposes.

14.2 Acceptability criteria for single or separate double specification limits

If single or separate specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \overline{x}}{s}$$

and/or

$$Q_L = \frac{\overline{x} - L}{s}$$

as appropriate,

then compare the quality statistic (Q_U and/or Q_L) with the acceptability constant k obtained from table II-A for normal inspection. If the appropriate quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if less, it is not acceptable.

Thus, if only the upper specification limit U is given, the lot is

acceptable if $Q_U \geqslant k$

not acceptable if $Q_U < k$

Or, if only the lower specification limit L is given, the lot is

acceptable if $Q_L \geqslant k$

not acceptable if $Q_L < k$

When both U and L are given (k values are different if the AQLs are different for the upper limit and the lower limit), the lot is

acceptable if both $Q_L \geqslant k_L$ and $Q_U \geqslant k_U$

not acceptable if either $Q_L < k_L$ or $Q_U < k_U$

Example

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From table I-A, the sample size code letter is F; from table II-A it is seen that a sample size of 10 is required and that the acceptability constant k is 1,41. Suppose the measurements are as follows : 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 55 °C; 50 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size : n	10
Sample mean $\overline{x}$: $\Sigma x/n$	54,9
Standard deviation s : $\sqrt{\sum_i (x_i - \overline{x})^2/(n-1)}$ (See A.1.2, annex A.)	3,414
Specification limit (upper) : U	60
$Q_U = (U - \overline{x})/s$	1,494
Acceptability constant : k (see table II-A)	1,41
Acceptability criterion : compare Q_U with k	1,494 > 1,41

The lot meets the acceptability criterion, and is therefore acceptable.

14.3 Graphical method for a single specification limit

When a graphical criterion is desired, draw the line

$$\overline{x} = U - k s \text{ (for an upper limit) or}$$

$$\overline{x} = L + k s \text{ (for a lower limit),}$$

as appropriate, on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. When the inspection concerns an upper specification limit the accept zone is the zone below the line. When a lower specification limit is considered, the accept zone is the zone above the line. Using the values of s and $\bar{x}$ calculated from the measurements obtained from a sample (see annex A for the calculation of s), plot the point $(s, \bar{x})$ on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

Example

Using the data given in the example in 14.2, mark the point $U = 60$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $- k$ [as $k = 1,41$, this means the line passes through points $(s = 1, \bar{x} = 58,59)$, $(s = 2, \bar{x} = 57,18)$, $(s = 3, \bar{x} = 55,77)$, etc.]. Select a suitable point and draw a straight line through it and $(s = 0, \bar{x} = 60)$, i.e. U. The accept zone is then the area under this line. The calculated values of s and $\bar{x}$ are 3,414 and 54,9. Plotting the point $(s, \bar{x})$, it will be seen from figure 1 that it lies just inside the accept zone; the lot is acceptable.

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot plot the point $(s, \bar{x})$ and decide whether or not the lot is acceptable.
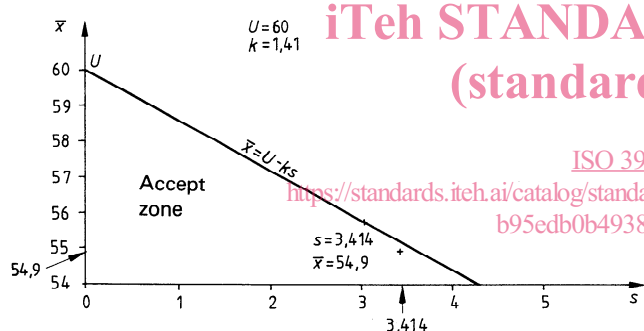

Figure 1 — Example of the use of an acceptance chart for a single specification limit: "*s*" method

14.4 Numerical method for single or separate double specification limits

Example

A certain pyrotechnic delay has a specified minimum delay time of 4,0 s and a maximum time of 9,0 s. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit and an AQL of 2,5 % applied to the upper limit. From table I-A it is seen that the sample size code letter is J; from table I-B it is seen that the sample size is 35 for the "*s*" method and from table II-A it is found that the upper and lower acceptability con-

stants are $k_U = 1,57$ and $k_L = 2,54$ respectively. Suppose the sample delay times are as follows:

6,95	6,04	6,68	6,63	6,65
6,40	6,44	6,34	6,04	6,15
6,44	7,15	6,70	6,59	6,51
6,35	7,17	6,83	6,25	6,96
6,80	5,84	6,15	6,25	6,57
6,52	6,59	6,86	6,57	6,91
6,29	6,63	6,70	6,67	6,67

Compliance with the acceptability criteria is to be determined.

Information needed	Value obtained
Sample size: n	35
Sample mean $\bar{x}$: $\Sigma x/n$	6,55 s
Sample standard deviation s: $\sqrt{\sum_i (x_i - \bar{x})^2/(n - 1)}$ (See A.1.2, annex A.)	0,31 s
Upper specification limit: U	9,0 s
$Q_U = (U - \bar{x})/s$	7,90
Acceptability constant: k_U (see table II-A)	1,57
Lower specification limit: L	4,0 s
$Q_L = (\bar{x} - L)/s$	8,23
Acceptability constant: k_L (see table II-A)	2,54
Acceptability criterion: is $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$?	7,90 > 1,57 and 8,23 > 2,54

The lot meets the acceptability criteria, and is acceptable.

14.5 Graphical method for separate double specification limits

When a graphical criterion is desired for separate double specification limits, draw the lines

$$\bar{x} = U - k_U s \text{ (for the upper limit) and}$$

$$\bar{x} = L + k_L s \text{ (for the lower limit)}$$

on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. Using the values of s and x calculated from the measurements obtained from a sample, plot the point $(s, \bar{x})$ on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

Example

Using the data given in the example in 14.4 mark the point $U = 9.0$ on the $\overline{x}$ (vertical) axis and draw a line through this point with a slope $- k_U$ [as $k_U = 1.57$, this means that the line will pass through the point $(s = 1, \overline{x} = 7.43)$]. Also mark the point $L = 4.0$ on the $\overline{x}$ axis and draw a line through this point with slope $+ k_L$ [as $k_L = 2.54$, this means that the line will pass through the point $(s = 1, \overline{x} = 6.54)$]. The accept zone is then the area bounded by these lines and the $\overline{x}$ axis. The calculated values of s and $\overline{x}$ are 0,31 and 6,55. Plotting the point $(s = 0.31, \overline{x} = 6.55)$ as shown in figure 2 it will be seen that it lies well within the accept zone and that the lot is acceptable.
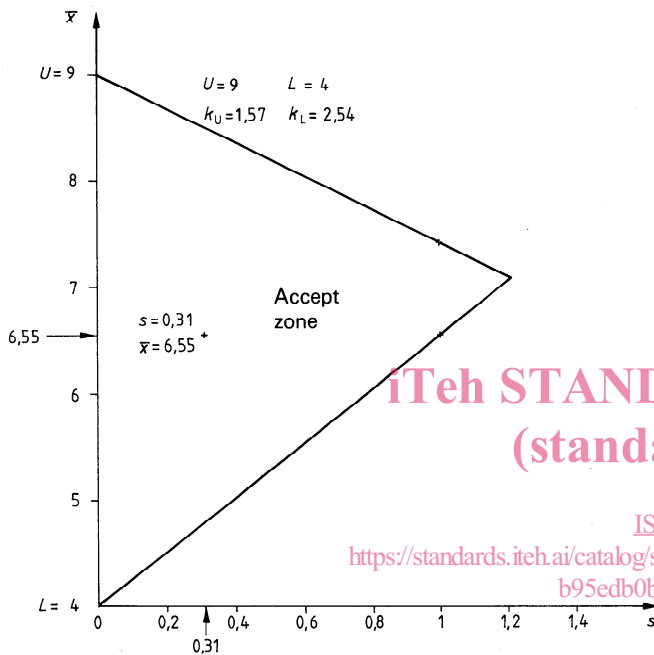

Figure 2 — Example of the use of an acceptance chart for separate double specification limits: "s" method

14.6 Acceptability criterion for a combined double specification limit

If a combined upper and lower specification limit has been given, it will be necessary to use a graphical method, unless s is greater than the value of the MSSD obtained from table IV-s, in which case the lot shall be immediately judged unacceptable.

Consult the chart in the "s" series which is labelled with the appropriate code letter and select the acceptance curve with the AQL specified for the two limits.

Then calculate the values of

$$\frac{s}{U - L} \text{ and } \frac{\overline{x} - L}{U - L}$$

and plot a point representing these values on the graph (or a copy of it). If the point lies inside the accept zone, the lot is acceptable; if outside, it is not acceptable.

For greater convenience it is recommended that, before the inspection operations begin, the acceptance curves for normal and tightened inspection be copied (or traced) onto graph paper. (A sheet of the special graph paper needed for this is given in annex D.) The scales should be adjusted so that s and $\overline{x}$ can be plotted directly (e.g. the upper limit is given instead of 1,0 and the lower limit instead of 0 on the $\overline{x}$ scale).

Then plot on the chart the values of s and $\overline{x}$ found from the sample and, if the point lies inside the accept zone, the lot is acceptable; if outside, it is not acceptable.

NOTE — For sample size code letters B and C (i.e. sample sizes 3 and 4), the accept zone is bounded by four straight lines: the $\overline{x}$ axis, the line $\overline{x} = U - k s$, a line parallel to the $\overline{x}$ axis through the MSSD (see table IV-s) and the line $\overline{x} = L + ks$. The value of k is obtained from the table II-A, II-B or II-C.

Example

The minimum temperature of operation for a certain device is specified as 60,0 °C and the maximum temperature as 70,0 °C. Production is in inspection lots of 96 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From table I-A, the sample size code letter is F; from table I-B it is seen that a sample size of 10 is required and from table IV-s that the value of f_s for the MSSD is 0,276. Suppose the measurements obtained are as follows : 63,5 °C; 62,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Sample size : n	10
Sample mean : $\overline{x} = \Sigma x/n$	64,57
Sample standard deviation s : $$\sqrt{\sum_i (x_i - \overline{x})^2/(n - 1)}$$ (See clause A.1.2, annex A.)	3,01
Value of f_s for MSSD (table IV-s)	0,276
MSSD $= f_s (U - L)$	2,76

The appropriate acceptance curve is taken from diagram s-F.

If, as on figure 3, the scales have been adjusted to the real measurements, plot the point $(s = 3.01, \overline{x} = 64.57)$. This lies outside the acceptance curve and the lot is not acceptable. It could have been judged not acceptable as soon as it was seen that s was greater than the MSSD.

If the scale of the chart has not been adjusted to the values of s and $\overline{x}$, the following additional calculations are required :

Standardized mean :
$(\overline{x} - L)/(U - L) = (64.57 - 60)/(70 - 60) = 0.457$

Standardized s : $s/(U - L) = 3.01/(70 - 60) = 0.301$

The point (0,301, 0,457) is plotted on figure 3.

As it lies outside the acceptance curve for AQL = 1,5 %, the lot is not acceptable.

NOTE — This lot is not acceptable even though all tested items **in the sample** are within specification limits.
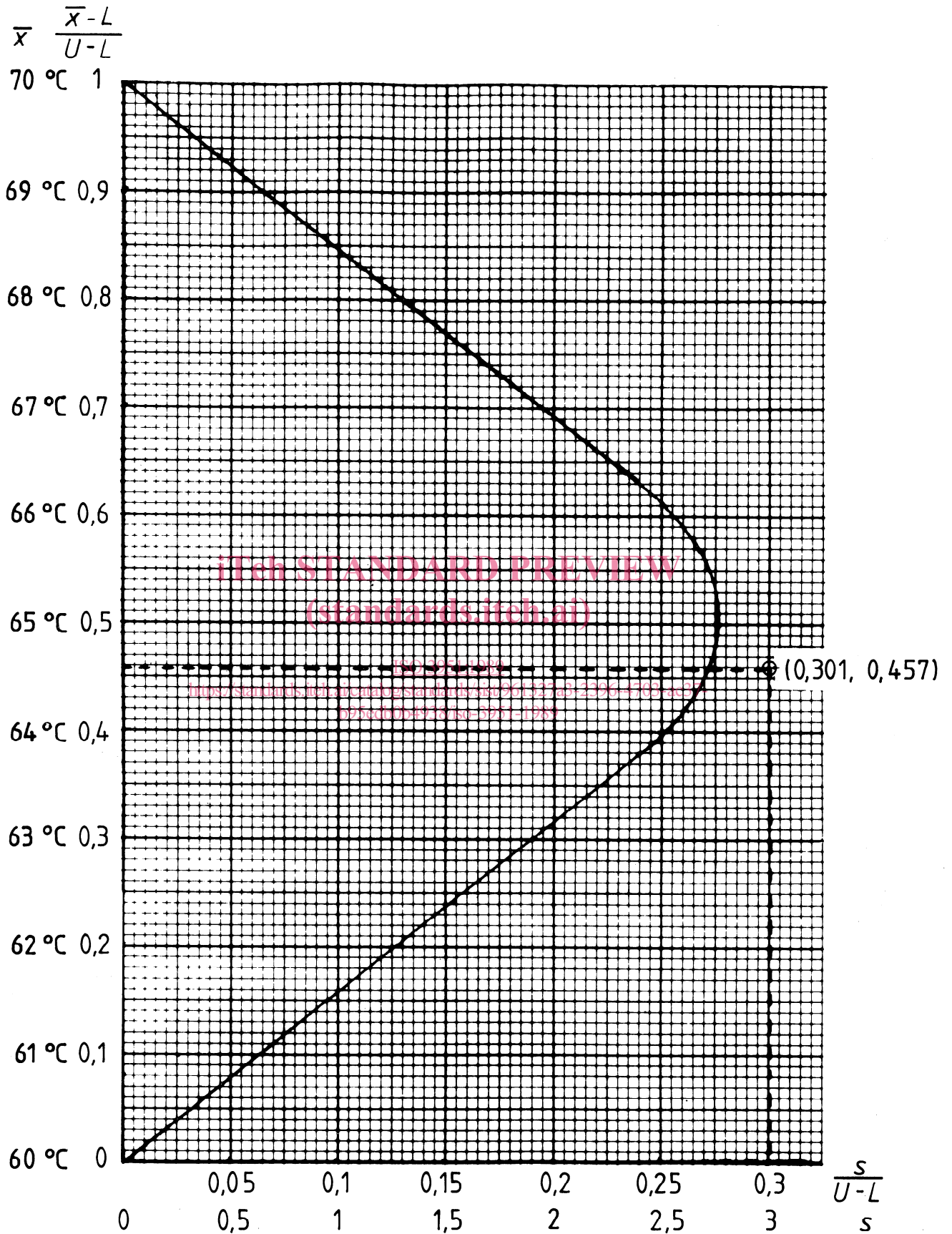
Figure 3 — Acceptance curve for sample size code letter F, AQL = 1,5 %, MSSD = 2,76 : "*s*" method