INTERNATIONAL

Designation: ~~E111 – 04 (Reapproved 2010)~~ E111 – 17

Standard Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus[1]

This standard is issued under the fixed designation E111; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the U.S. Department of Defense.

1. ~~Scope~~ Scope*

1.1 This test method covers the determination of Young's modulus, tangent modulus, and chord modulus of structural ~~materials.~~ materials, see Fig. 1. This test method is limited to materials in which and to temperatures and stresses at which creep is negligible compared to the strain produced immediately upon loading and to elastic behavior.

1.2 Because of experimental problems associated with the establishment of the origin of the stress-strain curve described in 8.1, the determination of the initial tangent modulus (that is, the slope of the stress-strain curve at the origin) and the secant modulus are outside the scope of this test method.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate ~~safety~~ safety, health, and ~~health~~environmental practices and determine the applicability of regulatory ~~requirements~~limitations prior to use.*

1.5 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

~~E4 Practices for Force Verification of Testing Machines~~
E6 Terminology Relating to Methods of Mechanical Testing
~~E8~~E8/E8M Test Methods for Tension Testing of Metallic Materials
E9 Test Methods of Compression Testing of Metallic Materials at Room Temperature
E21 Test Methods for Elevated Temperature Tension Tests of Metallic Materials
E83 Practice for Verification and Classification of Extensometer Systems
~~E231~~E177 ~~Method~~Practice for ~~Static Determination of Young's Modulus of Metals at Low and Elevated Temperatures~~Use of the Terms Precision and Bias in ASTM Test Methods ~~(Withdrawn 1985)~~
E1012 Practice for Verification of Testing Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application

2.2 *General Considerations*—While certain portions of the standards and practices listed are applicable and should be referred to, the precision required in this test method is higher than that required in general testing.

3. Terminology

3.1 *Definitions:* Terms common to mechanical testing.

3.1.1 ~~*accuracy*—the degree of agreement between an accepted standard value of Young's~~The definitions of mechanical testing terms that appear in Terminology E6 ~~modulus (the average of many observations made according to this method, preferably by many observers) and the value determined.~~apply to this test method. These terms include initial tangent modulus, secant modulus, gauge length, yield strength, tensile strength, stress-strain diagram, and extensometer.

[1] This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.
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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

***A Summary of Changes section appears at the end of this standard**

~~3.1.1.1 Increased accuracy is associated with decreased bias relative to the accepted standard value; two methods with equal bias relative to the accepted standard value have equal accuracy even if one method is more precise than the other. See also *bias* and *precision*.~~

~~3.1.1.2 The accepted standard value is the value of Young's modulus for the statistical universe being sampled using this method. When an accepted standard value is not available, accuracy cannot be established.~~

3.1.2 The terms accuracy, precision, and bias are used as defined in Practice E177.

3.1.3 ~~*bias, statistical*—a constant or systematic error in~~ In addition, the following common terms that appear in the Terminology E6~~test results.~~ apply to this test method.

~~3.1.2.1 Bias can exist between the accepted standard value and a test result obtained from this test method, or between two test results obtained from this test method, for example, between operators or between laboratories.~~

3.1.4 ~~*precision*—~~*chord modulus—*the ~~degree of mutual agreement among individual measurements made under prescribed like conditions.~~slope of the chord drawn between any two specified points on the stress-strain curve below the elastic limit of the material.

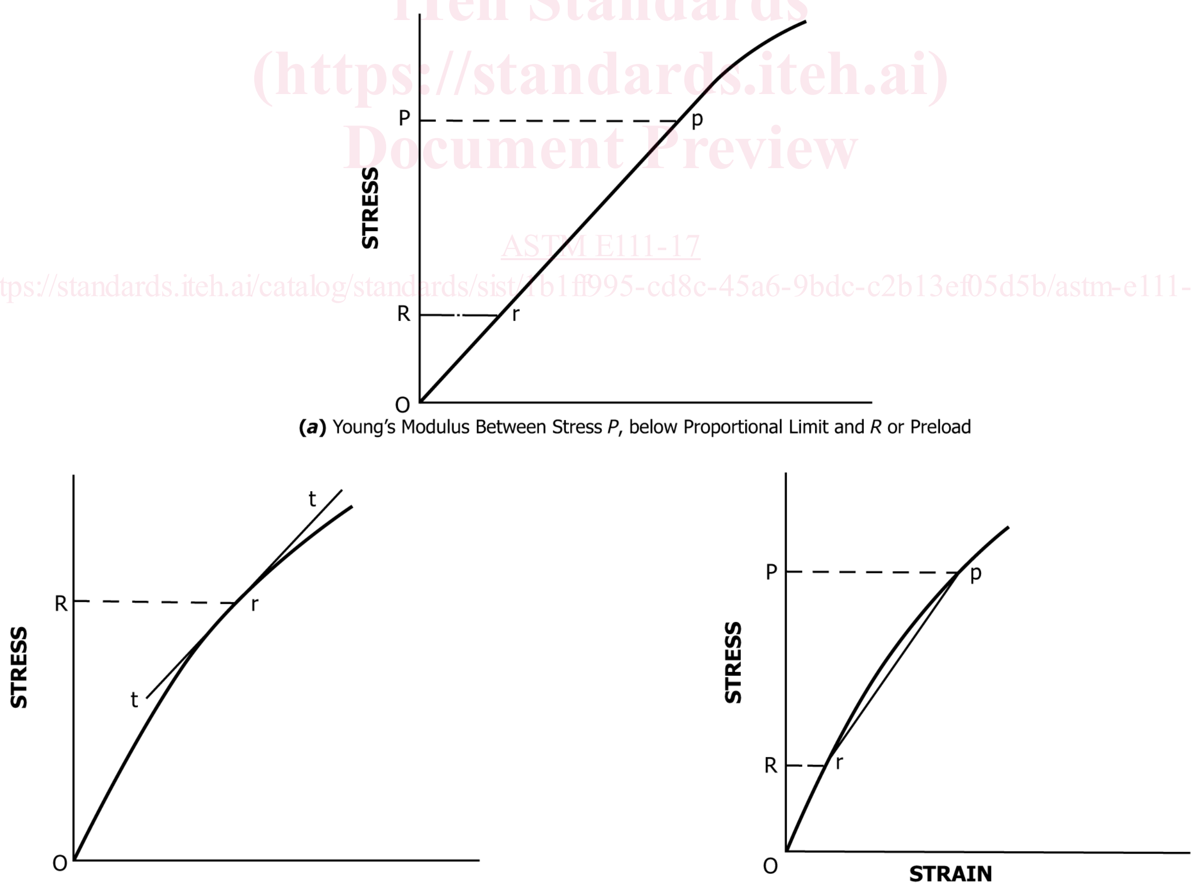
~~3.1.4 *Young's modulus*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit (see Fig. 1a).~~

~~3.1.4.1 *tangent modulus*—the slope of the stress-strain curve at any specified stress or strain (see Fig. 1b).~~

~~3.1.4.2 *chord modulus*—the slope of the chord drawn between any two specified points on the stress-strain curve (see Fig. 1c).~~

3.1.5 *elastic limit [FL^2], n—*the greatest stress that a material is capable of sustaining without any permanent strain remaining upon complete release of the stress.

3.1.5.1 *Discussion—*



(***a***) Young's Modulus Between Stress *P*, below Proportional Limit and *R* or Preload

(***b***) Tangent Modulus at Any Stress *R*

(***c***) Chord Modulus Between Any Two Stresses *P* and *R*

FIG. 1 Stress-Strain Diagrams Showing Straight Lines Corresponding to (*a*) Young's Modulus, (*b*) Tangent Modulus, and (*c*) Chord Modulus

Due to practical considerations in determining the elastic limit, measurements of strain using a small force, rather than zero force, are usually taken as the initial and final reference.

3.1.6 *indicated temperature, n*—the temperature indicated by a temperature measuring device using good pyrometric practice.

3.1.7 *nominal temperature, n*—the intended test temperature.

3.1.8 *proportional limit [FL^2], n*—the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law).

3.1.9 *tangent modulus*—the slope of the stress-strain curve at any specified stress or strain.

3.1.10 *Young's modulus*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit.

~~3.2 For definitions of other terms used in this test method, refer to Terminology E6.~~

4. Summary of Test Method

4.1 A uniaxial force is applied to the test specimen and the force and strain are measured, either incrementally or continuously. The axial stress is determined by dividing the indicated force by the specimen's original cross-sectional area. The appropriate slope is then calculated from the stress-strain curve, which may be derived under conditions of either increasing or decreasing forces (increasing from preload to maximum applied force or decreasing from maximum applied force to preload).

5. Significance and Use

5.1 The value of Young's modulus is a material property useful in design for calculating compliance of structural materials that follow Hooke's law when subjected to uniaxial loading (that is, the strain is proportional to the applied force).

5.2 For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord modulus is useful in estimating the change in strain for a specified range in stress.

5.3 Since for many materials, Young's modulus in tension is different from Young's modulus in compression, it shall be derived from test data obtained in the stress mode of interest.

5.4 The accuracy and precision of apparatus, test specimens, and procedural steps should be such as to conform to the material being tested and to a reference standard, if available.

5.5 Precise determination of Young's modulus requires due regard for the numerous variables that may affect such determinations. These include (*1*) characteristics of the specimen such as orientation of grains relative to the direction of the stress, grain size, residual stress, previous strain history, dimensions, and eccentricity; (*2*) testing conditions, such as alignment of the specimen, speed of testing, temperature, temperature variations, condition of test equipment, ratio of error in applied force to the range in force values, and ratio of error in extension measurement to the range in extension values used in the determination; and (*3*) interpretation of data (see Section 9).

5.6 When the modulus determination is made at strains in excess of 0.25 %, correction ~~should~~shall be made for changes in cross-sectional area and ~~gage~~gauge length, by substituting the instantaneous cross section and instantaneous ~~gage~~gauge length for the original values.

5.7 Compression results may be affected by barreling (see Test Methods E9). Strain measurements should therefore be made in the specimen region where such effects are minimal.

6. Apparatus

6.1 *Dead Weights*—Calibrated dead weights may be used. Any cumulative errors in the dead weights or the dead weight loading system shall not exceed 0.1 %.

6.2 *Testing Machines*—In determining the suitability of a testing machine, the machine shall be calibrated under conditions approximating those under which the determination is made. Corrections may be applied to correct for proven systematic errors.

6.3 *Loading Fixtures*—~~Grips and other devices for obtaining and maintaining axial alignment are shown in Test Methods~~ Loading ~~E8 and E9. It is essential that the loading fixtures~~ fixtures shall be properly designed and maintained. ~~Procedures for verifying the alignment are described in detail in Practice E1012.~~ The allowable bending as defined in Practice E1012 shall not exceed 5 %.

NOTE 1— Grips and other devices for obtaining and maintaining axial alignment are shown in Test Methods E8/E8M and E9. Procedures for verifying the alignment are described in detail in Practice E1012.

6.4 *Extensometers*—Class B-1 or better extensometers as described in Practice E83 shall be used. Corrections may be applied for proven systematic errors in strain and are not considered as a change in class of the extensometer. Either an averaging extensometer or the average of the strain measured by at least two extensometers arranged at equal intervals around the cross section shall be used. If two extensometers are used on other than round sections, they shall be mounted at ends of an axis of symmetry of the section. If a force-strain recorder, strain-transfer device, or strain follower is used with the extensometer, they shall be calibrated as a unit in the same manner in which they are used for determination of Young's modulus. The ~~gage~~gauge length shall be determined with an accuracy consistent with the precision expected from the modulus determination and from the extensometer.

NOTE 2—The accuracy of the modulus determination depends on the precision of the strain measurement. The latter can be improved by increasing the ~~gage~~gauge length. This may, however, present problems in maintaining specimen tolerances and temperature uniformity.

6.5 *Furnaces or Heating Devices*—When determining Young's modulus at elevated temperature, the furnace or heating device used shall be capable of maintaining a uniform indicated temperature in the reduced section of the test specimen so that a variation of not more than ±1.5°C for nominal temperatures up to and including 900°C, and not more than ±3.0°C for temperatures above 900°C, occurs. (Heating by self-resistance ~~is~~shall not ~~accepted.)~~ be used.) Minimize indicated temperature variations and control changes within the allowable ~~limits, since differences in thermal expansion between specimen and extensometer parts may cause significant errors in apparent strain.~~ limits. An instrumented sample representative of the real test ~~will~~ may be used demonstrate that the setup meets the above capabilities.

NOTE 3—Differences in thermal expansion between specimen and extensometer parts can cause significant errors in apparent strain.

6.6 *Low-Temperature Baths and Refrigeration Equipment*—When determining Young's modulus at ~~low temperatures,~~ temperatures below room temperature, an appropriate low-temperature bath or refrigeration system ~~is required~~ shall be used to maintain the specimen at the ~~specified~~nominal temperature during testing. For a low-temperature bath, the lower tension rod or adapter may pass through the bottom of an insulated container and be welded or fastened to it to prevent leakage. ~~For temperatures to about − 80°C, chipped dry ice may be used to cool an organic solvent such as ethyl alcohol in the low-temperature bath. Other organic solvents having lower solidification temperatures, such as methylcyclohexane or isopentane, may be cooled with liquid nitrogen to temperatures lower than − 80°C. Liquid nitrogen may be used to achieve a testing temperature of − 196°C. Lower testing temperatures may be achieved with liquid hydrogen and liquid helium, but special containers or cryostats are required to provide for minimum heat leakage to permit efficient use of these coolants. When liquid hydrogen is used, special precautions must be taken to avoid explosions of hydrogen gas and air mixtures. If refrigeration equipment is used to cool the specimens with air as the cooling medium, it is desirable to have forced air circulation to provide uniform cooling.~~

NOTE 4—~~At low temperatures, when using a coolant bath, immersion-type extensometers are recommended.~~For nominal temperatures to about − 80°C, chipped dry ice that cools an organic solvent such as ethyl alcohol in the low-temperature bath is suitable. Other organic solvents having lower solidification temperatures, such as methylcyclohexane or isopentane, cooled with liquid nitrogen are useful at temperatures lower than − 80°C. Liquid nitrogen can be used to achieve a nominal temperature of − 196°C. Lower nominal temperatures are possible with liquid hydrogen and liquid helium, with special containers or cryostats to minimize heat leakage and to permit efficient use of these coolants. Liquid hydrogen can produce explosive mixtures of hydrogen gas and air. If refrigeration equipment is used to cool the specimens with air as the cooling medium, it is desirable to have forced air circulation to provide uniform cooling.

6.6.1 At low temperatures, when using a coolant bath, immersion-type extensometers should be used.

6.7 Temperature measuring, controlling, and recording instruments shall be calibrated periodically against a secondary standard, such as a precision potentiometer. Lead-wire error should be checked with the lead wires in place as they normally are used.

7. Test Specimens

7.1 *Selection and Preparation of Specimens*—Special care shall be taken to obtain representative specimens ~~which~~that are straight and uniform in cross section. If straightening of the material for the specimen is required, ~~the~~then resultant residual stresses shall be removed by a subsequent stress relief ~~heat treatment which~~annealing procedure that shall be reported with the test results.

7.2 *Dimensions*—The ~~recommended~~ specimen length (and fillet radius in the case of tension specimens) ~~is~~ should be greater than the minimum requirements for general-purpose specimens. In addition, the ratio of length to cross section of compression specimens should be such as to avoid buckling (see Test Methods E9).

NOTE 5—For examples of tension and compression specimens, see Test Methods ~~E8~~E8/E8M and E9.

7.3 For tension specimens, the center lines of the grip sections and of the threads of threaded-end specimens shall be concentric with the center line of the ~~gage~~gauge section within close tolerances in order to obtain the degree of alignment required. If pin-loaded sheet-type specimens are used, the centers of the gripping holes shall be not more than 0.005 times the width of the ~~gage~~gauge section from its center line. For sheet-type specimens, ~~it may be necessary to provide~~ small tabs or notches for attaching the ~~extensometer.~~extensometer may be used.

NOTE 6—The effect of eccentric loading ~~may~~can be illustrated by calculating the bending moment and stress thus added. For a standard 12.5-mm diameter specimen, the stress increase is 1.5 % for each 0.025 mm of eccentricity. This error increases to about 2.5 % per 0.025 mm for a 9-mm diameter specimen and to about 3.2 % per 0.025 mm for a 6-mm diameter specimen.

7.4 The length of the reduced section of tension specimens shall exceed the ~~gage~~gauge length by at least twice the diameter or twice the width. The length of compression specimens shall be in accordance with Test Methods E9, and all specimens shall have a uniform cross-sectional area throughout the ~~gage~~gauge length.

~~NOTE 5—If a general-purpose tension specimen such as those shown in Test Methods E8, having a small amount of taper in the reduced section is used, the average cross-sectional area for the gage length should be used in computing stress.~~

7.4.1 If a general-purpose tension specimen such as those shown in Test Methods E8/E8M, having a small amount of taper in the reduced section is used, the average cross-sectional area for the gauge length should be used in computing stress.

7.5 For compression specimens, the ends shall be flat, parallel and perpendicular to the lateral surfaces as specified in Test Methods E9.

7.6 ~~This test method is intended to produce intrinsic materials properties. Therefore, the specimen needs to~~ The specimen shall be free of residual ~~stresses, which may require~~ stresses. The specimen may be subjected to an annealing procedure ~~at $T_m/3$ for 30 min~~ to relieve the ~~stresses in the material (where T~~residual stresses. ~~$_m$ is the melting point of the material in K). The procedure must be mentioned in the report section.~~ If the intent of the test is to verify the performance of a product, the ~~heat treatment~~ annealing procedure may be omitted. ~~Record~~Report the condition of the material tested, including any ~~heat treatment, in the test report.~~annealing procedure.

NOTE 7—An annealing procedure at $T_m/3$ for 30 min to relieve the stresses in the material (where T_m is the melting point of the material in K) has been used successfully.

8. Procedure

8.1 ~~For most loading systems and test specimens, effects of backlash, specimen curvature, initial grip alignment, etc., introduce significant errors in the extensometer output when applying a small force to the test specimen. Measurements shall therefore~~ Measurements shall be made from a small force or preload, known to be high enough to minimize ~~these effects,~~ extensomer output errors, to some higher applied force, still within either the proportional limit or elastic limit of the material. For linearly elastic materials, the slope of the straight-line portion of the stress-strain curve shall be established between the preload and the proportional limit to define Young's modulus. If the actual stress-strain curve is desired, this line ~~can~~may appropriately be shifted along the strain axis to coincide with the origin. For nonlinearly elastic materials the tangent or chord modulus may be established between the appropriate stress values on the stress strain curve.

NOTE 8—For most loading systems and test specimens, effects of backlash, specimen curvature, initial grip alignment, etc., introduce significant errors in the extensometer output when applying a small force to the test specimen.

8.2 *Measurement of Specimens*—~~Make the measurements for the determination of average cross-sectional area~~ Measure specimen dimensions at the ends of the ~~gage~~gauge length and at least at one intermediate ~~location. Use any means of measuring that is capable of producing area calculations~~ location to within 1 % accuracy.

8.3 *Alignment*—~~Take special care to ensure~~ Ensure as nearly axial loading as possible. The strain increments between the initial-load and the final-load measurement on opposite sides of the specimens should not differ from the average by more than 3 %.

8.4 *Soaking Time of Specimens at Testing Temperature*—After the specimen to be tested has reached the ~~testing~~nominal temperature, maintain the specimen at the ~~testing~~nominal temperature for a sufficient length of time to attain equilibrium conditions of the specimen and extensometer before applying force. Report the time to attain ~~test~~ the nominal temperature and the time at the nominal temperature before applying force.

NOTE 9—~~The recommended soak time at the test temperature is 1 hour per 25 mm (1 hour/inch) of specimen thickness or diameter.~~
The recommended soak time at the nominal temperature is 1 hour per 25 mm (1 hour/inch) of specimen thickness or diameter. If the temperature of the system is not uniform by the time loading of the specimen is started, variations in thermal expansion will be reflected in the modulus line. Furthermore, fluctuations in temperature of the extensometer extensions during testing which result from cycling of the furnace temperature or changes in the level of the cooling bath ~~may~~can also affect the slope of the modulus line.

8.5 *Speed of Testing*—The speed of testing shall be low enough that thermal effects of adiabatic expansion or contraction are negligible and that accurate determination of force and extension is possible, yet the speed shall be high enough that creep will be negligible. In loading with dead weights, avoid temporary overloading due to inertia of the weights. The strain rate should be reported.

8.6 *~~Speed~~Number of ~~Testing~~—runs*—~~The speed of testing shall be low enough that thermal effects of adiabatic expansion or contraction are negligible and that accurate determination of load and extension is possible, yet the speed shall be high enough that creep will be negligible. In loading with dead weights, avoid temporary overloading due to inertia of the weights. The strain rate should be reported.~~A minimum of three runs should be made for each specimen. Exercise care to not exceed the proportional limit in the case of Young's modulus, and the elastic limit in the case of the tangent or chord modulus. Report each of the three values or the average along with the method for determining them.

~~NOTE 7—A minimum of three runs are recommended for each specimen. Care must be taken not to exceed the proportional limit in the case of Young's modulus, and the elastic limit in the case of the tangent or chord modulus. Report each of the three values or the average alomg with the method for getting them.~~

8.6.1 Young's modulus, tangent modulus, or chord modulus for a given specimen may be determined along with yield strength and tensile strength using a single loading cycle. If modulus values are determined this way, report that only one loading cycle was used. Three cycles within the elastic region as recommended in 8.6, may be used to determine the modulus, before straining the specimen into the plastic range to determine yield and tensile strengths.

~~NOTE 8—It is recognized that Young's modulus, tangent modulus, or chord modulus for a given specimen may be determined along with yield strength and tensile strength using a single loading cycle. If modulus values are determined this way, report that only one loading cycle was used. Three cycles within the elastic region as recommended in Note 7, can be used to determine the modulus, before straining the specimen into the plastic range to determine yield and tensile strengths.~~

8.7 *Temperature Control*—~~The average~~ Keep the variation of the indicated temperature from the actual tempertaure as small as is practical through good practice and precise control. The average indicated temperature over the specimen ~~gage~~gauge length shall not deviate from the ~~indicated~~ nominal ~~test~~ temperature by more than ±2°C. In elevated-temperature tests, indicated temperature variations along the ~~gage~~gauge length of the specimen shall not exceed the following limits: up to and including 900 ± 1.5°C, above 900 ± 3.0°C. (See 6.5.) The test must be performed with the same setup and under similar conditions as those of the instrumented test described in 6.5. Temperature changes should be minimized while making strain measurements.

~~NOTE 9—The terms "indicated nominal temperature" or "indicated temperature" mean the temperature that is indicated by the temperature-measuring device using good pyrometric practice.[5]~~

NOTE 10—~~It is recognized that~~ The actual temperatures ~~may~~can vary more than the indicated temperatures. ~~The use of "indicated temperatures" for the limits of permissible variation in temperature are not to be construed as minimizing the importance of good practice and precise temperature control. All laboratories are obligated to keep the variation of indicated temperature from the actual temperature as small as is practical.~~ Temperature changes during the test, within the allowable limits, can cause significant strain errors due to differences in thermal expansion of the test specimen and extensometer parts. ~~Temperature changes should be minimized while making strain measurements.~~

8.8 In low-temperature testing in which the bath is cooled with dry ice or in which a refrigeration system is used, the indicated temperature of the medium around the specimen shall be maintained at temperatures within 1.5°C of the specified temperature. Bath indicated temperatures or the temperature of circulating air from a refrigeration system may be ~~done~~measured with a ~~copper-constantan~~ thermocouple or a suitable thermometer. If the specimen is submerged in a bath at the boiling point of the bath, allow sufficient soaking time (see Note ~~6~~9) ~~must be allowed~~ to provide equilibrium conditions. Specimens tested in boiling liquids ~~must~~shall meet the ~~temperature control~~ temperature-control requirements specified in ~~8.6~~8.7.

~~8.7.1 **Caution**—The boiling point of a commercial liquid gas may not be the same as the published temperature for the pure liquid gas.~~

NOTE 11—The boiling point of a commercial liquid gas may not be the same as the published temperature for the pure liquid gas.

8.9 *Temperature Measurement*—The method of temperature measurement ~~must~~shall be sufficiently sensitive and reliable to ensure that the temperature of the specimen is within the limits specified in ~~8.6~~8.7 and ~~8.7~~8.8. ~~Thermocouples in conjunction with potentiometers or millivolt meters are generally used to measure temperatures. A discussion of temperature measurement and the use of thermocouples is given in Test Methods E21.~~

NOTE 12—Thermocouples in conjunction with potentiometers or millivolt meters are generally used to measure temperatures. A discussion of temperature measurement and the use of thermocouples is given in Test Methods E21.

9. Interpretation of Data

9.1 When the modulus determination is made at strains in excess of 0.25 %, correct for changes in cross-sectional area and gauge length by substituting the instantaneous cross section and instantaneous gauge length for the original values.

9.2 ~~If a plot of load-versus-extension (force versus elongation) is obtained by means of an autographic recorder, the value for Young's modulus is obtained by determining the slope of the line for forces less than the force corresponding to the proportional limit. Choice of the lower force point depends on the limitations set forth in 8.1. Young's modulus is calculated from the force increment and corresponding extension increment, between two points on the line as far apart as possible, by use of the following equation:~~

$$E = \left(\frac{\Delta_p}{A_o}\right) / \left(\frac{\Delta_c}{L_o}\right) \quad (1)$$

~~where:~~

~~Δ_p = force increment,~~
~~A_o = original cross-sectional area,~~
~~Δ_c = extension increment, and~~
~~L_o = original gage length.~~

Graphical Data Method:

~~The precision of the value obtained for Young's modulus will depend upon the precision of each of the values used in the calculation. It is suggested that the report include an estimate of the precision of the reported value of Young's modulus based on the summation of the precisions of the respective values. When the modulus determination is made at strains in excess of 0.25 %, corrections shall be made for changes in cross-sectional area and gage length by substituting the instantaneous cross section and instantaneous gage length for the original values.~~

9.2.1 If a plot of force-versus-extension (force versus elongation) is obtained by graphically, compute the value for Young's modulus is obtained by determining the slope of the line for forces less than the force corresponding to the proportional limit.