



Designation: E111 – 17

# Standard Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus<sup>1</sup>

This standard is issued under the fixed designation E111; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reappraisal. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reappraisal.

*This standard has been approved for use by agencies of the U.S. Department of Defense.*

## 1. Scope\*

1.1 This test method covers the determination of Young's modulus, tangent modulus, and chord modulus of structural materials, see Fig. 1. This test method is limited to materials in which and to temperatures and stresses at which creep is negligible compared to the strain produced immediately upon loading and to elastic behavior.

1.2 Because of experimental problems associated with the establishment of the origin of the stress-strain curve described in 8.1, the determination of the initial tangent modulus (that is, the slope of the stress-strain curve at the origin) and the secant modulus are outside the scope of this test method.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.5 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

## 2. Referenced Documents

2.1 *ASTM Standards:*<sup>2</sup>

**E6 Terminology Relating to Methods of Mechanical Testing**  
**E8/E8M Test Methods for Tension Testing of Metallic Materials**

<sup>1</sup> This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

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<sup>2</sup> For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

- E9 Test Methods of Compression Testing of Metallic Materials at Room Temperature**
- E21 Test Methods for Elevated Temperature Tension Tests of Metallic Materials**
- E83 Practice for Verification and Classification of Extensometer Systems**
- E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods**
- E1012 Practice for Verification of Testing Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application**

2.2 *General Considerations*—While certain portions of the standards and practices listed are applicable and should be referred to, the precision required in this test method is higher than that required in general testing.

## 3. Terminology

3.1 *Definitions:* Terms common to mechanical testing.

3.1.1 The definitions of mechanical testing terms that appear in Terminology E6 apply to this test method. These terms include initial tangent modulus, secant modulus, gauge length, yield strength, tensile strength, stress-strain diagram, and extensometer.

3.1.2 The terms accuracy, precision, and bias are used as defined in Practice E177.

3.1.3 In addition, the following common terms that appear in the Terminology E6 apply to this test method.

3.1.4 *chord modulus*—the slope of the chord drawn between any two specified points on the stress-strain curve below the elastic limit of the material.

3.1.5 *elastic limit* [ $FL^2$ ],  $n$ —the greatest stress that a material is capable of sustaining without any permanent strain remaining upon complete release of the stress.

3.1.5.1 *Discussion*—Due to practical considerations in determining the elastic limit, measurements of strain using a small force, rather than zero force, are usually taken as the initial and final reference.

3.1.6 *indicated temperature*,  $n$ —the temperature indicated by a temperature measuring device using good pyrometric practice.

\*A Summary of Changes section appears at the end of this standard

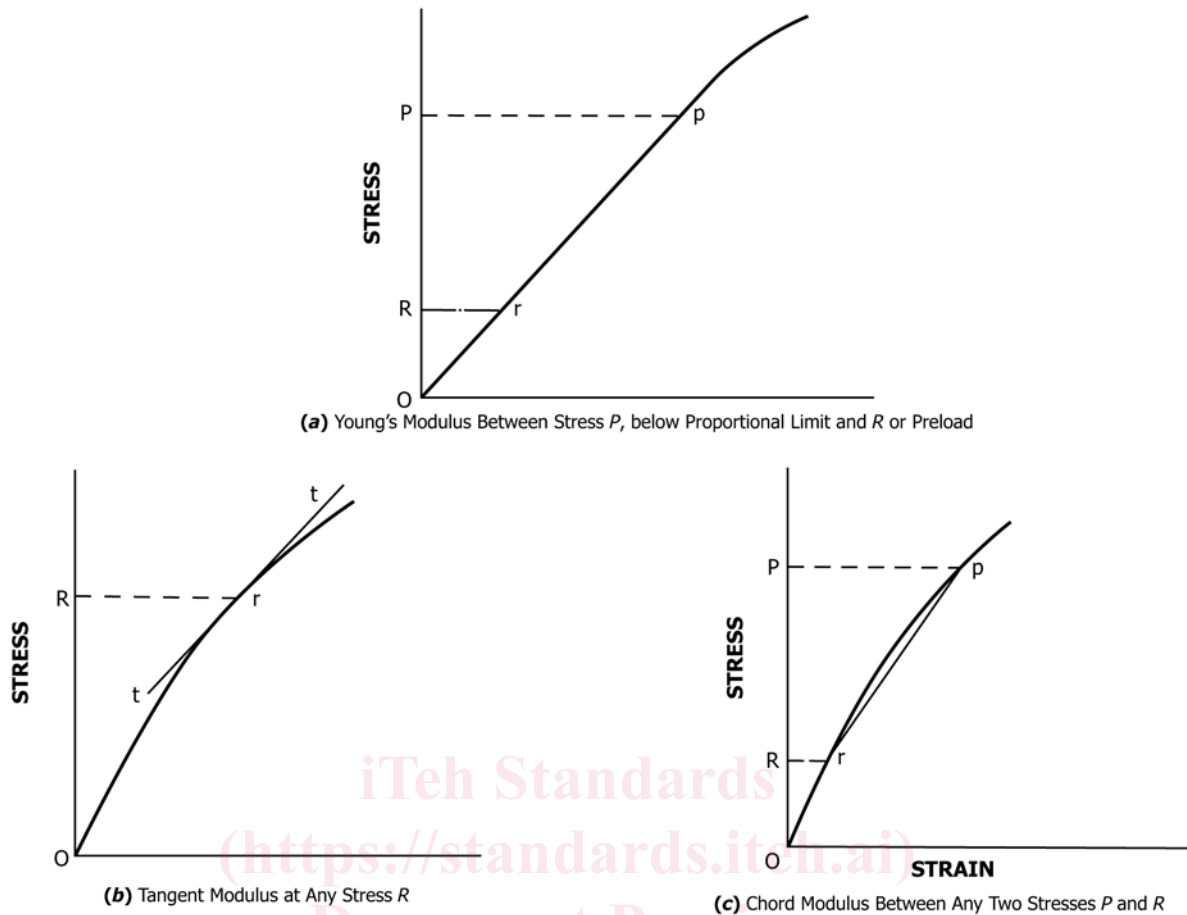


FIG. 1 Stress-Strain Diagrams Showing Straight Lines Corresponding to (a) Young's Modulus, (b) Tangent Modulus, and (c) Chord Modulus

3.1.7 *nominal temperature,  $n$* —the intended test temperature.

3.1.8 *proportional limit  $[FL^2]$ ,  $n$* —the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law).

3.1.9 *tangent modulus*—the slope of the stress-strain curve at any specified stress or strain.

3.1.10 *Young's modulus*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit.

#### 4. Summary of Test Method

4.1 A uniaxial force is applied to the test specimen and the force and strain are measured, either incrementally or continuously. The axial stress is determined by dividing the indicated force by the specimen's original cross-sectional area. The appropriate slope is then calculated from the stress-strain curve, which may be derived under conditions of either increasing or decreasing forces (increasing from preload to maximum applied force or decreasing from maximum applied force to preload).

#### 5. Significance and Use

5.1 The value of Young's modulus is a material property useful in design for calculating compliance of structural

materials that follow Hooke's law when subjected to uniaxial loading (that is, the strain is proportional to the applied force).

5.2 For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord modulus is useful in estimating the change in strain for a specified range in stress.

5.3 Since for many materials, Young's modulus in tension is different from Young's modulus in compression, it shall be derived from test data obtained in the stress mode of interest.

5.4 The accuracy and precision of apparatus, test specimens, and procedural steps should be such as to conform to the material being tested and to a reference standard, if available.

5.5 Precise determination of Young's modulus requires due regard for the numerous variables that may affect such determinations. These include (1) characteristics of the specimen such as orientation of grains relative to the direction of the stress, grain size, residual stress, previous strain history, dimensions, and eccentricity; (2) testing conditions, such as alignment of the specimen, speed of testing, temperature, temperature variations, condition of test equipment, ratio of error in applied force to the range in force values, and ratio of error in extension measurement to the range in extension values used in the determination; and (3) interpretation of data (see Section 9).

5.6 When the modulus determination is made at strains in excess of 0.25 %, correction shall be made for changes in cross-sectional area and gauge length, by substituting the instantaneous cross section and instantaneous gauge length for the original values.

5.7 Compression results may be affected by barreling (see Test Methods E9). Strain measurements should therefore be made in the specimen region where such effects are minimal.

## 6. Apparatus

6.1 *Dead Weights*—Calibrated dead weights may be used. Any cumulative errors in the dead weights or the dead weight loading system shall not exceed 0.1 %.

6.2 *Testing Machines*—In determining the suitability of a testing machine, the machine shall be calibrated under conditions approximating those under which the determination is made. Corrections may be applied to correct for proven systematic errors.

6.3 *Loading Fixtures*—Loading fixtures shall be properly designed and maintained. The allowable bending as defined in Practice E1012 shall not exceed 5 %.

NOTE 1—Grips and other devices for obtaining and maintaining axial alignment are shown in Test Methods E8/E8M and E9. Procedures for verifying the alignment are described in detail in Practice E1012.

6.4 *Extensometers*—Class B-1 or better extensometers as described in Practice E83 shall be used. Corrections may be applied for proven systematic errors in strain and are not considered as a change in class of the extensometer. Either an averaging extensometer or the average of the strain measured by at least two extensometers arranged at equal intervals around the cross section shall be used. If two extensometers are used on other than round sections, they shall be mounted at ends of an axis of symmetry of the section. If a force-strain recorder, strain-transfer device, or strain follower is used with the extensometer, they shall be calibrated as a unit in the same manner in which they are used for determination of Young's modulus. The gauge length shall be determined with an accuracy consistent with the precision expected from the modulus determination and from the extensometer.

NOTE 2—The accuracy of the modulus determination depends on the precision of the strain measurement. The latter can be improved by increasing the gauge length. This may, however, present problems in maintaining specimen tolerances and temperature uniformity.

6.5 *Furnaces or Heating Devices*—When determining Young's modulus at elevated temperature, the furnace or heating device used shall be capable of maintaining a uniform indicated temperature in the reduced section of the test specimen so that a variation of not more than  $\pm 1.5^\circ\text{C}$  for nominal temperatures up to and including  $900^\circ\text{C}$ , and not more than  $\pm 3.0^\circ\text{C}$  for temperatures above  $900^\circ\text{C}$ , occurs. (Heating by self-resistance shall not be used.) Minimize indicated temperature variations and control changes within the allowable limits. An instrumented sample representative of the real test may be used demonstrate that the setup meets the above capabilities.

NOTE 3—Differences in thermal expansion between specimen and extensometer parts can cause significant errors in apparent strain.

6.6 *Low-Temperature Baths and Refrigeration Equipment*—When determining Young's modulus at temperatures below room temperature, an appropriate low-temperature bath or refrigeration system shall be used to maintain the specimen at the nominal temperature during testing. For a low-temperature bath, the lower tension rod or adapter may pass through the bottom of an insulated container and be welded or fastened to it to prevent leakage.

NOTE 4—For nominal temperatures to about  $-80^\circ\text{C}$ , chipped dry ice that cools an organic solvent such as ethyl alcohol in the low-temperature bath is suitable. Other organic solvents having lower solidification temperatures, such as methylcyclohexane or isopentane, cooled with liquid nitrogen are useful at temperatures lower than  $-80^\circ\text{C}$ . Liquid nitrogen can be used to achieve a nominal temperature of  $-196^\circ\text{C}$ . Lower nominal temperatures are possible with liquid hydrogen and liquid helium, with special containers or cryostats to minimize heat leakage and to permit efficient use of these coolants. Liquid hydrogen can produce explosive mixtures of hydrogen gas and air. If refrigeration equipment is used to cool the specimens with air as the cooling medium, it is desirable to have forced air circulation to provide uniform cooling.

6.6.1 At low temperatures, when using a coolant bath, immersion-type extensometers should be used.

6.7 Temperature measuring, controlling, and recording instruments shall be calibrated periodically against a secondary standard, such as a precision potentiometer. Lead-wire error should be checked with the lead wires in place as they normally are used.

## 7. Test Specimens

7.1 *Selection and Preparation of Specimens*—Special care shall be taken to obtain representative specimens that are straight and uniform in cross section. If straightening of the material for the specimen is required, then resultant residual stresses shall be removed by a subsequent stress relief annealing procedure that shall be reported with the test results.

7.2 *Dimensions*—The specimen length (and fillet radius in the case of tension specimens) should be greater than the minimum requirements for general-purpose specimens. In addition, the ratio of length to cross section of compression specimens should be such as to avoid buckling (see Test Methods E9).

NOTE 5—For examples of tension and compression specimens, see Test Methods E8/E8M and E9.

7.3 For tension specimens, the center lines of the grip sections and of the threads of threaded-end specimens shall be concentric with the center line of the gauge section within close tolerances in order to obtain the degree of alignment required. If pin-loaded sheet-type specimens are used, the centers of the gripping holes shall be not more than 0.005 times the width of the gauge section from its center line. For sheet-type specimens, small tabs or notches for attaching the extensometer may be used.

NOTE 6—The effect of eccentric loading can be illustrated by calculating the bending moment and stress thus added. For a standard 12.5-mm diameter specimen, the stress increase is 1.5 % for each 0.025 mm of eccentricity. This error increases to about 2.5 % per 0.025 mm for a 9-mm diameter specimen and to about 3.2 % per 0.025 mm for a 6-mm diameter specimen.