INTERNATIONAL

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Standard Practice for Obtaining Colorimetric Data from a Visual Display Unit Using Tristimulus Colorimeters[1]

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INTRODUCTION

This practice provides directions for correcting the results obtained with tristimulus colorimeters when measuring the tristimulus values or chromaticity coordinates of colored displays. Tristimulus colorimeters approximate the CIE color matching functions $\bar{x}(\lambda),\bar{y}(\lambda),\bar{z}(\lambda)$ to make these measurements. The errors generated in measuring colors on a display may be minimized using this practice.

1. Scope

1.1 This practice is intended as an aid for improving the accuracy of colorimetric measurements made with tristimulus colorimeters on visual display units, such as cathode ray tubes (CRTs) and self-luminous flat-panel displays. It explains a useful step in the analysis of colorimetric data that takes advantage of the fact that light from such displays consists of an additive mixture of three primary colored lights. However, it is not a complete specification of how such measurements should be made.

1.2 This practice is limited to display devices and colorimetric instruments that meet linearity criteria as defined in the practice. It is not concerned with effects that might cause measurement bias such as temporal or geometric differences between the instrument being optimized and the instrument used for reference.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate ~~safety~~ safety, health and ~~health~~environmental practices and determine the applicability of regulatory limitations prior to use.*

1.4 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E284 Terminology of Appearance

E1336 Test Method for Obtaining Colorimetric Data From a Visual Display Unit by Spectroradiometry

E1341 Practice for Obtaining Spectroradiometric Data from Radiant Sources for Colorimetry

2.2 *ISO/CIE Standard:*

ISO 11664–1:2007(E)/CIE S 014–1/E:2006 Joint Standard ISO/CIE Standard: Colorimetry Part 1 – CIE Standard Colorimetric Observers[3]

3. Terminology

3.1 *Definitions*—Unless otherwise stated, definitions of appearance terms in Terminology E284 are applicable to this practice.

3.2 *Definitions of Terms Specific to This Standard:*

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[1] This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.06 on Display, Imaging and Imaging Colorimetry.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Currently available from CIE (Commission International on Illumination), http://www.techstreet.com.

3.2.1 *calibration, n—in reference to a tristimulus colorimeter*, the process performed outside of this practice to adjust the tristimulus colorimeter to provide the best possible results for average or predefined conditions.

3.2.2 *optimization, n—in reference to a tristimulus colorimeter*, the process performed pursuant to this practice to adjust the tristimulus colorimeter or to interpret its readings to provide better results when applied to a particular display device.

3.2.3 *compatible, adj—in reference to a tristimulus colorimeter*, one so designed as to automate the procedure described in this practice.

4. Summary of Practice

4.1 Tristimulus colorimeters comprised of three or four detector channels are, in general, not amenable to accurate calibration that holds for all manner of usage with different illuminated devices and objects. This is because the spectral responsivities of their detector channels do not exactly match the defined Commission Internationale de L'Éclairage (CIE) $\bar{x}(\lambda),\bar{y}(\lambda),\bar{z}(\lambda)$ functions. Factory or subsequent calibration reflects judgments and compromises that may not be readily apparent. Nevertheless, this practice provides guidance on how such a tristimulus colorimeter may be optimized for use with a particular video display device, providing better accuracy with that device than its more general calibration provides. An optimization matrix transforms the instrumental (measured) CIE *X, Y, Z* values into adjusted *X, Y, Z* values that are closer to the ideal. This matrix is determined by reference to a colorimeter with higher intrinsic accuracy. The method derives from the fact that the color stimulus functions from display devices are linear combinations of three primary functions and are not entirely arbitrary.

5. Significance and Use

5.1 This practice may be applied when tristimulus colorimeters are used to measure the colors produced on self-luminous video display devices such as CRTs and flat-panel displays, including electroluminescent (EL) panels, light emitting diodes (LEDs) field emission displays (FEDs), and back-lit liquid crystal displays (LCDs). This practice is not meant to be a complete description of a procedure to measure the color coordinates of a display. Rather, it provides a method for obtaining more accurate results when certain conditions are met. It may be used by any person engaged in the measurement of color on display devices who has access to the requisite equipment.

5.2 This practice defines a class of tristimulus colorimeters that may be said to be compatible with this practice.

6. Background of Practice

6.1 *Colorimetry:*

6.1.1 Color measurement instruments consist, in general, of means to measure radiometric power as transmitted through a number of bandpass filters. Most commonly, electrical devices are used to measure the filtered light. They may be used with different filters in succession, or multiple devices may be used concurrently. In instruments called spectroradiometers, the radiometric power is measured through a large number (typically 30 to 500) of narrowband filters. (Practice E1341 describes how a monochromator or polychromator (spectrograph) may be employed to filter and measure light in separate bands on the order of 1-nm wide.) In instruments called tristimulus colorimeters, the radiometric power is measured through three or four wideband filters. These filters may be constructed from dispersive elements (prisms and gratings) or from materials with selective spectral transmission or reflection. The latter may be either uniform or comprised of different patches, in a mosaic pattern, that provide the desired overall effect.

6.1.2 No matter how many filters are used, or in what manner, the goal of the measurement process is to determine tristimulus values *X, Y,* ~~*Z,*~~*Z.* ~~as defined by ISO in its Standard ISO 11664–1:2007(E)/CIE S 014–1E:2006 and the CIE in its publication Publ. 015(1).~~For light with a color stimulus function Φ(λ),

$$X = k \int_{360\,nm}^{830\,nm} \Phi(\lambda)\bar{x}(\lambda)d\lambda \quad (1)$$

$$Y = k \int_{360\,nm}^{830\,nm} \Phi(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = k \int_{360\,nm}^{830\,nm} \Phi(\lambda)\bar{z}(\lambda)d\lambda$$

where:

k is 683 lm/W for emissive devices, such as displays, and $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are color-matching functions. While the standard definition of *X, Y, Z* requires the use of the CIE 1931 2° color-matching functions, the mathematics described in this practice would also be applicable to any other set of color-matching functions, such as the CIE 1964 10° functions.

These equations (Eq 1) are defined by ISO in its Standard ISO 11664–1:2007(E)/CIE S 014–1E:2006 and the CIE in its publication Publ. 015(**1**).[4]

6.1.3 In practice, color measurement instruments compute *X, Y, Z* by the summation of the signals as measured through the various filters, each signal being multiplied by an appropriate calibration factor. In matrix notation:

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} C_{X1} \, C_{X2} \, C_{X3} \ldots C_{Xf} \\ C_{Y1} \, C_{Y2} \, C_{Y3} \ldots C_{Yf} \\ C_{Z1} \, C_{Z2} \, C_{Z3} \ldots C_{Zf} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_f \end{bmatrix} \quad (2)$$

where:

F_1, F_2, F_3, through F_f are the electrical signals from the f filtered detectors and the C_{ij} are calibration coefficients. X_m, Y_m, Z_m have subscripts to indicate that they are measured values rather than ideal ones.

6.1.4 In this practice, we presume that the color measuring instrument is linear: that each signal F_a is strictly proportional to the received optical power, that any zero-offset (background in darkness) is removed, that the proportionality for signal F_a is not affected by the value of signal F_b, and in the case of closely packed detectors (such as charge-coupled device (CCD) detector elements) no signal F_a spills over and affects signal F_b as it approaches saturation. These presumptions are amenable to experimental verification using methods beyond the scope of this practice (**2**).

6.1.5 The values of the matrix elements C_{ij} may be determined using criteria that depends on the design and intended application of the instrument. The full extent of this subject is beyond the scope of this practice. However, in general, for spectroradiometers ($f \approx 30$ to 500), C_{Xj} reflects the tabulated value of *(λ)* near the center wavelength of Filter *j* as well as the spectral responsivity of the corresponding detector channel. (Likewise, C_{Yj} and C_{Zj} vary with *(λ)*.) For tristimulus colorimeters, the choice of C_{ij} is discussed further, below. As a general matter, the instrument designer should choose passbands and matrix elements that balance accuracy, sensitivity, and other design requirements.

6.1.6 Tristimulus colorimeters are generally designed with filters that are intended to match the spectral responsivities of their detector channels to the CIE color matching functions. For such an instrument,

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} C_{X1} & 0 & 0 \\ 0 & C_{Y2} & 0 \\ 0 & 0 & C_{Z3} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \quad (3)$$

where:

the non-zero C_{ij} matrix elements represent adjustable gains of the detector channels. However, the $\bar{x}(\lambda)$ function has two distinct lobes. This may be dealt with by splitting the lobes into two functions, $\bar{x}_{short}(\lambda)$ and $\bar{x}_{long}(\lambda)$, each with a separate filter (F_1 and F_2, respectively). For such an instrument,

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} C_{X1} & C_{X2} & 0 & 0 \\ 0 & 0 & C_{Y3} & 0 \\ 0 & 0 & 0 & C_{Z4} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \quad (4)$$

Alternatively, the $\bar{z}(\lambda)$ function properly scaled may serve as the $\bar{x}_{short}(\lambda)$, since they have a similar shape,

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} C_{X1} & 0 & C_{X3} \\ 0 & C_{Y2} & 0 \\ 0 & 0 & C_{Z3} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \quad (5)$$

In all of these cases, it is difficult to realize an exact match between the CIE color-matching functions and the actual spectral responsivities of the corresponding detector channels. This means that no choice of C_{ij} will provide perfect calibration for all applications of the instrument. The criteria for setting the C_{ij} might not be well documented for a particular instrument.

6.1.7 It is generally believed that spectroradiometers, with their many detector channels, may be calibrated to yield superior measurements of *X, Y, Z* for diverse applications. Nevertheless, the relative simplicity of tristimulus colorimeters and their commensurately lower cost have made them popular where the highest accuracy is not required.

6.2 *Self-Luminous Displays:*

6.2.1 A self-luminous display, such as a CRT, an electroluminescent (EL) panel, a field emission display (FED), light emitting diodes (LED) or a back-lit liquid crystal display (LCD) generates colored light by the proportional superposition (addition) of primary colored lights $\Phi_r(\lambda)$, $\Phi_g(\lambda)$, $\Phi_b(\lambda)$. The subscripts represent red, green, and blue, the primary colors of an additive set. An arbitrarily colored patch on the visual display has one and only one color stimulus function $\Phi(\lambda)$,

$$\Phi(\lambda) = a\Phi_r(\lambda) + b\Phi_g(\lambda) + c\Phi_b(\lambda) \quad (6)$$

[4] The boldface numbers in parentheses refer to the list of references at the end of this standard.

where *a, b, c* are coefficients that are determined by the display electronics.

6.2.2 The display electronics vary *a, b, c* over the face of the display in order to generate a colored image. For this practice, we presume that the display electronics may be set to make *a, b, c* uniform (perhaps after averaging nonobvious fine-structure) over a sufficient area of the display to permit measurements to be made on that area.

6.2.3 It is a requirement for the applicability of this practice that the display device behaves as stated in Eq 6. This practice does not represent that any particular display device will act as predicted by Eq 6, though those within the mentioned classes of devices might do so. The procedure for experimental verification of this property for a specific display device is beyond the scope of this practice (**3**).

6.3 *Colorimetric Measurement of Displays:*

6.3.1 Each of the primary color stimulus functions $\Phi_r(\lambda)$, $\Phi_g(\lambda)$, $\Phi_b(\lambda)$ stimulates responses in the *f* detector channels that may be represented by a vector $\boldsymbol{F}$ (that is, $\boldsymbol{F}_r$, $\boldsymbol{F}_g$, $\boldsymbol{F}_b$). Given their construction, these vectors are linearly independent. (None of the three can be expressed as a linear combination of the other two.) While $\boldsymbol{F}$ is an element of an *f*-dimensional vector space, it is clear that only a three-dimensional subspace is spanned by the $\boldsymbol{F}$'s of all possible color stimulus functions following Eq 6. Further, the mapping of $\boldsymbol{F}$ into (X_m, Y_m, Z_m) space by Eq 2 remains three dimensional. In other words, there is a one-to-one mapping of the vector (*a, b, c*) onto (*X, Y, Z*) by application of Eq 1; and, for a particular instrument with a fixed calibration matrix $\boldsymbol{C}$, there is also a one-to-one mapping of the vector (*a, b, c*) onto (X_m, Y_m, Z_m). From this we deduce that a matrix $\boldsymbol{R}$ exists that can be used to translate (X_m, Y_m, Z_m) values into actual (*X, Y, Z*) values.

6.3.2 A colorimeter that takes advantage of this fact must provide means for implementing the matrix $\boldsymbol{R}$. That is, all *f* filtered detector signals should contribute linearly toward the computation of each output, X_m, Y_m, Z_m, instead of using different detectors for each output. This idea was reported as long ago as 1973 by Wagner (**4**), and it has been expanded upon and rediscovered by others since then (**5-10**).

6.3.3 On the basis of this property, a tristimulus colorimeter can be optimized for use on a self-luminous display by the proper derivation of a matrix $\boldsymbol{R}$ for that display. We proceed on the assumptions that the components are sufficiently stable, and that similarly built displays have similar enough spectral primaries to make a derivation of $\boldsymbol{R}$ worthwhile. However, these assumptions should be quantified before accuracy claims are made in any specific situation.

6.3.4 On the basis of this property, a tristimulus colorimeter designed for use with displays need not produce signals $\boldsymbol{F}$ that are close to CIE tristimulus values. Signal/noise may be improved by matching the spectral responsivities of the filtered detectors to the emission spectra of the primary colors. In such designs, it is especially important to use a matrix $\boldsymbol{R}$ that is specific to the particular $\Phi_r(\lambda)$, $\Phi_g(\lambda)$, $\Phi_b(\lambda)$.

7. Optimization

7.1 *General:*

7.1.1 Given the existence of a matrix $\boldsymbol{R}$, how is it determined? Experimentally, the problem is one of comparing the ~~data~~ *X, Y, Z* data from a reference colorimeter with the ~~data~~ X_m, Y_m, Z_m data from the colorimeter being optimized, for a number of color samples at different display settings. From these data, $\boldsymbol{R}$ is ~~calculated.~~ evalulated.

7.1.2 This practice is not directly concerned with the absolute accuracy of the measurements. It concerns the transfer of calibration from a reference instrument to another instrument, regardless of the absolute accuracy of the reference.

7.2 *Noiseless Data:*

7.2.1 For clarity, we first consider the case in which the measuring instruments are free of noise. We define vectors $\boldsymbol{n}$ and $\boldsymbol{m}$ as:

$$\boldsymbol{n} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad ; \boldsymbol{m} = \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} \tag{7}$$

where:

each is an element of its corresponding vector space, and both derive from the same (*a, b, c*) setting on a display. The matrix $\boldsymbol{R}$ maps between them:

$$\boldsymbol{n} = \boldsymbol{R}\,\boldsymbol{m} \tag{8}$$

7.2.2 This relationship may be stated for more than one such pairs of vectors (for multiple display settings) at the same time:

$$\boldsymbol{N} = \boldsymbol{R}\,\boldsymbol{M} \tag{9}$$

where:

$$\boldsymbol{N} = \begin{bmatrix} X_1\, X_2\, X_3 \ldots X_i \\ Y_1\, Y_2\, Y_3 \ldots Y_i \\ Z_1\, Z_2\, Z_3 \ldots Z_i \end{bmatrix} \tag{10}$$

and

$$\boldsymbol{M} = \begin{bmatrix} X_{m1} & X_{m2} & X_{m3} & \ldots & X_{mi} \\ Y_{m1} & Y_{m2} & Y_{m3} & \ldots & Y_{mi} \\ Z_{m1} & Z_{m2} & Z_{m3} & \ldots & Z_{mi} \end{bmatrix} \quad (11)$$

7.2.3 When matrices *N* and *M* have exactly three columns, and when their columns are linearly independent, ***R*** may be easily determined:

$$\boldsymbol{R} = \boldsymbol{N}\,\boldsymbol{M}^{-1} \quad (12)$$

The determination of matrix **R** in this case requires the reference values and the test colorimeter measurements of exactly three distinct colors on the display. In other words, ***R*** can be obtained by measuring ***M*** for each primary color (red, green, blue) of a display.

7.3 *Real-World Transformations of X, Y, Z:*

7.3.1 In practice, neither measurements of ***n*** nor ***m*** are made with perfect accuracy. Noise affects the measurements, the linearity presumptions in 6.1.4 and 6.2.3 may not be perfectly true, and there may be other unexpected systematic effects that affect the data. One way to reduce such effects of measurement imperfection in applying the matrix correction is to determine ***R*** by using more than three color samples and by using statistical methods based on the least square minimization, as demonstrated by obtaining ***R*** by measurement of eight colors of a display (Practice E1455-92).[5] This method, however, was found to be not effective enough to reduce the effect of measurement noise. This method was improved by applying corrections for the absolute scale of measured tristimulus values for each color of the display, based on the first results from applying Practice E1455-92, and performing another path of calculation, introducing weights for each color in the minimization process (Practice E1455-96).[5] This method improved the performance significantly from the Practice E1455-92. However, the calculation process has become considerably complicated. Another method has become available, in which the effect of the fluctuations of signals common to all detector channels is theoretically eliminated (**9**). For example, the effects of measurement errors due to display flicker (causing variation in measured luminance) and instability of display (causing a change of display luminance when reference instrument is measured and when target instrument is measured) are eliminated. This method requires measurements of four colors of a display—each primary color plus white. The ***R*** matrix is obtained in such a way that the chromaticity errors are reduced to zero for all the four colors, and the correction is valid for all other colors. Correction for *Y* can also be implemented optionally. This method is theoretically more robust than the earlier ASTM methods for correction of measured chromaticity, and shown to perform equal to or better than Practice E1455-96, while the measurement and calculation process are much simpler. This method (commonly called Four-Color method) is described in the sections below.

7.3.2 *Principles of the Four-Color Method*—The primary colors (red, green, and blue) and white of a display are measured by a target instrument (a colorimeter being optimized) and a reference instrument (a calibrated colorimeter or spectroradiometer). From the chromaticity coordinates ($x_{m,R}$, $y_{m,R}$), ($x_{m,G}$, $y_{m,G}$), ($x_{m,B}$, $y_{m,B}$) of red, green, and blue, measured by the target instrument, the relative tristimulus values $\boldsymbol{M}_{rel,RGB}$ of the primary colors from the target instrument are expressed as:

$$\boldsymbol{M}_{rel,RGB} = \begin{bmatrix} X_{rel,m,R} & X_{rel,m,G} & X_{rel,m,B} \\ Y_{rel,m,R} & Y_{rel,m,G} & Y_{rel,m,B} \\ Z_{rel,m,R} & Z_{rel,m,G} & Z_{rel,m,B} \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} x_{m,R} & x_{m,G} & x_{m,B} \\ y_{m,R} & y_{m,G} & y_{m,B} \\ z_{m,R} & z_{m,G} & z_{m,B} \end{bmatrix} \begin{bmatrix} k_{m,R} & 0 & 0 \\ 0 & k_{m,G} & 0 \\ 0 & 0 & k_{m,B} \end{bmatrix}$$

where:

$$z_{m,R} = 1 - x_{m,R} - y_{m,R}$$
$$z_{m,G} = 1 - x_{m,G} - y_{m,G}$$
$$z_{m,B} = 1 - x_{m,B} - y_{m,B}$$

$k_{m,R}$, $k_{m,G}$, $k_{m,B}$ are the relative factors to relate the measured chromaticity coordinates to the relative tristimulus values.

Likewise, from the chromaticity coordinates ($x_{r,R}$, $y_{r,R}$), ($x_{r,G}$, $y_{r,G}$), ($x_{r,B}$, $y_{r,B}$) of red, green, and blue measured by the reference instrument, the relative tristimulus values $\boldsymbol{N}_{rel,RGB}$ of the primary colors from the reference instrument are expressed as:

$$\boldsymbol{N}_{rel,RGB} = \begin{bmatrix} X_{rel,r,R} & X_{rel,r,G} & X_{rel,r,B} \\ Y_{rel,r,R} & Y_{rel,r,G} & Y_{rel,r,B} \\ Z_{rel,r,R} & Z_{rel,r,G} & Z_{rel,r,B} \end{bmatrix} \quad (14)$$

$$= \begin{bmatrix} x_{r,R} & x_{r,G} & x_{r,B} \\ y_{r,R} & y_{r,G} & y_{r,B} \\ z_{r,R} & z_{r,G} & z_{r,B} \end{bmatrix} \begin{bmatrix} k_{r,R} & 0 & 0 \\ 0 & k_{r,G} & 0 \\ 0 & 0 & k_{r,B} \end{bmatrix}$$

[5] A copy of this standard is available from: https://global.ihs.com/standards.cfm?publisher=ASTM.

where:

$$z_{r,R} = 1 - x_{r,R} - y_{r,R}$$
$$z_{r,G} = 1 - x_{r,G} - y_{r,G}$$
$$z_{r,B} = 1 - x_{r,B} - y_{r,B}$$

$k_{r,R}$, $k_{r,G}$, $k_{r,B}$ are the relative factors to relate the measured chromaticity coordinates to the relative tristimulus values.

Based on the additivity of tristimulus values, when the chromaticity coordinates ($x_{m,W}$, $y_{m,W}$) and ($x_{r,W}$, $y_{r,W}$) for white of the display is measured by the target instrument and the reference instrument, respectively, the following relationships hold:

$$\begin{bmatrix} x_{m,W} \\ y_{m,W} \\ z_{m,W} \end{bmatrix} = \begin{bmatrix} x_{m,R} & x_{m,G} & x_{m,B} \\ y_{m,R} & y_{m,G} & y_{m,B} \\ z_{m,R} & z_{m,G} & z_{m,B} \end{bmatrix} \begin{bmatrix} k_{m,R} \\ k_{m,G} \\ k_{m,B} \end{bmatrix} \quad (15)$$

and

$$\begin{bmatrix} x_{r,W} \\ y_{r,W} \\ z_{r,W} \end{bmatrix} = \begin{bmatrix} x_{r,R} & x_{r,G} & x_{r,B} \\ y_{r,R} & y_{r,G} & y_{r,B} \\ z_{r,R} & z_{r,G} & z_{r,B} \end{bmatrix} \begin{bmatrix} k_{r,R} \\ k_{r,G} \\ k_{r,B} \end{bmatrix} \quad (16)$$

The white color of the display can be of any intensity combination of the three primary colors. The values of $k_{m,R}$, $k_{m,G}$, $k_{m,B}$ and $k_{r,R}$, $k_{r,G}$, $k_{r,B}$ are obtained by solving Eq 15 and 16 as:

$$\begin{bmatrix} k_{m,R} \\ k_{m,G} \\ k_{m,B} \end{bmatrix} = \begin{bmatrix} x_{m,R} & x_{m,G} & x_{m,B} \\ y_{m,R} & y_{m,G} & y_{m,B} \\ z_{m,R} & z_{m,G} & z_{m,B} \end{bmatrix}^{-1} \begin{bmatrix} x_{m,W} \\ y_{m,W} \\ z_{m,W} \end{bmatrix} \quad (17)$$

and

$$\begin{bmatrix} k_{r,R} \\ k_{r,G} \\ k_{r,B} \end{bmatrix} = \begin{bmatrix} x_{r,R} & x_{r,G} & x_{r,B} \\ y_{r,R} & y_{r,G} & y_{r,B} \\ z_{r,R} & z_{r,G} & z_{r,B} \end{bmatrix}^{-1} \begin{bmatrix} x_{r,W} \\ y_{r,W} \\ z_{r,W} \end{bmatrix} \quad (18)$$

By entering the values of $k_{m,R}$, $k_{m,G}$, $k_{m,B}$ and $k_{r,R}$, $k_{r,G}$, $k_{r,B}$ into Eq 13 and 14, $\boldsymbol{M}_{rel,RGB}$ and $\boldsymbol{N}_{rel,RGB}$ are determined, then, the correction matrix $\boldsymbol{R}_{rel}$ is obtained by:

$$\boldsymbol{R}_{rel} = \boldsymbol{N}_{rel,RGB} \cdot \boldsymbol{M}_{rel,RGB}{}^{-1} \quad (19)$$

If luminance value Y is not available or of no interest, $\boldsymbol{R}_{rel}$ is used to correct the chromaticity coordinates (x_m, y_m) of any color of the display measured by the target instrument. In this case, the relative tristimulus values of the measured color is taken as:

$$\boldsymbol{M}_{rel} = \begin{bmatrix} X_{rel,m} \\ Y_{rel,m} \\ Z_{rel,m} \end{bmatrix} = \begin{bmatrix} x_m \\ y_m \\ (1 - x_m - y_m) \end{bmatrix} \quad (20)$$

Then the corrected relative tristimulus values $\boldsymbol{M}'_{rel}$ of the measured color is obtained by:

$$\boldsymbol{M}'_{rel} = \begin{bmatrix} X'_{rel,m} \\ Y'_{rel,m} \\ Z'_{rel,m} \end{bmatrix} = \boldsymbol{R}_{rel} \cdot \boldsymbol{M}_{rel} \quad (21)$$

The corrected chromaticity coordinate is obtained by:

$$\begin{bmatrix} x'_m \\ y'_m \end{bmatrix} = \begin{bmatrix} X'_{rel,m}/(X'_{rel,m}+Y'_{rel,m}+Z'_{rel,m}) \\ Y'_{rel,m}/(X'_{rel,m}+Y'_{rel,m}+Z'_{rel,m}) \end{bmatrix} \quad (22)$$

If the luminance values for the four colors ($Y_{m,R}$, $Y_{m,G}$, $Y_{m,B}$, $Y_{m,W}$) and ($Y_{r,R}$, $Y_{r,G}$, $Y_{r,B}$, $Y_{r,W}$) are also measured by the target instrument and the reference instrument, respectively, a correction matrix can be obtained for correction of absolute tristimulus values. In this case, The matrix $\boldsymbol{R}_{rel}$ needs only to be scaled by an additional factor.

First, the absolute tristimulus values of the four colors measured by the target instrument are obtained by: