



Designation: **D5850 – 95 (Reapproved 2012) D5850 – 18**

Standard Test Method for (Analytical Procedure) Determining Transmissivity, Storage Coefficient, and Anisotropy Ratio from a Network of Partially Penetrating Wells¹

This standard is issued under the fixed designation D5850; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope—Scope*

1.1 This test method covers an analytical procedure for determining the transmissivity, storage coefficient, and ratio of vertical to horizontal hydraulic conductivity of a confined aquifer using observation well drawdown measurements from a constant-rate pumping test. This test method uses data from a minimum of four partially penetrating, ~~properly recommended to be positioned~~ observation wells around a partially penetrating control well.

1.2 The analytical procedure is used in conjunction with the field procedure in Test Method **D4050**.

1.3 *Limitations*—The limitations of the technique for determination of the horizontal and vertical hydraulic conductivity of aquifers are primarily related to the correspondence between the field situation and the simplifying assumption of this test method.

1.4 *Units*—The values stated in inch-pound units are to be regarded as the standard. The SI units given in parentheses are ~~for information only~~ mathematical conversions, which are provided for information purposes only and are not considered standard.

1.5 All observed and calculated values shall conform to the guidelines for significant digits and rounding established in Practice **D6026**, unless superseded by this standard.

1.6 The procedures used to specify how data are collected/recorded or calculated in this standard are regarded as the industry standard. In addition, they are representative of the significant digits that generally should be retained. The procedures used do not consider material variation, purpose for obtaining the data, special purpose studies, or any considerations for the user's objective; and it is common practice to increase or reduce the significant digits of reported data to be commensurate with these considerations. It is beyond the scope of this standard to consider significant digits used in analysis method or engineering design.

1.7 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.8 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*²

D653 Terminology Relating to Soil, Rock, and Contained Fluids

D3740 Practice for Minimum Requirements for Agencies Engaged in Testing and/or Inspection of Soil and Rock as Used in Engineering Design and Construction

D4050 Test Method for (Field Procedure) for Withdrawal and Injection Well Testing for Determining Hydraulic Properties of Aquifer Systems

D5473/**D5473M** Test Method for (Analytical Procedure) for Analyzing the Effects of Partial Penetration of Control Well and Determining the Horizontal and Vertical Hydraulic Conductivity in a Nonleaky Confined Aquifer

¹ This test method is under the jurisdiction of ASTM Committee **D18** on Soil and Rock and is the direct responsibility of Subcommittee **D18.21** on Groundwater and Vadose Zone Investigations.

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² For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

*A Summary of Changes section appears at the end of this standard

D6026 Practice for Using Significant Digits in Geotechnical Data

3. Terminology

3.1 Definitions:

3.1.1 For definitions of common technical terms in this standard, see Terminology [D653](#).

3.2 ~~Definitions:~~The following definitions from Terminology [D653](#) are used in this standard and are presented for the convenience of the user.

3.2.1 *anisotropy*—having different properties in different directions.

3.2.2 *aquifer, confined*—*confined aquifer*—*in hydrogeology*, an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *confining bed*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.2.3 *control well*—*in aquifer testing*, well by which the head and flow in the aquifer is changed, stressed, for example, by pumping, injection, or imposing a constant change of head.

3.2.4 *drawdown*—*drawdown [L]*—*in field aquifer tests*, vertical distance the static head is lowered free water elevation is lowered or the pressure head is reduced due to the removal of free water.

3.2.5 *hydraulic conductivity*—~~(field in field aquifer test)tests~~, the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.2.6 *observation well*—*monitoring well (observation well), n*—*in hydrogeology*, a well open to all or part of an aquifer, installed, usually of small diameter, for measuring water levels, collecting water samples, or determining other groundwater characteristics.

3.2.6.1 Discussion—

The well may be cased or uncased, but if cased the casing should have openings to allow flow of groundwater into or out of the casing, such as a well screen.

3.1.7 *piezometer*—a device so constructed and sealed as to measure hydraulic head at a point in the subsurface.

3.2.7 *storage coefficient*—*in aquifers*, the volume of water that an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. For a confined aquifer, the storage coefficient is equal to the product of the specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.

3.2.8 *transmissivity*—*in aquifers*, the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.2.8.1 Discussion—

It is equal to an integration of the hydraulic conductivities across the saturated part of the aquifer perpendicular to the flow paths.

3.1.10 For definitions of other terms used in this test method, see Terminology [D653](#).

3.3 Symbols and Dimensions:

3.3.1 A — K_z/K_r , anisotropy ratio [nd].

3.3.2 b —thickness of aquifer [L].

3.3.3 C_f —drawdown correction factor, equal to the ratio of the drawdown for a fully penetrating well network to the drawdown for a partially penetrating well network ($W(u)/(W(u) + f_s)$).

3.3.4 d —distance from top of aquifer to top of screened interval of control well [L].

3.3.5 d' —distance from top of aquifer to top of screened interval of observation well [L].

3.3.6 f_s —incremental dimensionless drawdown component resulting from partial penetration [nd].

3.3.7 K —hydraulic conductivity [LT^{-1}].

3.3.7.1 Discussion—

The use of symbol K for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol k is commonly used for this term in the rock and soil mechanics literature.

3.3.8 K_o —modified Bessel function of the second kind and zero order.

3.3.9 K_r —hydraulic conductivity in the plane of the aquifer, radially from the control well (horizontal hydraulic conductivity) [LT^{-1}].

- 3.3.10 K_z —hydraulic conductivity normal to the plane of the aquifer (vertical hydraulic conductivity) [LT^{-1}].
- 3.3.11 l —distance from top of aquifer to bottom of screened interval of control well [L].
- 3.3.12 l' —distance from top of aquifer to bottom of screened interval of observation well [L].
- 3.3.13 Q —discharge [L^3T^{-1}].
- 3.3.14 r —radial distance from control well [L].
- 3.3.15 S —storage coefficient [nd].
- 3.3.16 s —drawdown observed in partially penetrating well network [L].
- 3.3.17 s_f —drawdown observed in fully penetrating well network [L].
- 3.3.18 T —transmissivity [L^2T^{-1}].
- 3.3.19 t —time since pumping began [T].
- 3.3.20 u — $(r^2S)/(4Tt)$ [nd].
- 3.3.21 $W(u)$ —an exponential integral known in hydrology as the Theis well function of $u[nd]$.

4. Summary of Test Method

4.1 This test method makes use of the deviations in drawdown near a partially penetrating control well from those that would occur near a control well fully penetrating the aquifer. In general, drawdown within the screened horizon of a partially penetrating control well tends to be greater than that which would have been observed near a fully penetrating well, whereas the drawdown above or below the screened horizon of the partially penetrating control well tends to be less than the corresponding fully penetrating case. Drawdown deviations due to partial penetration are amplified when the vertical hydraulic conductivity is less than the horizontal hydraulic conductivity. The effects of partial penetration diminish with increasing distance from the pumped well, becoming negligible at a distance of about $1.5b/(K_z/K_r)^{1/2}$. This test method relies on obtaining drawdown measurements at a minimum of two locations within this distance of the pumped well and at each location obtaining data from observation wells completed to two different depths. At each location, one observation well should be screened at about the same elevation as the screen in the pumped well, while the other observation well should be screened in sediments not screened by the pumped well.

4.2 According to Theis (1),³ the drawdown around a fully penetrating control well pumped at a constant rate and tapping a homogeneous, confined aquifer is as follows:

$$s_f = \frac{Q}{4\pi T} W(u) \quad (1)$$

where:

$$W(u) = \int_u^\infty \frac{e^{-x}}{x} dx \quad (2)$$

4.2.1 Drawdown near a partially penetrating control well pumped at a constant rate and tapping a homogeneous, anisotropic, confined aquifer is presented by Hantush (2, 3, 4):

$$s = \frac{Q}{4\pi T} (W(u) + f_s) \quad (3)$$

According to Hantush (2, 3, 4), at late pumping times, when $t > b^2S/(2TA)$, f_s can be expressed as follows:

$$f_s = \frac{4b^2}{\pi^2(l-d)(l'-d)} \sum_{n=1}^{\infty} \left(\frac{1}{n^2} \right) K_0 \left(\frac{n\pi r \sqrt{K_z/K_r}}{b} \right) \left[\sin \left(\frac{n\pi l}{b} \right) - \sin \left(\frac{n\pi d}{b} \right) \right] \left[\sin \left(\frac{n\pi l'}{b} \right) - \sin \left(\frac{n\pi d'}{b} \right) \right] \quad (4)$$

4.2.2 For a given observed drawdown, it is possible/practicable to compute a correction factor, C_f , defined as the ratio of the drawdown for a fully penetrating well to the drawdown for a partially penetrating well:

$$C_f = \frac{W(u)}{W(u) + f_s} \quad (5)$$

The observed drawdown for each observation well may be corrected to the fully penetrating equivalent drawdown by multiplying by the correction factor:

$$s_f = C_f s \quad (6)$$

³ The boldface numbers given in parentheses refer to a list of references at the end of the text.

The drawdown values corresponding to the fully penetrating case may then be analyzed by conventional distance-drawdown methods to compute transmissivity and storage coefficient.

4.2.3 The correction factors are a function of both transmissivity and storage coefficient, that are the parameters being sought. Because of this, the test method relies on an iterative procedure in which an initial estimate of T and S are made from which initial correction factors are computed. Using these correction factors, fully penetrating drawdown values are computed and analyzed using distance-drawdown methods to determine revised values for T and S . The revised T and S values are used to compute revised correction factors, C_f . This process is repeated until the calculated T and S values change only slightly from those obtained in the previous iteration.

4.2.4 The correction factors are also a function of the anisotropy ratio, A . For this reason, all of the calculations described above must be performed for several different assumed anisotropy ratios. The assumed anisotropy value that leads to the best solution, that is, best straight line fit or best curve match, is deemed to be the actual anisotropy ratio.

5. Significance and Use

5.1 This test method is one of several available for determining vertical anisotropy ratio. Among other available methods are Weeks ((5); see Test Method [D5473D5473/D5473M](#)), that relies on distance-drawdown data, and Way and McKee (6), that utilizes time-drawdown data. An important restriction of the Weeks distance-drawdown method is that the observation wells must need to have identical construction (screened intervals) and two or more of the observation wells must need to be located at a distance from the pumped well beyond the effects of partial penetration. The procedure described in this test method general distance-drawdown method, in that it works in theory for anymost observation well configurationconfigurations incorporating three or more wells, provided some of the wells are within the zone where flow is affected by partial penetration.

5.2 Assumptions:

5.2.1 Control well discharges at a constant rate, Q .

5.2.2 Control well is of infinitesimal diameter and partially penetrates the aquifer.

5.2.3 Data are obtained from a number of partially penetrating observation wells, some screened at elevations similar to that in the pumped well and some screened at different elevations.

5.2.4 The aquifer is confined, homogeneous and areally extensive. The aquifer may be anisotropic, and, if so, the directions of maximum and minimum hydraulic conductivity are horizontal and vertical, respectively.

5.2.5 Discharge from the well is derived exclusively from storage in the aquifer.

5.3 *Calculation Requirements*—Application of this method is computationally intensive. The function, f_s , shown in (Eq 4) must be evaluated numerous times using arbitrary input parameters. It is not practical to use existing, somewhat limited, tables of values for f_s and, because this equation is rather formidable, it is may not readily be easily tractable by hand. Because of this, it is assumed the practitioner using this test method will have available a computerized procedure for evaluating the function f_s . This can be accomplished using commercially available mathematical software including some spreadsheet applications, or by writing programsprograms. in (7languages) such as Fortran or C.

NOTE 1—The quality of the result produced by this standard is dependent on the competence of the personnel performing it, and the suitability of the equipment and facilities used. Agencies that meet the criteria of Practice [D3740](#) are generally considered capable of competent and objective testing/sampling/inspection/etc. Users of this standard are cautioned that compliance with Practice [D3740](#) does not in itself assure reliable results. Reliable results depend on many factors; Practice [D3740](#) provides a means of evaluating some of those factors.

NOTE 2—Most fractured (unconfined) aquifers, even noncarbonates, will have some form of convergent flow to master fissures or channels (Worthington et al., 2016). A relationship is known to occur in carbonates where potentiometric troughs correspond with sub-surface conduits or channels (Quinlan and Ewers, 1989).

6. Apparatus

6.1 Apparatus for withdrawal tests is given in Test Method [D4050](#). The apparatus described below are those components of the apparatus that require special attributes for this specific test.

6.2 *Construction of the Control Well*—Screen the control well through only part of the vertical extent of the aquifer to be tested. The exact distances from the top of the aquifer to the top and bottom of the pumped well screen interval must be known.

6.3 *Construction and Placement of Observation Wells*—The procedure will work for arbitrary positioning of observation wells and placement of their screens, as long as three or more observation wells are used and some of the observation wells fall inside the zone where flow is affected by partial penetration, that is, the area where significant vertical flow components exists. However, strategic selection of the number and location of observation wells will maximizeincrease the potential quality of the data set and improve the reliability of the interpretation.

6.3.1 *Optimum results will It is recommended that results* be obtained by using a minimum of four observation wells incorporating two pairs of observation wells located at two different distances from the pumped well, both within the zone where flow is affected by partial penetration. Each well pair should consist of a shallow well and a deep well, that span vertically the area in which vertical anisotropy is sought. For each well pair, one observation well screen should be at the same elevation as the screen in the pumped well, whereas the other observation well screen should be at a different elevation than the screen in the pumped well.

6.3.2 This test method relies on choosing several arbitrary anisotropy ratios, correcting the observed drawdowns for partial penetration, and evaluating the results. If ~~at~~the observation wells are screened at the same elevation, the quality of the data trace produced by correcting the observed drawdown measurements is not sensitive to the choice of anisotropy, making it difficult to determine this parameter accurately. If, however, observation well screens are located both within the pumped zone (where drawdown is greater than the fully penetrating case) and the unpumped zone (where drawdown is less than the fully penetrating case), the quality of the corrected data is sensitive to the choice of anisotropy ratio, making it easier to quantify this parameter.

7. Procedure

7.1 Pre-test preparations, pumping test guidelines, and post-test procedures associated with the pumping test itself are described in Test Method **D4050**.

7.2 Verify the quality of the data set. Review the record of measured flow rates to make sure the rate was held constant during the test. Check to see that hand measurements of drawdown agree well with electronically measured values. Finally, check the background water-level fluctuations observed prior to or following the pumping test to see if adjustments ~~must~~need to be made to the observed drawdown values to account for background fluctuations. If appropriate, adjust the observed drawdown values accordingly.

7.3 Analysis of the field data is described in Section **88**.

8. Calculation and Interpretation of Results

8.1 *Initial Estimates of Transmissivity and Storage Coefficient*—This test method requires that initial estimates of T and S be obtained. These estimates can be made using a wide variety of procedures, including time-drawdown analysis, recovery analysis, distance-drawdown analysis, estimation of T using specific capacity, grain-size analyses of formation samples, or results of laboratory permeability tests, and estimation of storage coefficient based on geology, sediment type, and aquifer thickness.

8.2 *Select Data for Analysis*—This test method requires a single drawdown observation for each observation well used in the test. The drawdowns used should ~~all~~correspond to the same time since pumping began, usually near or at the end of the test. Select a time, t , late enough in the test so that it satisfies the relationship $t > b^2S/(2TA)$.

8.3 *Distance-Drawdown Analysis Methods*—The selected drawdown values will be corrected for partial penetration and the corrected drawdown will be analyzed using distance-drawdown methods. Use either a semilog procedure or a log-log procedure. The semilog procedure requires that u be small. For distant observation wells, this condition may be violated and the semilog method may be invalid. If u is not sufficiently small, the logarithmic approximation of the Theis well function, $W(u)$, is not accurate. Examples of errors for some u values are as follows:

u	Error, %
0.01	0.25
0.03	1.01
0.05	2.00
0.10	5.35

The log-log method is more general, being valid for all values of u .

8.3.1 Semilog Method:

8.3.1.1 If this method is used, plot the corrected drawdown, s_p , on the linear scale versus distance, r , on the log scale. Construct a straight line of best fit through the data points and record the slope of the line, Δs , and the zero drawdown intercept, R ,

where:

Δs = change in drawdown over one log cycle, and

R = distance where line of best fit crosses 0 drawdown.

8.3.1.2 Using these input parameters, calculate transmissivity and storage coefficient as follows:

$$T = \frac{2.3026Q}{2\pi\Delta s} \quad (7)$$

$$S = \frac{2.25 Tt}{R^2} \quad (8)$$

8.3.2 *Log-Log Method*—If the log-log method is selected, plot corrected drawdown, s_p , on the vertical logarithmic axis versus the reciprocal of the distance squared, $1/r^2$, on the horizontal logarithmic axis. On a separate graph having the same scale as the data plot, prepare a standard Theis type curve by plotting $W(u)$ on the vertical axis versus $1/u$ on the horizontal axis (see **Fig. 1**). Overlay the data plot on the type curve and, while keeping the coordinate axes of the two plots parallel, shift the data plot to align with the type curve effecting a match position. Select and record the values of an arbitrary point, referred to as the match point, anywhere on the overlapping part of the plots. Record the match-point coordinates— $W(u)$, $1/u$, s_p , $1/r^2$. For convenience, the match point may be selected where $W(u)$ and $1/u$ are integer values. Using these match-point values, compute transmissivity and storage coefficient as follows:

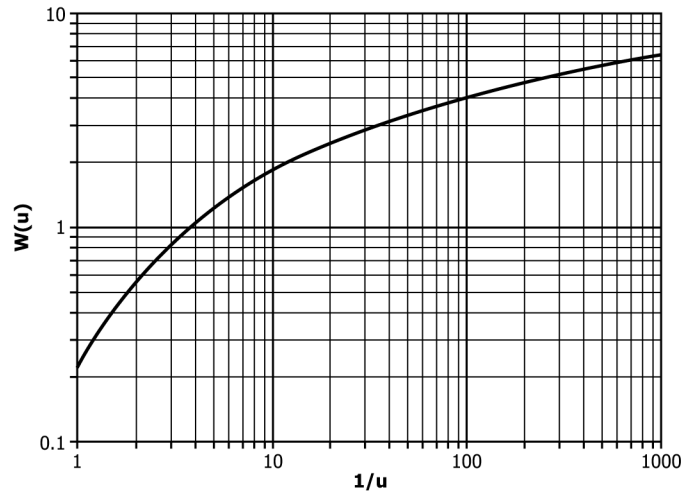


FIG. 1 Theis Type Curve

$$T = \frac{Q}{4\pi s} W(u) \tag{9}$$

$$S = \frac{4Tu}{r^2} \tag{10}$$

8.4 *Iterative Calculations*—Use the following steps to estimate vertical anisotropy ratio and refine the values for transmissivity and storage coefficient.

8.4.1 Select several arbitrary anisotropy ratios, spanning a range likely to include the actual anisotropy of the aquifer. Usually four or five values will suffice.

8.4.2 For each assumed anisotropy value, use the estimated T and S values to calculate correction factors, C_f , and corrected drawdowns, s_f , for each observation well. Use Eq 2, Eq 4, Eq 5, and Eq 6

8.4.3 Using the corrected drawdowns, prepare a distance-drawdown graph for each value of assumed anisotropy. Compare the graphs to determine which one provides the best data trace. For semilog graphs, this is the plot that best describes a straight line. For log-log graphs, it is the plot that best fits the Theis type curve. Record the corresponding anisotropy value as the best estimate for A .

8.4.4 Using the selected distance-drawdown graph, calculate T and S as described in 8.3. The values obtained are considered revised estimates of transmissivity and storage coefficient.

8.4.5 Select several new, arbitrary anisotropy values spanning a range that is narrower than the previous one and that includes the previous estimate for A . Go back to 8.4.2 to repeat the iteration process. Each iteration will generate new values for correction factors and corrected drawdowns, new distance-drawdown graphs and revised estimates for A , T , and S .

8.5 *Example Calculation:*

8.5.1 A test well screened in the bottom 10 ft (3.05 m) of a 50-ft (15.24 m) thick aquifer was pumped at a rate of 2 gpm (385 cubic feet per day [cfd]) for one day. The corresponding data parameters are as follows:

- $Q = 385$ cfd (10.9 cmd)
- $b = 50$ ft (15.24 m)
- $d = 40$ ft (12.19 m)
- $l = 50$ ft (15.24 m)
- $t =$ one day

8.5.2 Table 1 shows well geometry and drawdown data for four observation wells that were monitored during the pumping test.

TABLE 1 Well Geometry and Drawdown Information

Observation Well	r , Distance from Pumped Well, in ft (m)	d , Distance from Top of Aquifer to Top of Screen, in ft (m)	l , Distance from Top of Aquifer to Bottom of Screen, in ft (m)	s , Drawdown after 1 Day, in ft (m)
1	10 (3.05)	0 (0)	10 (3.05)	3.11 (0.95)
2	11 (3.35)	30 (9.14)	40 (12.19)	7.49 (2.28)
3	50 (15.24)	40 (12.19)	50 (15.24)	4.56 (1.39)
4	60 (18.29)	0 (0)	10 (3.05)	2.65 (0.81)