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Standard Practices for
Cycle Counting in Fatigue Analysis[1]

This standard is issued under the fixed designation E 1049; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 These practices are a compilation of acceptable procedures for cycle-counting methods employed in fatigue analysis. This standard does not intend to recommend a particular method.

1.2 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*
E 912 Definitions of Terms Relating to Fatigue Loading[2]

3. Terminology

3.1 *Definitions:*

3.1.1 *constant amplitude loading—in fatigue loading*, a loading in which all of the peak loads are equal and all of the valley loads are equal.

3.1.2 *cycle—in fatigue loading*, under constant amplitude loading, the load variation from the minimum to the maximum and then to the minimum load.

NOTE 1—In spectrum loading, definition of cycle varies with the counting method used.

3.1.3 *mean crossings—in fatigue loading*, the number of times that the load-time history crosses the mean-load level with a positive slope (or a negative slope, or both, as specified) during a given length of the history (see Fig. 1).

3.1.3.1 *Discussion*—For purposes related to cycle counting, a mean crossing may be defined as a crossing of the reference load level.

3.1.4 *mean load, P_m —in fatigue loading*, the algebraic average of the maximum and minimum loads in constant amplitude loading, or of individual cycles in spectrum loading,

$$P_m = (P_{\max} + P_{\min})/2 \tag{1}$$

or the integral average of the instantaneous load values or the algebraic average of the peak and valley loads of a spectrum loading history.

3.1.5 *peak—in fatigue loading*, the point at which the first derivative of the load-time history changes from a positive to a negative sign; the point of maximum load in constant amplitude loading (see Fig. 1).

3.1.6 *range—in fatigue loading*, the algebraic difference between successive valley and peak loads (positive range or increasing load range), or between successive peak and valley loads (negative range or decreasing load range); see Fig. 1.

NOTE 2—In spectrum loading, range may have a different definition, depending on the counting method used; for example, "overall range" is defined by the algebraic difference between the largest peak and the smallest valley of a given load-time history.

3.1.6.1 *Discussion*—In cycle counting by various methods, it is common to employ ranges between valley and peak loads, or between peak and valley loads, which are not necessarily successive events. In these practices, the definition of the word "range" is broadened so that events of this type are also included.

3.1.7 *reversal—in fatigue loading*, the point at which the first derivative of the load-time history changes sign (see Fig. 1).

NOTE 3—In constant amplitude loading, a cycle is equal to two reversals.

3.1.8 *spectrum loading—in fatigue loading*, a loading in which all of the peak loads are not equal or all of the valley loads are not equal, or both. (Also known as variable amplitude loading or irregular loading.)

3.1.9 *valley—in fatigue loading*, the point at which the first derivative of the load-time history changes from a negative to a positive sign (also known as trough); the point of minimum load in constant amplitude loading (see Fig. 1).

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *load*—used in these practices to denote force, stress, strain, torque, acceleration, deflection, or other parameters of interest.

3.2.2 *reference load—for spectrum loading*, used in these practices to denote the loading level that represents a steady-state condition upon which load variations are superimposed.
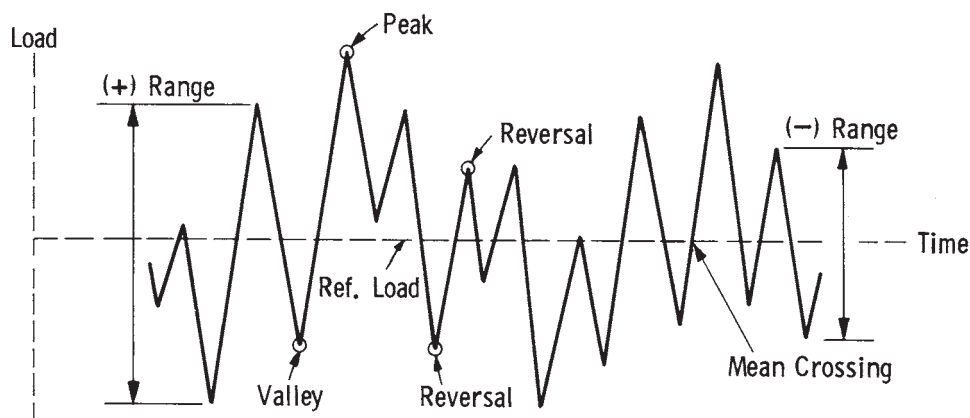

FIG. 1 Basic Fatigue Loading Parameters

The reference load may be identical to the mean load of the history, but this is not required.

3.3 For other definitions of terms used in these practices refer to Definitions E 912.

4. Significance and Use

4.1 Cycle counting is used to summarize (often lengthy) irregular load-versus-time histories by providing the number of times cycles of various sizes occur. The definition of a cycle varies with the method of cycle counting. These practices cover the procedures used to obtain cycle counts by various methods, including level-crossing counting, peak counting, simple-range counting, range-pair counting, and rainflow counting. Cycle counts can be made for time histories of force, stress, strain, torque, acceleration, deflection, or other loading parameters of interest.

5. Procedures for Cycle Counting

5.1 *Level-Crossing Counting*:

5.1.1 Results of a level-crossing count are shown in Fig. 2(*a*). One count is recorded each time the positive sloped portion of the load exceeds a preset level above the reference load, and each time the negative sloped portion of the load exceeds a preset level below the reference load. Reference load crossings are counted on the positive sloped portion of the loading history. It makes no difference whether positive or negative slope crossings are counted. The distinction is made only to reduce the total number of events by a factor of two.

5.1.2 In practice, restrictions on the level-crossing counts are often specified to eliminate small amplitude variations which can give rise to a large number of counts. This may be accomplished by filtering small load excursions prior to cycle counting. A second method is to make no counts at the reference load and to specify that only one count be made between successive crossings of a secondary lower level associated with each level above the reference load, or a secondary higher level associated with each level below the reference load. Fig. 2(*b*) illustrates this second method. A variation of the second method is to use the same secondary level for all counting levels above the reference load, and another for all levels below the reference load. In this case the levels are generally not evenly spaced.

5.1.3 The most damaging cycle count for fatigue analysis is derived from the level-crossing count by first constructing the largest possible cycle, followed by the second largest, etc., until all level crossings are used. Reversal points are assumed to occur halfway between levels. This process is illustrated by Fig. 2(*c*). Note that once this most damaging cycle count is obtained, the cycles could be applied in any desired order, and this order could have a secondary effect on the amount of damage. Other methods of deriving a cycle count from the level-crossings count could be used.

5.2 *Peak Counting*:

5.2.1 Peak counting identifies the occurrence of a relative maximum or minimum load value. Peaks above the reference load level are counted, and valleys below the reference load level are counted, as shown in Fig. 3(*a*). Results for peaks and valleys are usually reported separately. A variation of this method is to count all peaks and valleys without regard to the reference load.

5.2.2 To eliminate small amplitude loadings, mean-crossing peak counting is often used. Instead of counting all peaks and valleys, only the largest peak or valley between two successive mean crossings is counted as shown in Fig. 3(*b*).

5.2.3 The most damaging cycle count for fatigue analysis is derived from the peak count by first constructing the largest possible cycle, using the highest peak and lowest valley, followed by the second largest cycle, etc., until all peak counts are used. This process is illustrated by Fig. 3(*c*). Note that once this most damaging cycle count is obtained, the cycles could be applied in any desired order, and this order could have a secondary effect on the amount of damage. Alternate methods of deriving a cycle count, such as randomly selecting pairs of peaks and valleys, are sometimes used.

5.3 *Simple-Range Counting*:

5.3.1 For this method, a range is defined as the difference between two successive reversals, the range being positive when a valley is followed by a peak and negative when a peak is followed by a valley. The method is illustrated in Fig. 4. Positive ranges, negative ranges, or both, may be counted with this method. If only positive or only negative ranges are counted, then each is counted as one cycle. If both positive and negative ranges are counted, then each is counted as one-half
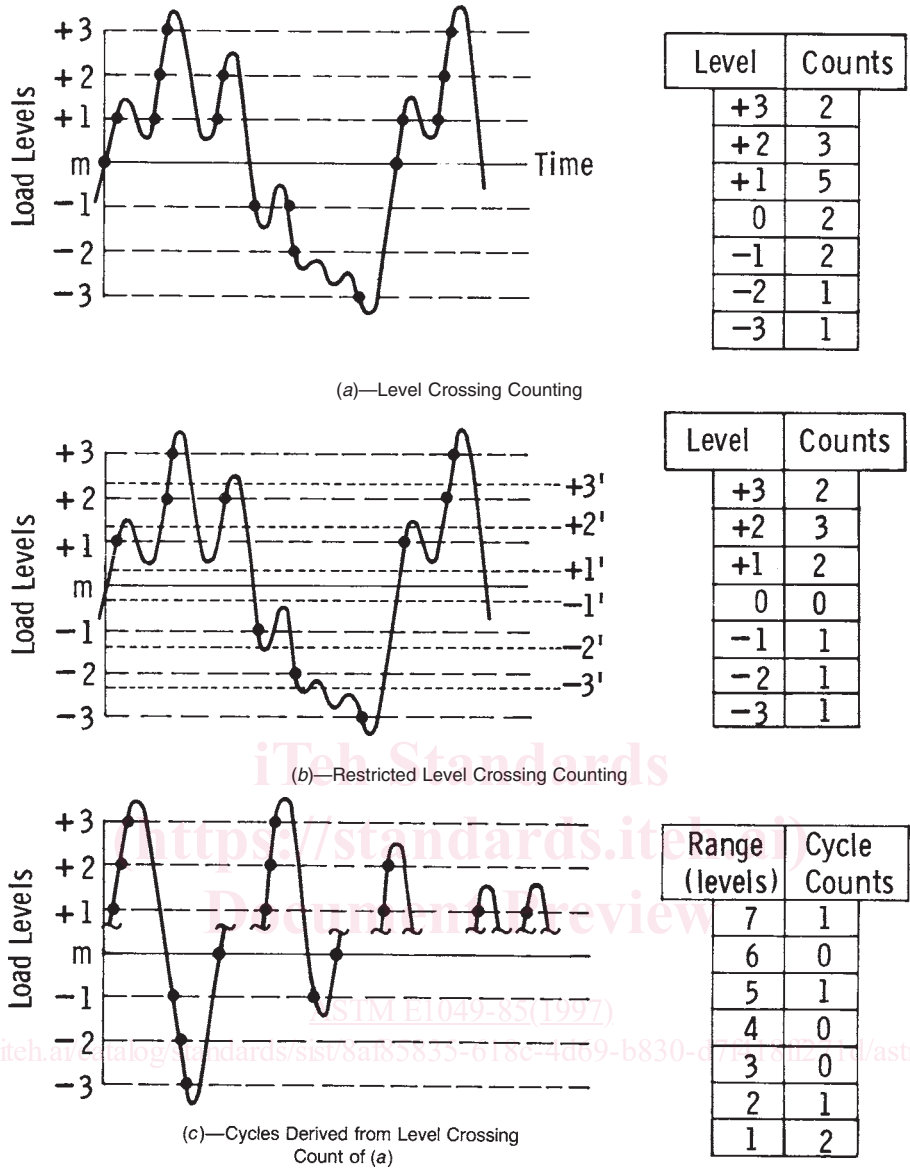
2

Level	Counts
+3	2
+2	3
+1	5
0	2
−1	2
−2	1
−3	1

(a)—Level Crossing Counting

Level	Counts
+3	2
+2	3
+1	2
0	0
−1	1
−2	1
−3	1

(b)—Restricted Level Crossing Counting

Range (levels)	Cycle Counts
7	1
6	0
5	1
4	0
3	0
2	1
1	2

(c)—Cycles Derived from Level Crossing Count of (a)

FIG. 2 Level-Crossing Counting Example

cycle. Ranges smaller than a chosen value are usually eliminated before counting.

5.3.2 When the mean value of each range is also counted, the method is called simple range-mean counting. For the example of Fig. 4, the result of a simple range-mean count is given in X1.1 in the form of a range-mean matrix.

5.4 *Rainflow Counting and Related Methods*:

5.4.1 A number of different terms have been employed in the literature to designate cycle-counting methods which are similar to the rainflow method. These include range-pair counting (**1, 2**),[3] the Hayes method (**3**), the original rainflow method (**4-6**), range-pair-range counting (**7**), ordered overall range counting (**8**), racetrack counting (**9**), and hysteresis loop counting (**10**). If the load history begins and ends with its maximum peak, or with its minimum valley, all of these give identical counts. In other cases, the counts are similar, but not generally identical. Three methods in this class are defined here: range-pair counting, rainflow counting, and a simplified method for repeating histories.

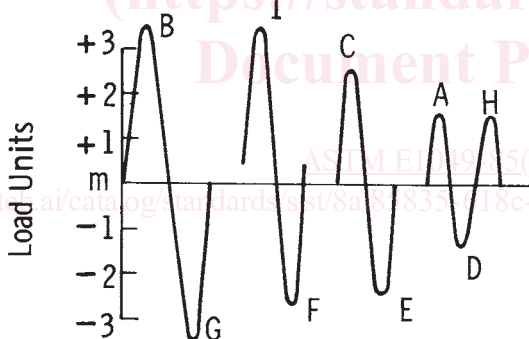[3] The boldface numbers in parentheses refer to the list of references appended to these practices.

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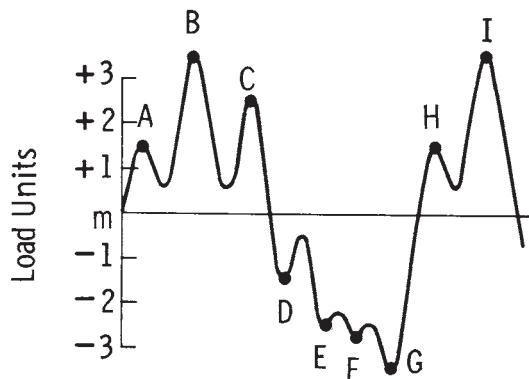
Peak	Counts
+3.5	2
+2.5	1
+1.5	2
−1.5	1
−2.5	1
−2.7	1
−3.5	1

(a)—Peak Counting
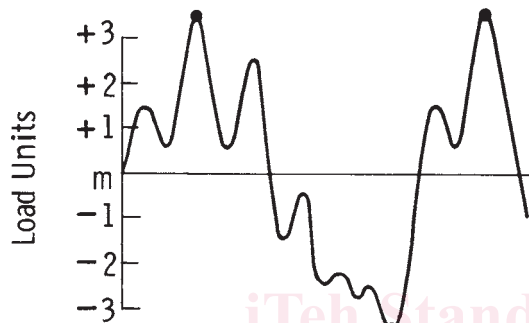
Peak	Counts
+3.5	2
−3.5	1

(b)—Mean Crossing Peak Counting

Range (units)	Cycle Counts
7	1
6.2	1
5	1
3	1.5

(c)—Cycles Derived from Peak Count of (a)

FIG. 3 Peak Counting Example

5.4.2 The various methods similar to the rainflow method may be used to obtain cycles and the mean value of each cycle; they are referred to as two-parameter methods. When the mean value is ignored, they are one-parameter methods, as are simple-range counting, peak counting, etc.

5.4.3 *Range-Pair Counting*—The range-paired method counts a range as a cycle if it can be paired with a subsequent loading in the opposite direction. Rules for this method are as follows:

5.4.3.1 Let X denote range under consideration; and Y, previous range adjacent to X.

(1) Read next peak or valley. If out of data, go to Step 5.

(2) If there are less than three points, go to Step 1. Form ranges X and Y using the three most recent peaks and valleys that have not been discarded.

(3) Compare the absolute values of ranges X and Y.

*(a)* If $X < Y$, go to Step 1.

(b) If $X \geq Y$, go to Step 4.

(4) Count range Y as one cycle and discard the peak and valley of Y; go to Step 2.

(5) The remaining cycles, if any, are counted by starting at the end of the sequence and counting backwards. If a single range remains, it may be counted as a half or full cycle.

5.4.3.2 The load history in Fig. 4 is replotted as Fig. 5(a) and is used to illustrate the process. Details of the cycle counting are as follows:

(1) $Y = |A\text{-}B|$; $X = |B\text{-}C|$; and $X > Y$. Count $|A\text{-}B|$ as one cycle and discard points A and B. (See Fig. 5(b). Note that a cycle is formed by pairing range A-B and a portion of range B-C.)

4
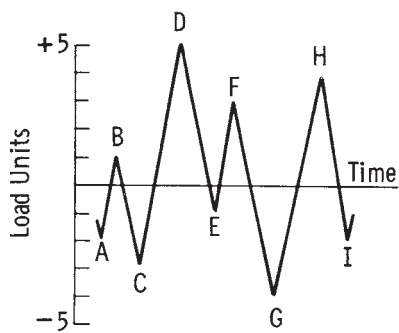
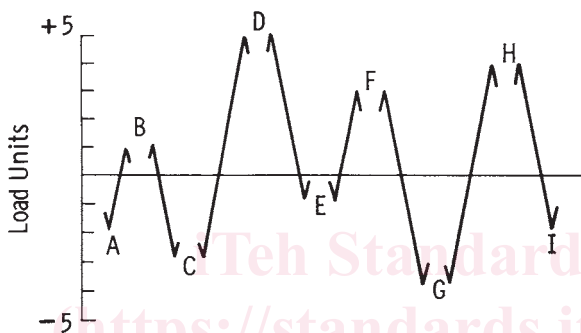| Range (units)	Cycle Counts	Events
10	0	
9	0	
8	1.0	C-D, G-H
7	0.5	F-G
6	1.0	D-E, H-I
5	0	
4	1.0	B-C, E-F
3	0.5	A-B
2	0	
1	0	

FIG. 4 Simple Range Counting Example—Both Positive and Negative Ranges Counted

(2) $Y = |C\text{-}D|$; $X = |D\text{-}E|$; and $X < Y$.

(3) $Y = |D\text{-}E|$; $X = |E\text{-}F|$; and $X < Y$.

(4) $Y = |E\text{-}F|$; $X = |F\text{-}G|$; and $X > Y$. Count $|E\text{-}F|$ as one cycle and discard points E and F. (See Fig. 5(*c*).)

(5) $Y = |C\text{-}D|$; $X = |D\text{-}G|$; and $X > Y$. Count $|C\text{-}D|$ as one cycle and discard points C and D. (See Fig. 5(*d*).)

(6) $Y = |G\text{-}H|$; $X = |H\text{-}I|$; and $X < Y$. Go to the end and count backwards.

(7) $Y = |H\text{-}I|$; $X = |G\text{-}H|$; and $X > Y$. Count $|H\text{-}I|$ as one cycle and discard points H and I. (See Fig. 5(*e*).)

(8) End of counting. See the table in Fig. 5 for a summary of the cycles counted in this example, and see Appendix X1.2 for this cycle count in the form of a range-mean matrix.

5.4.4 *Rainflow Counting*:

5.4.4.1 Rules for this method are as follows: let X denote range under consideration; Y, previous range adjacent to X; and S, starting point in the history.

(1) Read next peak or valley. If out of data, go to Step 6.

(2) If there are less than three points, go to Step 1. Form ranges X and Y using the three most recent peaks and valleys that have not been discarded.

(3) Compare the absolute values of ranges X and Y.

(*a*) If $X < Y$, go to Step 1.

(*b*) If $X \geq Y$, go to Step 4.

(4) If range Y contains the starting point S, go to Step 5; otherwise, count range Y as one cycle; discard the peak and valley of Y; and go to Step 2.

(5) Count range Y as one-half cycle; discard the first point (peak or valley) in range Y; move the starting point to the second point in range Y; and go to Step 2.

(6) Count each range that has not been previously counted as one-half cycle.

5.4.4.2 The load history of Fig. 4 is replotted as Fig. 6(*a*) and is used to illustrate the process. Details of the cycle counting are as follows:

(1) $S = A$; $Y = |A\text{-}B|$; $X = |B\text{-}C|$; $X > Y$. Y contains S, that is, point A. Count $|A\text{-}B|$ as one-half cycle and discard point A; $S = B$. (See Fig. 6(*b*).)

(2) $Y = |B\text{-}C|$; $X = |C\text{-}D|$; $X > Y$. Y contains S, that is, point B. Count $|B\text{-}C|$ as one-half cycle and discard point B; $S = C$. (See Fig. 6(*c*).)

(3) $Y = |C\text{-}D|$; $X = |D\text{-}E|$; $X < Y$.

(4) $Y = |D\text{-}E|$; $X = |E\text{-}F|$; $X < Y$.

(5) $Y = |E\text{-}F|$; $X = |F\text{-}G|$; $X > Y$. Count $|E\text{-}F|$ as one cycle and discard points E and F. (See Fig. 6(*d*). Note that a cycle is formed by pairing range $E\text{-}F$ and a portion of range $F\text{-}G$.)

(6) $Y = |C\text{-}D|$; $X = |D\text{-}G|$; $X > Y$; Y contains S, that is, point C. Count $|C\text{-}D|$ as one-half cycle and discard point C. $S = D$. (See Fig. 6(*e*).)

(7) $Y = |D\text{-}G|$; $X = |G\text{-}H|$; $X < Y$.

(8) $Y = |G\text{-}H|$; $X = |H\text{-}I|$; $X < Y$. End of data.

(9) Count $|D\text{-}G|$ as one-half cycle, $|G\text{-}H|$ as one-half cycle, and $|H\text{-}I|$ as one-half cycle. (See Fig. 6(*f*).)

FIG. 5 Range-Pair Counting Example

The table in Fig. 5:

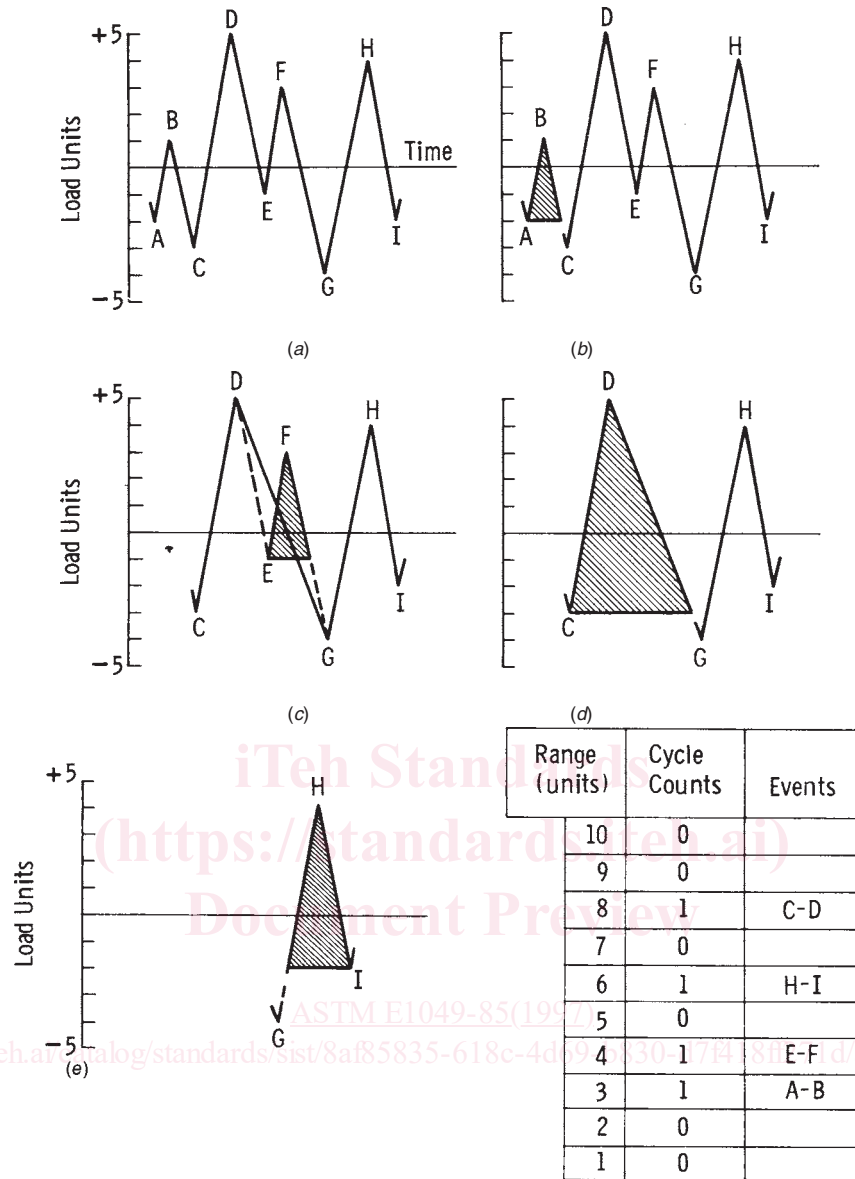| Range (units)	Cycle Counts	Events
10	0	
9	0	
8	1	C-D
7	0	
6	1	H-I
5	0	
4	1	E-F
3	1	A-B
2	0	
1	0	

(10) End of counting. See the table in Fig. 6 for a summary of the cycles counted in this example, and see Appendix X1.3 for this cycle count in the form of a range-mean matrix.

5.4.5 *Simplified Rainflow Counting for Repeating Histories*:

5.4.5.1 It may be desirable to assume that a typical segment of a load history is repeatedly applied. Here, once either the maximum peak or minimum valley is reached for the first time, the range-pair count is identical for each subsequent repetition of the history. The rainflow count is also identical for each subsequent repetition of the history, and for these subsequent repetitions, the rainflow count is the same as the range-pair count. Such a repeating history count contains no half cycles, only full cycles, and each cycle can be associated with a closed stress-strain hysteresis loop **(4, 10-12).** Rules for obtaining such a repeating history cycle count, called "simplified rain-flow counting for repeating histories" are as follows:

5.4.5.2 Let *X* denote range under consideration; and *Y*, previous range adjacent to *X*.

(1) Arrange the history to start with either the maximum peak or the minimum valley. (More complex procedures are available that eliminate this requirement; see **(Ref 12)**.

(2) Read the next peak or valley. If out of data, STOP.

(3) If there are less than three points, go to Step 2. Form ranges *X* and *Y* using the three most recent peaks and valleys that have not been discarded.

(4) Compare the absolute values of ranges *X* and *Y*.

(a) If *X* < *Y*, go to Step 2.

(b) If *X* ≥ *Y*, go to Step 5.

(5) Count range *Y* as one cycle; discard the peak and valley of *Y*; and go to Step 3.

5.4.5.3 The loading history of Fig. 4 is plotted as a repeating load history in Fig. 7(*a*) and is used to illustrate the process. Rearranging the history to start with the maximum peak gives Fig. 7(*b*), Reversal Points *A*, *B*, and *C* being moved to the end of the history. Details of the cycle counting are as follows:

(1) $Y = |D\text{-}E|$; $X = |E\text{-}F|$; $X < Y$.