



Basic quantities in cutting and grinding — Part 2 : Geometry of the active part of cutting tools — General conversion formulae to relate tool and working angles

Définitions de base pour la coupe et la rectification — Partie 2 : Géométrie de la partie active des outils coupants — Formules de conversion générales liant les angles de l'outil en main et les angles en travail

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Basic quantities in cutting and grinding — Part 2 : Geometry of the active part of cutting tools — General conversion formulae to relate tool and working angles

0 Introduction

This part of ISO 3002 presents formulae which can be used to convert tool angles to working angles and vice versa. The formulae are general and can be used for all possible cutting conditions. Tool angles (angles in the tool-in-hand system) and working angles (angles in the tool-in-use system) are defined in ISO 3002/1, together with the sign conventions for these angles.

The tool-in-hand reference system of planes (used to define tool angles) rotates with the cutting tool whenever the orientation of the cutting tool is changed relative to the machine tool. Similarly, the orientation of the tool-in-use reference system of planes (used to define working angles) changes with changes in the resultant cutting direction. It is therefore necessary, in order to relate the tool-in-hand and tool-in-use reference system of planes, to relate both of them to a third reference system of planes, the machine reference system of planes, which does not rotate when the tool is re-oriented or when the resultant cutting direction changes.

The relationship between the tool-in-hand and the machine reference systems of planes defines the setting of the tool in the machine. The relative position of the tool-in-use and the machine reference systems of planes is defined by the motion of the tool relative to the workpiece.

Certain general principles have been taken into consideration for the establishment of the conversion formulae :

- a) The definitions of the machine reference system of axes as defined in ISO 841 have been adopted.
- b) The origin of each coordinate system is considered to be at the "selected point" on the cutting edge and at a given moment in time.
- c) Transformation angles are defined in such a manner as to be computable readily from available workshop data.

1 Scope and field of application

This part of ISO 3002 deals with the establishment and application of the conversion formulae under the following headings :

- a) Definitions of the coordinate axes for the tool-in-hand and for the tool-in-use derived from the appropriate planes defined in ISO 3002/1. Definitions for the machine axes and planes are based on ISO 841.
- b) Definitions of the setting angles and motion angles.
- c) Conversion formulae.
- d) Practical examples.

NOTE — In addition to the terms given in the three official ISO languages (English, French, Russian), this part of ISO 3002 gives the equivalent terms in German and Dutch; these terms have been included at the request of Technical Committee ISO/TC 29, and are published under the responsibility of the committee members for Germany, F.R. (DIN) and the Netherlands (NNI). However, only the terms given in the official languages can be considered as ISO terms.

2 Definitions of the coordinates axes

2.1 Tool-in-hand and tool-in-use axes

Axes are defined for the tool-in-hand using the tool-in-hand reference system of planes and for the tool-in-use using the tool-in-use reference system of planes. The axes are defined by the intersection of planes; the appropriate pairs of planes are given in parentheses below.

2.1.1 Tool-in-hand axes

f-Set of coordinate axes : $X_f (P_r P_p)$, $Y_f (P_p P_t)$, $Z_f (P_f P_r)$.

The X_f -axis is positive in a direction which is away from both the reference corner at the tool, (as defined in ISO 3002/1) and the assumed machined surface; the Y_f -axis is positive in a direction opposite to the assumed direction of primary motion; the Z_f -axis is positive in a direction away from the assumed transient surface on the workpiece. [See figures 1 and 2.]

NOTE — The resulting defined axes may conform to either a right hand or a left hand coordinate axis system depending upon the type of tool considered, its intended use and the location of the selected point on the cutting edges.

In a similar way, other sets of tool-in-hand axes can be defined. However, for the practical application of the transformation formulae, only the f-set of coordinate axes is required. Therefore, in what follows, the more general term "tool-in-hand axes" will be used to designate the "f-set of coordinate axes".

NOTE — For a given tool, the set of tool-in-hand axes is unique. The axes are determined only by the operation the tool is "assumed" to perform (for example, for a cylindrical turning tool, the tool-in-hand axes do not change relative to the tool when the tool is used for a facing operation).

2.1.2 Tool-in-use axes

f_e-Set of coordinate axes : $X_{fe} (P_{re} P_{pe})$, $Y_{fe} (P_{pe} P_{fe})$, $Z_{fe} (P_{fe} P_{re})$.

Similarly in this system, the X_{fe} -axis is positive in a direction which is away from both the reference corner at the tool, (as defined in ISO 3002/1) and the assumed machined surface; the Y_{fe} -axis is positive in the direction opposite to the resultant cutting direction; the Z_{fe} -axis is positive in a direction away from the transient surface on the workpiece. [See figures 1 and 2.]

NOTE — The resulting defined axes may conform to either a right hand or a left hand coordinate axis system depending upon the type of tool considered, its intended use and the location of the selected point on the cutting edges.

In a similar way, other sets of tool-in-use axes can be defined. However, for the practical application of the transformation formulae, only the f_e-set of coordinate axes is required. Therefore, in what follows, the more general term "tool-in-use axes" will be used to designate the "f_e-set of coordinate axes".

NOTE — The orientation of the tool-in-use axes, relative to the tool, may vary depending on the resultant cutting direction and the orientation of the tool in the machine.

2.2 Definitions of the machines axes and planes

For each type of machine tool, a machine reference system of axes is defined in ISO 841.

Unfortunately, the defined axis nomenclature is not suitable for direct use in this part of ISO 3002, however, it has been used to determine the orientation of a machine reference system of planes.

With the tool in its "zero position" in the machine such that the tool-in-hand reference system of planes ($P_r-P_f-P_p$) coincides with the machine reference system of planes (defined by the $X-Y-Z$ axes of ISO 841) and with the tool in its "most natural position", so that the assumed working plane P_f is oriented to be parallel to the direction of component feed motion predominant in the particular machining operation considered, a set of fixed machine setting axes ($X_m-Y_m-Z_m$) is defined to coincide with the tool-in-hand axes. Thus, with the tool in its "zero position" :

- machine setting axis X_m coincides with tool-in-hand axis X_f ;
- machine setting axis Y_m coincides with tool-in-hand axis Y_f ;
- machine setting axis Z_m coincides with tool-in-hand axis Z_f .

3 Definition of the setting and motion angles

Two sets of three angles (Euler angles) are required, one set to define the orientation of the tool-in-hand axes (X_f, Y_f, Z_f) in relation to the machine setting axes (X_m, Y_m, Z_m) (see figure 3) and the second set to define the orientation of the tool-in-use axes (X_{fe}, Y_{fe}, Z_{fe}) in relation to the same machine setting axes (see figure 4).

It should be noted that the first set of angles corresponds to the practical procedure of positioning the tool with a classical tool holder [see figure 3].

In some practical cases the second set of angles is not directly related to the available workshop data. For these cases an alternative way of proceeding is outlined in 3.3, using the components of feed speed and cutting speed with respect to the machine setting axes in order to evaluate the auxiliary angles in 4.3.

3.1 The setting angles

The position of the tool-in-hand reference system of planes defined by X_f, Y_f, Z_f , with respect to the machine reference system of planes defined by X_m, Y_m, Z_m is determined by three angles : the plan setting angle G , the elevation setting angle H , the roll setting angle L . The definitions of these angles are given below. The setting angles are illustrated in figures 3 a) and 3 b). It is assumed that the tool is initially in its "zero" position on the machine tool, such that the tool-in-hand reference system of planes coincides with the machine reference system of planes and the tool is in its "most natural" position on the machine tool (see 2.2). The tool is then rotated successively through the angles G, H, L , as illustrated by the graduated scales of the tool holder shown in figure 3 a). The angles G, H, L thus define the positioning of the tool-in-hand reference system of planes with respect to the machine reference system of planes.

The angles may be applied in any order; however, the following explanations of the rotations which are embodied in the definitions assume that the angles are applied in the sequence G, H, L .

3.1.1 The plan setting angle G

The angle between the assumed working plane P_f in its zero position and the assumed working plane P_f in its final position, measured in the tool reference plane P_r in its zero position.

It corresponds to a rotation around the machine setting Y_m -axis. Performing this rotation causes the tool-in-hand coordinate axes to take up an intermediate position in which they are designated X'_f, Y_m, Z'_f . [See figures 3 a) and 3 b).]

The sign convention is defined thus :

Increasing the plan setting angle G in the positive direction decreases the angle κ_{re} and increases the angle ψ_{re} .

3.1.2 The elevation setting angle H

The angle between the machine setting Y_m -axis and its projection on the final position of the assumed working plane P_f of the tool. It corresponds to a rotation around the intermediate Z'_f -axis.

This rotation gives the final position of the tool-in-hand X_f -axis an intermediate position of the tool-in-hand Y_f -axis which is designated Y'_f , and Z'_f is unchanged. Thus, the tool-in-hand axes in this second intermediate position are designated X_f, Y'_f, Z'_f .

The sign convention is defined thus :

Increasing the elevation setting angle H in the positive direction increases the angle γ_{pe} in the positive direction.

3.1.3 The roll setting angle L

The angle between the tool reference plane P_r in its zero position and the tool reference plane P_r in its final position and measured in the assumed working plane P_f in its final position.

It corresponds to a rotation around the final tool-in-hand X_f -axis. This rotation gives the final position of the tool-in-hand axes X_f, Y_f, Z_f in relation to the machine reference axes.

The sign convention is defined thus :

Increasing the roll setting angle L in the positive direction increases the angle α_{fe} .

3.2 The motion angles

The position of the tool-in-use reference system of planes defined by X_{fe} , Y_{fe} , Z_{fe} with respect to the machine reference system of planes defined by X_m , Y_m , Z_m can be determined by three angles : the plan motion angle M , the elevation motion angle N and the roll motion angle T . An alternative method of determining the relative positions is described in 3.3. The definitions of the motion angles are given below. It should be noted that the orientation of the tool-in-use reference system of planes relative to the machine reference system of planes is determined by the direction and magnitudes of cutting speed and feed speed.

The motion angles are illustrated in figure 4. Assume a set of coordinate axes x , y , z , initially coincident with the set of the machine axes X_m , Y_m , Z_m and which can be made coincident with the tool-in-use axes X_{fe} , Y_{fe} , Z_{fe} by means of three Eulerian rotations.

3.2.1 Plan motion angle M

The angle between the Y_m – Z_m machine plane and the working plane P_{fe} measured in the X_m – Z_m machine plane. It corresponds to the auxiliary coordinate system x , y , z being rotated through angle M about the machine setting Y_m -axis into an intermediate position. The coordinate axes in this intermediate position are designated x' , Y_m , z' (see figure 4).

The sign convention is defined thus :

Increasing the plan motion angle M in the positive direction increases the angle κ_{re} and decreases the angle ψ_{re} .

In conventional turning, M is the angle between the rotational axis of the workpiece and the direction of feed motion, thus :

- for cylindrical turning $M = 0$ (see figure 5);
- for turning a cone with top angle T : $M = \frac{T}{2}$;
- for most other operations the plan motion angle M is equal to zero.

3.2.2 Elevation motion angle N

The angle between the machine setting Y_m -axis and its projection on the working plane P_{fe} .

It corresponds to the intermediate coordinate system being rotated about the " Z' "-axis from its position x' , Y_m , z' into a second intermediate position. The coordinate axes in this second intermediate position are designated X_{fe} , y' , z' . This rotation gives the final position of the X_{fe} -axis and an intermediate position of the y' -axis. The designation of z' is unchanged since rotation is about this axis.

The sign convention is defined thus :

Increasing the elevation motion angle N in the positive direction decreases the angle γ_{pe} in the positive direction.

In conventional cylindrical turning, if the selected point on the cutting edge has an offset height h below the rotational axis [see figure 5 b)], the angle N is found from

$$\sin N = \frac{2h}{d}$$

where d is the effective diameter of the workpiece at the selected point on the cutting edge.

In drilling, N is equal to the elevation setting angle H , since for this operation the assumed working plane P_f and the working plane P_{fe} remain coincident and N and H are the angular position of a radius, drawn through the selected point, with respect to the zero position.

In most cases, the elevation motion angle N is zero.

3.2.3 Roll motion angle T

The angle between the X_m – Z_m machine plane and the working reference plane P_{re} measured in the working plane P_{fe} .

It corresponds to a rotation about the X_{fe} -axis of the second intermediate system X_{fe} , y' , z' into the tool-in-use system X_{fe} , Y_{fe} , Z_{fe} .

The sign convention is defined thus :

Increasing the roll motion angle T in the positive direction decreases the angle α_{fe} .

In conventional turning (cylindrical, conical, plunge and face turning), if the direction of primary motion is parallel to the machine Y_m -axis, it is the angle between the direction of primary motion and the resultant cutting direction (i.e. resultant cutting speed angle η),

$$\operatorname{tg} T = \frac{f}{\pi d}$$

where

d is the effective diameter, in millimetres, of the workpiece at the selected point on the cutting edge;

f is the feed, in millimetres per revolution.

In roll (slab) milling and face milling, it is the sum of the roll setting angle L and the resultant cutting speed angle η (see figure 6).

In drilling, T is equal to the resultant cutting speed angle η .

3.3 Alternative method of relating the tool-in-use reference system of planes to the machine reference system of planes

If the value of the motion angles cannot be deduced easily from available workshop data (i.e. conical turning with a non-centred cutting tool) the position of the tool-in-use reference system of planes defined by X_{fe}, Y_{fe}, Z_{fe} with respect to the machine reference system of planes defined by X_m, Y_m, Z_m can be determined from the X_m, Y_m and Z_m components of the feed speed v_f and cutting speed v_c , which are designated :

components of feed speed $(v_f)_{X_m}, (v_f)_{Y_m}, (v_f)_{Z_m}$

components of cutting speed $(v_c)_{X_m}, (v_c)_{Y_m}, (v_c)_{Z_m}$

and used to evaluate auxiliary angles as presented in 4.3.

4 List of conversion formulae — General case

Direct transformation is the expression giving the working angles as a function of the tool angles, setting angles and motion angles. Inverse transformation is the expression giving the tool angles as a function of the working angles, setting angles and motion angles.

In the equations, auxiliary angles are used and are listed in 4.3. They are designated by the general notation (i, j) and are defined as functions of the tool setting angles and the motion angles (respectively components of feed speed and cutting speed).

Although the expressions are complex, in practice they are simplified considerably since certain angles are equal usually to either zero or 90° .

In addition to the conversion formulae below, the table "Relations between the angles in the 'tool-in-hand system' " included in 6.6 of ISO 3002/1 should be used. The same table can be used for the working angles provided that the suffix "e" is added to all angles.

4.1 Direct transformation (working angles as a function of tool angles)

4.1.1 Working cutting edge inclination

$$\sin \lambda_{se} = \cos \lambda_s \sin \kappa_r \cos (1,2) + \sin \lambda_s \cos (2,2) - \cos \lambda_s \cos \kappa_r \cos (3,2)$$

4.1.2 Working cutting edge angle

$$\operatorname{tg} \kappa_{re} = - \frac{\cos \lambda_s \sin \kappa_r \cos (1,1) + \sin \lambda_s \cos (2,1) - \cos \lambda_s \cos \kappa_r \cos (3,1)}{\cos \lambda_s \sin \kappa_r \cos (1,3) + \sin \lambda_s \cos (2,3) - \cos \lambda_s \cos \kappa_r \cos (3,3)}$$

4.1.3 Working normal rake

$$\sin \gamma_{ne} = \frac{1}{\cos \lambda_{se}} \left\{ [-\sin \gamma_n \sin \lambda_s \sin \kappa_r + \cos \gamma_n \cos \kappa_r] \cos (1,2) + \right. \\ \left. + [\sin \gamma_n \sin \lambda_s \cos \kappa_r + \cos \gamma_n \sin \kappa_r] \cos (3,2) + \sin \gamma_n \cos \lambda_s \cos (2,2) \right\}$$

Alternatively :

$$\cos (\gamma_{ne} - \gamma_n) = [\cos \kappa_r \cos (1,1) + \sin \kappa_r \cos (3,1)] \cos \kappa_{re} + [\cos \kappa_r \cos (1,3) + \sin \kappa_r \cos (3,3)] \sin \kappa_{re}$$

NOTE — To make use of this latter relationship it is essential to have previous knowledge of whether the working normal rake is larger or smaller than the tool normal rake.

4.1.4 Working normal clearance

$$\cos \alpha_{ne} = \frac{1}{\cos \lambda_{se}} \left\{ [\sin \alpha_n \cos \kappa_r - \cos \alpha_n \sin \lambda_s \sin \kappa_r] \cos (1,2) + \right. \\ \left. + [\cos \alpha_n \sin \lambda_s \cos \kappa_r + \sin \alpha_n \sin \kappa_r] \cos (3,2) + \cos \alpha_n \cos \lambda_s \cos (2,2) \right\}$$

Alternatively :

$$\cos (\alpha_{ne} - \alpha_n) = [\cos \kappa_r \cos (1,1) + \sin \kappa_r \cos (3,1)] \cos \kappa_{re} + [\cos \kappa_r \cos (1,3) + \sin \kappa_r \cos (3,3)] \sin \kappa_{re}$$

NOTE — To make use of this latter relationship it is essential to have previous knowledge of whether the working normal clearance is larger or smaller than the tool normal clearance.

It should be further noted that if either the working normal rake γ_{ne} or the working normal clearance α_{ne} has been determined, the other can be derived from the relationship :

$$\alpha_{ne} + \beta_{ne} + \gamma_{ne} = 90^\circ \quad \text{and} \quad \beta_{ne} = \beta_n$$

4.1.5 Working side rake

$$\tan \gamma_{fe} = \frac{\tan \gamma_f \cos (3,3) + \tan \gamma_p \cos (1,3) - \cos (2,3)}{\cos (2,2) - \tan \gamma_f \cos (3,2) - \tan \gamma_p \cos (1,2)}$$

4.1.6 Working back rake

$$\tan \gamma_{pe} = \frac{\tan \gamma_f \cos (3,1) + \tan \gamma_p \cos (1,1) - \cos (2,1)}{\cos (2,2) - \tan \gamma_f \cos (3,2) - \tan \gamma_p \cos (1,2)}$$

4.1.7 Working side clearance

$$\cot \alpha_{fe} = \frac{\cot \alpha_f \cos (3,3) + \cot \alpha_p \cos (1,3) - \cos (2,3)}{\cos (2,2) - \cot \alpha_f \cos (3,2) - \cot \alpha_p \cos (1,2)}$$

4.1.8 Working back clearance

$$\cot \alpha_{pe} = \frac{\cot \alpha_f \cos (3,1) + \cot \alpha_p \cos (1,1) - \cos (2,1)}{\cos (2,2) - \cot \alpha_f \cos (3,2) - \cot \alpha_p \cos (1,2)}$$

4.2 Inverse transformation (tool angles as a function of working angles)

4.2.1 Tool cutting edge inclination

$$\sin \lambda_s = \cos \lambda_{se} \sin \kappa_{re} \cos (2,1) + \sin \lambda_{se} \cos (2,2) - \cos \lambda_{se} \cos \kappa_{re} \cos (2,3)$$

4.2.2 Tool cutting edge angle

$$\tan \kappa_r = - \left[\frac{\cos \lambda_{se} \sin \kappa_{re} \cos (1,1) + \sin \lambda_{se} \cos (1,2) - \cos \lambda_{se} \cos \kappa_{re} \cos (1,3)}{\cos \lambda_{se} \sin \kappa_{re} \cos (3,1) + \sin \lambda_{se} \cos (3,2) - \cos \lambda_{se} \cos \kappa_{re} \cos (3,3)} \right]$$

4.2.3 Tool normal rake

$$\sin \gamma_n = \frac{1}{\cos \lambda_s} \left\{ [-\sin \gamma_{ne} \sin \lambda_{se} \sin \kappa_{re} + \cos \gamma_{ne} \cos \kappa_{re}] \cos (2,1) + \right. \\ \left. + [\sin \gamma_{ne} \sin \lambda_{se} \cos \kappa_{re} + \cos \gamma_{ne} \sin \kappa_{re}] \cos (2,3) + \sin \gamma_{ne} \cos \lambda_{se} \cos (2,2) \right\}$$

Alternatively, the tool normal rake γ_n may be derived from the expression for $\cos (\gamma_{ne} - \gamma_n)$ which is listed in 4.1. However, to use this relationship it is essential to have previous knowledge of whether the tool normal rake is larger or smaller than the working normal rake.

4.2.4 Tool normal clearance

$$\cos \alpha_n = \frac{1}{\cos \lambda_s} \left\{ [-\cos \alpha_{ne} \sin \lambda_{se} \sin \kappa_{re} + \sin \alpha_{ne} \cos \kappa_{re}] \cos (2,1) + \right. \\ \left. + [\cos \alpha_{ne} \sin \lambda_{se} \cos \kappa_{re} + \sin \alpha_{ne} \sin \kappa_{re}] \cos (2,3) + \cos \alpha_{ne} \cos \lambda_{se} \cos (2,2) \right\}$$

Alternatively, the tool normal clearance α_n may be derived from the expression for $\cos (\alpha_{ne} - \alpha_n)$ which is listed in 4.1. However, to use this relationship it is essential to have previous knowledge of whether the tool normal clearance is larger or smaller than the working normal clearance.

It should be further noted that if either the tool normal rake or tool normal clearance has been determined, the other can be derived from the relationship :

$$\alpha_n + \beta_n + \gamma_n = 90^\circ \quad \text{and} \quad \beta_n = \beta_{ne}$$

4.2.5 Tool side rake

$$\operatorname{tg} \gamma_f = \frac{\operatorname{tg} \gamma_{fe} \cos (3,3) + \operatorname{tg} \gamma_{pe} \cos (3,1) - \cos (3,2)}{-\operatorname{tg} \gamma_{fe} \cos (2,3) - \operatorname{tg} \gamma_{pe} \cos (2,1) + \cos (2,2)}$$

4.2.6 Tool back rake

$$\operatorname{tg} \gamma_p = \frac{\operatorname{tg} \gamma_{fe} \cos (1,3) + \operatorname{tg} \gamma_{pe} \cos (1,1) - \cos (1,2)}{-\operatorname{tg} \gamma_{fe} \cos (2,3) - \operatorname{tg} \gamma_{pe} \cos (2,1) + \cos (2,2)}$$

4.2.7 Tool side clearance

$$\operatorname{ctg} \alpha_f = \frac{\operatorname{ctg} \alpha_{fe} \cos (3,3) + \operatorname{ctg} \alpha_{pe} \cos (3,1) - \cos (3,2)}{-\operatorname{ctg} \alpha_{fe} \cos (2,3) - \operatorname{ctg} \alpha_{pe} \cos (2,1) + \cos (2,2)}$$

4.2.8 Tool back clearance

$$\operatorname{ctg} \alpha_p = \frac{\operatorname{ctg} \alpha_{fe} \cos (1,3) + \operatorname{ctg} \alpha_{pe} \cos (1,1) - \cos (1,2)}{-\operatorname{ctg} \alpha_{fe} \cos (2,3) - \operatorname{ctg} \alpha_{pe} \cos (2,1) + \cos (2,2)}$$

4.3 Auxiliary angles

The geometrical meaning of the auxiliary angles (i, j) is as follows :

The angle (1,1) is the angle between the axes X_f and X_{fe}

The angle (2,1) is the angle between the axes Y_f and X_{fe}

The angle (3,1) is the angle between the axes Z_f and X_{fe}

The angle (1,2) is the angle between the axes X_f and Y_{fe}

The angle (2,2) is the angle between the axes Y_f and Y_{fe}

The angle (3,2) is the angle between the axes Z_f and Y_{fe}

The angle (1,3) is the angle between the axes X_f and Z_{fe}

The angle (2,3) is the angle between the axes Y_f and Z_{fe}

The angle (3,3) is the angle between the axes Z_f and Z_{fe}

The values of $\cos(i, j)$ can be expressed as a combination of the cosines of the angles between the tool-in-hand axes (X_f, Y_f, Z_f) and the machine setting axes (X_m, Y_m, Z_m) on one hand and the cosines of the angles between the tool-in-use axes (X_{fe}, Y_{fe}, Z_{fe}) and the machine setting axes (X_m, Y_m, Z_m) on the other hand :

$$\begin{aligned}\cos(1,1) &= \cos(X_f, X_m) \cos(X_{fe}, X_m) + \cos(X_f, Y_m) \cos(X_{fe}, Y_m) + \cos(X_f, Z_m) \cos(X_{fe}, Z_m) \\ \cos(2,1) &= \cos(Y_f, X_m) \cos(X_{fe}, X_m) + \cos(Y_f, Y_m) \cos(X_{fe}, Y_m) + \cos(Y_f, Z_m) \cos(X_{fe}, Z_m) \\ \cos(3,1) &= \cos(Z_f, X_m) \cos(X_{fe}, X_m) + \cos(Z_f, Y_m) \cos(X_{fe}, Y_m) + \cos(Z_f, Z_m) \cos(X_{fe}, Z_m) \\ \cos(1,2) &= \cos(X_f, X_m) \cos(Y_{fe}, X_m) + \cos(X_f, Y_m) \cos(Y_{fe}, Y_m) + \cos(X_f, Z_m) \cos(Y_{fe}, Z_m) \\ \cos(2,2) &= \cos(Y_f, X_m) \cos(Y_{fe}, X_m) + \cos(Y_f, Y_m) \cos(Y_{fe}, Y_m) + \cos(Y_f, Z_m) \cos(Y_{fe}, Z_m) \\ \cos(3,2) &= \cos(Z_f, X_m) \cos(Y_{fe}, X_m) + \cos(Z_f, Y_m) \cos(Y_{fe}, Y_m) + \cos(Z_f, Z_m) \cos(Y_{fe}, Z_m) \\ \cos(1,3) &= \cos(X_f, X_m) \cos(Z_{fe}, X_m) + \cos(X_f, Y_m) \cos(Z_{fe}, Y_m) + \cos(X_f, Z_m) \cos(Z_{fe}, Z_m) \\ \cos(2,3) &= \cos(Y_f, X_m) \cos(Z_{fe}, X_m) + \cos(Y_f, Y_m) \cos(Z_{fe}, Y_m) + \cos(Y_f, Z_m) \cos(Z_{fe}, Z_m) \\ \cos(3,3) &= \cos(Z_f, X_m) \cos(Z_{fe}, X_m) + \cos(Z_f, Y_m) \cos(Z_{fe}, Y_m) + \cos(Z_f, Z_m) \cos(Z_{fe}, Z_m)\end{aligned}$$

The angles between the tool-in-hand axes and machine setting axes can be expressed as a function of the setting angles :

$$\begin{aligned}\cos(X_f, X_m) &= \cos G \cos H \\ \cos(X_f, Y_m) &= \sin H \\ \cos(X_f, Z_m) &= -\sin G \cos H \\ \cos(Y_f, X_m) &= -\cos G \sin H \cos L + \sin G \sin L \\ \cos(Y_f, Y_m) &= \cos H \cos L \\ \cos(Y_f, Z_m) &= \sin G \sin H \cos L + \cos G \sin L \\ \cos(Z_f, X_m) &= \cos G \sin H \sin L + \sin G \cos L \\ \cos(Z_f, Y_m) &= -\cos H \sin L \\ \cos(Z_f, Z_m) &= -\sin G \sin H \sin L + \cos G \cos L\end{aligned}$$

The angles between the tool-in-use axes and the machine setting axes can be expressed in two different ways : as a function of the motion angles (1st way) or as a function of the components of cutting speed and feed speed (2nd way).

1st way

$$\begin{aligned}\cos(X_{fe}, X_m) &= \cos M \cos N \\ \cos(X_{fe}, Y_m) &= \sin N \\ \cos(X_{fe}, Z_m) &= -\sin M \cos N \\ \cos(Y_{fe}, X_m) &= -\cos M \sin N \cos T + \sin M \sin T \\ \cos(Y_{fe}, Y_m) &= \cos N \cos T \\ \cos(Y_{fe}, Z_m) &= \sin M \sin N \cos T + \cos M \sin T \\ \cos(Z_{fe}, X_m) &= \cos M \sin N \sin T + \sin M \cos T \\ \cos(Z_{fe}, Y_m) &= -\cos N \sin T \\ \cos(Z_{fe}, Z_m) &= -\sin M \sin N \sin T + \cos M \cos T\end{aligned}$$