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**Information technology — Language  
independent arithmetic —**

Part 3:

**Complex integer and floating point  
arithmetic and complex elementary  
numerical functions**

*Technologies de l'information — Arithmétique indépendante des  
langages —*

*Partie 3: Arithmétique des nombres complexes entiers et en virgule  
flottante et fonctions numériques élémentaires complexes*

ISO/IEC 10967-3:2006

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# Contents

<b>Foreword</b>	<b>vii</b>
<b>Introduction</b>	<b>viii</b>
<b>1 Scope</b>	<b>1</b>
1.1 Inclusions . . . . .	1
1.2 Exclusions . . . . .	2
<b>2 Conformity</b>	<b>3</b>
<b>3 Normative references</b>	<b>4</b>
<b>4 Symbols and definitions</b>	<b>4</b>
4.1 Symbols . . . . .	4
4.1.1 Sets and intervals . . . . .	4
4.1.2 Operators and relations . . . . .	4
4.1.3 Mathematical functions . . . . .	5
4.1.4 Exceptional values . . . . .	6
4.1.5 Datatypes and special values . . . . .	6
4.1.6 Complex value constructors and complex datatype constructors . . . . .	8
4.2 Definitions of terms . . . . .	9
<b>5 Specifications for imaginary and complex datatypes and operations</b>	<b>14</b>
5.1 Imaginary and complex integer datatypes and operations . . . . .	14
5.1.1 The complex integer <i>result</i> helper function . . . . .	15
5.1.2 Imaginary and complex integer operations . . . . .	15
5.1.2.1 Complex integer comparisons . . . . .	15
5.1.2.2 Multiplication by the imaginary unit . . . . .	17
5.1.2.3 The real and imaginary parts of a complex value . . . . .	17
5.1.2.4 Formation of a complex integer from two real valued integers . . . . .	18
5.1.2.5 Basic complex integer arithmetic . . . . .	18
5.1.2.6 Absolute value and signum of integers and imaginary integers . . . . .	21
5.1.2.7 Divisibility interrogation . . . . .	21
5.1.2.8 Integer division and remainder extended to imaginary and complex integers . . . . .	22
5.1.2.9 Maximum and minimum . . . . .	27
5.2 Imaginary and complex floating point datatypes and operations . . . . .	28
5.2.1 Maximum error requirements . . . . .	28
5.2.2 Sign requirements . . . . .	29
5.2.3 Monotonicity requirements . . . . .	30
5.2.4 The complex floating point <i>result</i> helper functions . . . . .	30
5.2.5 Basic arithmetic for complex floating point . . . . .	31
5.2.5.1 Complex floating point comparisons . . . . .	31
5.2.5.2 Multiplication by the imaginary unit . . . . .	33
5.2.5.3 The real and imaginary parts of a complex value . . . . .	34
5.2.5.4 Formation of a complex floating point from two floating point values . . . . .	34
5.2.5.5 Fundamental complex floating point arithmetic . . . . .	34

5.2.5.6	Absolute value, phase and signum of complex floating point values	38
5.2.5.7	Floor, round, and ceiling	39
5.2.5.8	Maximum and minimum	39
5.2.6	Complex sign, multiplication, and division	40
5.2.6.1	Complex signum	41
5.2.6.2	Complex multiplication	41
5.2.6.3	Complex division	42
5.2.7	Operations for conversion from polar to Cartesian	43
5.3	Elementary transcendental imaginary and complex floating point operations	44
5.3.1	Operations for exponentiations and logarithms	44
5.3.1.1	Exponentiation of imaginary base to integer power	44
5.3.1.2	Natural exponentiation	45
5.3.1.3	Complex exponentiation of argument base	45
5.3.1.4	Complex square root	46
5.3.1.5	Natural logarithm	47
5.3.2	Operations for radian trigonometric elementary functions	49
5.3.2.1	Radian angle normalisation	49
5.3.2.2	Radian sine	49
5.3.2.3	Radian cosine	50
5.3.2.4	Radian tangent	50
5.3.2.5	Radian cotangent	51
5.3.2.6	Radian secant	52
5.3.2.7	Radian cosecant	52
5.3.2.8	Radian arc sine	53
5.3.2.9	Radian arc cosine	54
5.3.2.10	Radian arc tangent	56
5.3.2.11	Radian arc cotangent	57
5.3.2.12	Radian arc secant	58
5.3.2.13	Radian arc cosecant	59
5.3.3	Operations for hyperbolic elementary functions	60
5.3.3.1	Hyperbolic normalisation	61
5.3.3.2	Hyperbolic sine	61
5.3.3.3	Hyperbolic cosine	61
5.3.3.4	Hyperbolic tangent	61
5.3.3.5	Hyperbolic cotangent	62
5.3.3.6	Hyperbolic secant	62
5.3.3.7	Hyperbolic cosecant	62
5.3.3.8	Inverse hyperbolic sine	62
5.3.3.9	Inverse hyperbolic cosine	63
5.3.3.10	Inverse hyperbolic tangent	63
5.3.3.11	Inverse hyperbolic cotangent	64
5.3.3.12	Inverse hyperbolic secant	64
5.3.3.13	Inverse hyperbolic cosecant	65
5.4	Operations for conversion between imaginary and complex numeric datatypes	65
5.4.1	Integer to complex integer conversions	65
5.4.2	Floating point to complex floating point conversions	66
5.5	Support for imaginary and complex numerals	67

<b>6</b>	<b>Notification</b>	<b>68</b>
6.1	Continuation values . . . . .	68
<b>7</b>	<b>Relationship with language standards</b>	<b>68</b>
<b>8</b>	<b>Documentation requirements</b>	<b>69</b>
<b>Annex A</b>	<b>(normative) Partial conformity</b>	<b>71</b>
A.1	Maximum error relaxation . . . . .	71
A.2	Extra accuracy requirements relaxation . . . . .	71
A.3	Relationships to other operations relaxation . . . . .	72
A.4	Part 1 and part 2 requirements relaxation . . . . .	72
<b>Annex B</b>	<b>(informative) Rationale</b>	<b>73</b>
B.1	Scope . . . . .	73
B.1.1	Inclusions . . . . .	73
B.1.2	Exclusions . . . . .	73
B.2	Conformity . . . . .	73
B.3	Normative references . . . . .	74
B.4	Symbols and definitions . . . . .	74
B.4.1	Symbols . . . . .	74
B.4.1.1	Sets and intervals . . . . .	74
B.4.1.2	Operators and relations . . . . .	74
B.4.1.3	Mathematical functions . . . . .	74
B.4.1.4	Exceptional values . . . . .	75
B.4.1.5	Datatypes and special values . . . . .	75
B.4.1.6	Complex value constructors and complex datatype constructors . . . . .	75
B.4.2	Definitions of terms . . . . .	75
B.5	Specifications for the imaginary and complex datatypes and operations . . . . .	76
B.5.1	Imaginary and complex integer datatypes and operations . . . . .	76
B.5.2	Imaginary and complex floating point datatypes and operations . . . . .	76
B.5.2.1	Maximum error requirements . . . . .	76
B.5.2.2	Sign requirements . . . . .	76
B.5.2.3	Maximum error requirements . . . . .	76
B.5.2.4	Basic arithmetic for complex floating point . . . . .	77
B.5.3	Elementary transcendental imaginary and complex floating point operations . . . . .	78
B.5.3.1	Operations for exponentiations and logarithms . . . . .	78
B.5.3.2	Operations for radian trigonometric elementary functions . . . . .	78
B.5.3.2.1	Radian angle normalisation . . . . .	78
B.5.3.2.2	Radian sine . . . . .	79
B.5.3.2.3	Radian cosine . . . . .	79
B.5.3.2.4	Radian tangent . . . . .	79
B.5.3.2.5	Radian cotangent . . . . .	80
B.5.3.2.6	Radian secant . . . . .	80
B.5.3.2.7	Radian cosecant . . . . .	80
B.5.3.2.8	Radian arc sine . . . . .	80
B.5.3.2.9	Radian arc cosine . . . . .	81
B.5.3.2.10	Radian arc tangent . . . . .	81
B.5.3.2.11	Radian arc cotangent . . . . .	81

B.5.3.2.12	Radian arc secant . . . . .	81
B.5.3.2.13	Radian arc cosecant . . . . .	81
B.5.3.3	Operations for hyperbolic elementary functions . . . . .	82
B.5.3.3.1	Hyperbolic normalisation . . . . .	82
B.5.3.3.2	Hyperbolic sine . . . . .	82
B.5.3.3.3	Hyperbolic cosine . . . . .	82
B.5.3.3.4	Hyperbolic tangent . . . . .	82
B.5.3.3.5	Hyperbolic cotangent . . . . .	83
B.5.3.3.6	Hyperbolic secant . . . . .	83
B.5.3.3.7	Hyperbolic cosecant . . . . .	83
B.5.3.3.8	Inverse hyperbolic sine . . . . .	83
B.5.3.3.9	Inverse hyperbolic cosine . . . . .	83
B.5.3.3.10	Inverse hyperbolic tangent . . . . .	84
B.5.3.3.11	Inverse hyperbolic cotangent . . . . .	84
B.5.3.3.12	Inverse hyperbolic secant . . . . .	84
B.5.3.3.13	Inverse hyperbolic cosecant . . . . .	84
B.5.4	Operations for conversion between imaginary and complex numeric datatypes . . . . .	84
B.5.5	Support for imaginary and complex numerals . . . . .	84
B.6	Notification . . . . .	84
B.6.1	Continuation values . . . . .	84
B.7	Relationship with language standards . . . . .	84
B.8	Documentation requirements . . . . .	84
<b>Annex C</b>	<b>(informative) Example bindings for specific languages</b> . . . . .	<b>85</b>
C.1	Ada . . . . .	86
C.2	C . . . . .	96
C.3	C++ . . . . .	105
C.4	Fortran . . . . .	113
C.5	Common Lisp . . . . .	122
<b>Annex D</b>	<b>(informative) Bibliography</b> . . . . .	<b>133</b>
<b>Annex E</b>	<b>(informative) Cross reference</b> . . . . .	<b>135</b>
<b>Annex F</b>	<b>(informative) Possible changes to part 2</b> . . . . .	<b>147</b>

## Foreword

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ISO/IEC 10967-3 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology, Subcommittee SC 22, Programming languages, their environments and system software interfaces*.

ISO/IEC 10967 consists of the following parts, under the general title *Information technology — Language independent arithmetic*:

— *Part 1: Integer and floating point arithmetic*

— *Part 2: Elementary numerical functions*

— *Part 3: Complex integer and floating point arithmetic and complex elementary numerical functions*