8 Spatial reference frames

8.1 Overview

A *spatial reference frame* is a means of specifying a spatial coordinate system for a region of an object-space. The relationship between a spatial reference frame and the corresponding spatial coordinate system is discussed in 8.2. Spatial reference frame specifications are discussed in 8.3.

Many applications need to perform operations involving multiple spatial reference frames. Relationships among spatial reference frames are commonly derived from corresponding relationships among the spatial objects of interest within the application domain. Spatial reference frame relationships are discussed in 8.4.

A spatial reference frame template provides an abstraction of spatial reference frames that share common elements. Spatial reference frame templates are used to realise instances of spatial reference frames. This document defines a collection of spatial reference frame templates in 8.5. This document also defines a collection of standardized spatial reference frames in 8.6.

Spatial reference frames may be organized into specified sets to form an atlas for a large region. This document defines a collection of standardized spatial reference frame sets, as well as the members of those sets, in 8.7.

8.2 Spatial reference frame structure

A spatial reference frame uses a spatial coordinate system (see 5.4) to assign a unique coordinate n-tuple to each point in a region of an object-space. A spatial coordinate system is defined as the functional composition of an abstract coordinate system generating function and a normal embedding. The abstract coordinate system generating function $\boldsymbol{G}$ associates coordinates in coordinate-space to positions in position-space. A normal embedding $\boldsymbol{E}$ maps those positions in position-space to points in object-space. Different normal embeddings produce different spatial coordinate systems. If $\boldsymbol{c}$ is a coordinate for the coordinate system, then $\boldsymbol{c}$ identifies the object-space point $\boldsymbol{p} = \boldsymbol{E} \circ \boldsymbol{G}(\boldsymbol{c})$.

A spatial reference frame uses an object reference model (see 7.4) to determine a unique normal embedding $\boldsymbol{E}$ to map the position-space orthonormal frame to a corresponding orthonormal frame embedded in the object-space of the spatial object that it models (see 5.2.5). All spatial reference frames rely on their respective embedded frame in object-space to provide a reference for measurements and computations. The object reference model is a set of reference datums that bind geometric primitives (points, directed curves, or oriented surfaces) in position-space to corresponding geometrics aspects of the spatial object in object-space.

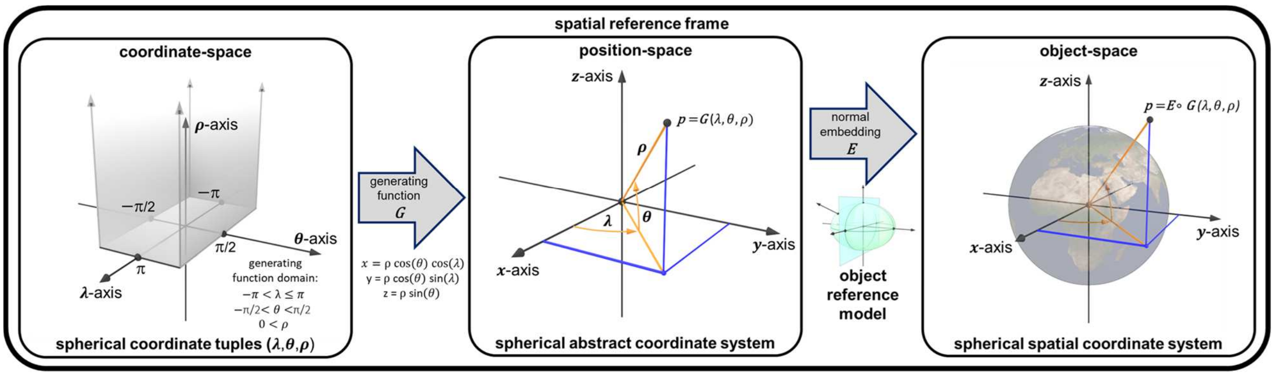

Figure 8.1 — The components of a spatial reference frame

Figure 8.1 illustrates a spatial reference frame in which a spherical spatial coordinate system is derived from a spherical abstract coordinate system and a normal embedding determined by an object reference model of the Earth. In this illustration, a coordinate (λ, θ, ρ) in coordinate-space is assigned to a position-vector $[x, y, z]$ in the orthonormal frame of position-space. That position is mapped to a location in the object-space of the Earth via the unique normal embedding that is determined by the object reference model and its associated reference datums. This embedding provides an embedded frame in object-space that serves as the uniform reference for measurements.

8.3 Spatial reference frame specification

8.3.1 SRF definition

A *spatial reference frame* (SRF) is a specification of a spatial coordinate system that is constructed from an ORM and a compatible abstract CS, such that coordinates uniquely specify positions (including time-dependent positions) with respect to the spatial object of the ORM. A specification of an SRF includes:

a) an ORM,

b) an abstract CS compatible with the ORM,

c) a binding of all parameters of the CS,

d) (optionally) k^{th} coordinate-component names,

e) (optionally) a description or specification of the region of object-space to which the SRF applies, expressed in terms of geographic name(s) and/or restrictions on the coordinate domain, and

f) (optionally) a coordinate-component of a CS of type 3D may be identified as the vertical coordinate-component (see 5.2.1).

An SRF specifies a spatial CS defined by the functional composition of the abstract CS generating function and the normal embedding associated with the ORM. All SRFs rely on the embedded frame, which provides a uniform reference for measuring distances and angles in object-space.

Spatial CS compatibility and the other elements of the specification of an SRF are defined in the following subclauses.

8.3.2 SRF specification elements

8.3.2.1 ORM and CS compatibility

The compatible CS type of the CS element of an SRF depends on the dimension of the ORM. The *dimension of an ORM* is defined as the dimension of the RD components of the specification of the ORM. The compatible CS types by ORM dimension are specified in Table 8.1.

Table 8.1 — Compatible CS types

ORM dimension	Compatible CS types
1D	1D CS
2D	Curve CS 2D CS
3D	Curve CS Surface CS 3D CS

The use of surface CSs or 3D CSs that are based on an oblate ellipsoid (or sphere) are restricted to ORMs that are based on an oblate ellipsoid (or respectively, sphere) RD.

The standardized surface CSs that are based on an oblate ellipsoid (or sphere) are:

a) surface geodetic,

b) surface planetodetic, and

c) all map projections.

The 3D CSs that are based on an oblate ellipsoid (or sphere) are:

a) geodetic 3D,

b) planetodetic 3D, and

c) all augmented map projections.

As a further restriction, some CSs are based on spheres only. CS OBLIQUE_MERCATOR_SPHERICAL has this restriction.

An SRF may be described in terms of the properties and other characteristics of the CS that is specified by the SRF. An SRF is said to be a *3D SRF*, *surface SRF*, or *2D SRF* if the CS of the SRF is of the corresponding CS type. Similarly, the CS properties of linearity, orthogonality, and handedness may be used as descriptors of an SRF corresponding to the properties of the CS that is specified by the SRF. Thus, an SRF is said to be a *linear SRF* or a *curvilinear SRF* if the CS of the SRF has the respective linearity property. For all SRFs, the normal embedding determined by the ORM provides an embedded frame with an inherent Cartesian basis that establishes a uniform reference from which distances and angles for the coordinate-components of the SRF are measured and specified in object-space. Every 3D SRF in this document is a right-handed SRF in consequence to the CS handedness restriction imposed in 5.2.3.

8.3.2.2 CS parameter binding

All CS parameter values shall be specified. In the case of a combination of a CS and an ORM based on an oblate ellipsoid (or sphere), the major semi-axis and minor semi-axis (or equivalently, the inverse flattening) (or respectively, sphere radius) of the ORM and CS shall match.

8.3.2.3 Coordinate-component names

A CS specification (see 5.3.8) includes the coordinate-component symbols with common names (if any). A specification of an SRF may optionally assign SRF-specific names to the k^{th} coordinate-components. The name assignment shall reflect the common use in the intended application domain.

EXAMPLE For an equatorial spherical CS, the assignment of SRF-specific names to the k^{th} coordinate-components of “right ascension” for λ, “declination” for θ, and “radius” for ρ.

8.3.2.4 Applicable region

A CS specification (see 5.3.8) includes the specification of the CS domain and CS range where the generating function (or mapping equations) and its inverse(s) are defined. An SRF specification may further restrict the CS domain. An *applicable region* is a restriction of the CS domain as used in an SRF. An *extended region* is a second region that contains the applicable region as a subset. The specification of these restrictions is important for several (SRF specific) reasons:

a) If the ORM is local, the restrictions are used to model, in coordinate-space, the local region of the object-space.

b) If the CS is a map projection or an augmented map projection, the restrictions are used to bound or otherwise limit distortions (see 5.3.7.3.3).

c) The SRF may be used in conjunction with other SRFs to form an atlas for a large region (see 8.7 SRF sets). In this case, the restrictions are used to control the pair-wise overlap of the spatial coverage of members of the SRF collection.

d) If the CS generating function (or map projection mapping equations) or the inverse function(s) have been implemented with a numerical approximation, the restrictions are used to control error bounds.

The extended region is used primarily for overlapping regions in forming an atlas as in (c) above. Not all properties of the SRF that are true in the applicable region will necessarily be true in the extended region. A distortion error bound that holds in the applicable region may not hold in the extended region.

Applicable regions and extended regions may be described and/or specified. An *applicable region description* is a statement that describes the spatial extent of the region such as in terms of named geographic areas or political entities.

EXAMPLE 1 “The German state of Baden-Wurttemberg” and “The Baltic Sea” are applicable region descriptions.

An *applicable region specification* consists of a set of constraints that specifies the spatial extent of the region. The spatial extent of the region may always be specified in terms of coordinate-component value ranges in the coordinate system of the SRF. Such a specification is termed to be of type *coordinate-region*.

If the ORM of the SRF is based on an oblate ellipsoid (or sphere), the spatial extent of the region may alternatively be specified in terms of coordinate-component value ranges in the geodetic coordinate system of the Celestiodetic SRF for that ORM (see 8.5.4). When the coordinate-component value ranges are specified in terms of geodetic coordinates in this way, the specification is termed to be of type *geodetic-region*. To avoid loss of precision, such geodetic coordinate values may be specified in arc degrees. Applicable region specifications of type geodetic-region may be useful for local Euclidean or map projection-based SRFs.

Each coordinate-component value range may be fully bounded, with both upper and lower bounds specified, semi-bounded, with only one (either upper or lower) bound specified, or unbounded, with no bounds specified. Together, the coordinate-component value ranges specify a full or partial bounding box, in terms of either the coordinate system of the SRF (type coordinate-region), or in terms of geodetic coordinates of the Celestiodetic SRF for that ORM (type geodetic-region).

If an applicable region has been specified, an *extended region specification* of the same type (type coordinate-region or type geodetic-region) may also be specified. The ranges specified for an extended region shall contain the corresponding applicable region ranges.

A coordinate is *within the applicable region* if it is contained in the CS domain of the SRF and satisfies all constraints in the applicable region specification. In the case of an applicable region specification of type geodetic-region, the coordinate is considered to be within the applicable region if the corresponding Celestiodetic SRF coordinate satisfies all geodetic constraints in the applicable region specification.

A coordinate is *within the extended region* if it is contained in the CS domain of the SRF and satisfies the constraints in the extended region specification. In the case of an extended region specification of type geodetic-region, the coordinate is considered to be within the extended region if the corresponding Celestiodetic SRF coordinate satisfies all geodetic constraints in the extended region specification.

EXAMPLE 2 The SRF is based on a transverse Mercator map projection (see SRF Template Transverse Mercator).
Applicable region specification of type coordinate-region: $167\ 000 \le u \le 833\ 000$, $0 \le v \le 9\ 500\ 000$
Extended region specification of type coordinate-region: $0 < u$, $-100 < v$
Note that the extended region is partially bounded.

EXAMPLE 3 The SRF is based on a transverse Mercator map projection (see SRF Template Transverse Mercator).
Applicable region specification of type geodetic-region: $-78° \leq \lambda < -72°$, $0° \leq \varphi < 84°$
Extended region specification of type geodetic-region: $-78{,}5° \leq \lambda < -71{,}5°$

Note that the extended region is partially bounded since no constraint on φ is specified.

8.4 SRF relationships

8.4.1 Overview

Many applications need to perform operations involving multiple SRFs. These SRFs can be related to one another in several different ways. In general, the relationships involve at least one SRF acting as the reference SRF for one or more other SRFs. Usually, SRF relationships are based on the mathematical relationships between specific SRFs or reflect corresponding relationships among the spatial objects of interest within the application domain. The SRFs can be in the same object-space or in different object-spaces. Such SRF relationships are discussed in 8.4.2.

In some applications dealing with curvilinear 3D SRFs, it is useful to reduce the dimensionality of the coordinate system of an SRF by fixing the values of one or more of its coordinate-components to induce a new SRF. Instances of such relationships include surface SRFs that are induced from corresponding 3D SRFs when one coordinate-component is fixed. This type of SRF relationship is discussed in 8.4.3.

In some applications, it is necessary to emplace an SRF at a convenient location within the object-space to measure or specify positions of interest to the application. Such SRFs can be instanced at the desired location, the lococentre, with the position of the lococentre and the orientation of the SRF specified using localization methods. These methods provide a local orthonormal frame. When the lococentre is on a coordinate surface, and the horizontal axes are tangent to that surface at that point, the orthonormal frame is termed a local tangent frame. Such lococentric SRFs are discussed in 8.4.4.

Operations on directions and vector quantities require a Cartesian coordinate system. When an SRF does not provide such a coordinate system, this requirement can be met by a lococentric SRF that has a Cartesian coordinate system. Use of SRFs for vector operations are discussed in 8.4.5.

8.4.2 Application-based SRF relationships

In many applications it is useful or necessary to relate information expressed in one SRF in terms of another SRF. The SRFs may be associated with the same spatial object or may be associated with different spatial objects. These SRF relationships are usually asymmetrical, with one SRF serving as the reference for the other.

An SRF may be associated with a specified region of a spatial object of interest. The applicable region of such an SRF is often a subset of the applicable region of a reference SRF associated with that same spatial object. Regional SRFs are commonly specified in terms of coordinates and parameters of the reference SRF. Thus, regional SRF relationships can include cases such as:

a) An SRF for a Mars rover operating area has its origin specified by the coordinates of the rover's landing site in the Mars planetocentric SRF (see 8.6.13), which serves as the reference SRF (see Figure 8.2), its primary axis direction aligned with local east at the landing site, and its applicable region specified by a maximum distance from the landing site.

b) Members of SRF sets that cover adjacent regions within larger geographic areas depend on reference SRFs based on global Earth reference models and geodetic coordinates. Such SRF sets include the zones of the Japan national plane coordinate system (see 8.7.4 and Figure 8.2), US state plane coordinate systems (see 8.7.2 or 8.7.8), or the universal transverse Mercator system (see 8.7.7). The applicable regions of the SRF set members are specified in terms of either geographic names or geodetic coordinate boundaries.

c) An SRF for a construction site for a building or bridge has its origin specified by a point in the Japan national plane coordinate system (see 8.7.4), which serves as the reference SRF (see Figure 8.2). The building site SRF’s primary axis direction is specified by an azimuth, and its applicable region is specified by a sequence of points forming a closed polygon.

SRFs can be associated with objects that are components of an assembly. Such component object SRFs are commonly specified in terms of coordinates and parameters of a reference SRF associated with the assembly. It is common for component object SRFs to be linked, forming chains based on the hierarchical structure of an assembly with articulated components. Thus, component object SRF relationships can include cases such as:

a) An SRF for a radar or sensor is specified in terms of the reference SRF of the vehicle to which it is attached (see Figure 8.2).

b) A series of SRFs for individual robot arm segments are specified in terms of a reference SRF, which can be either the SRF of the preceding robot arm segment, or the SRF of the base of the robot arm (see Figure 8.2).

c) Several SRFs with different coordinate systems (Euclidean, cylindrical, etc.) associated with a large, complex piece of agricultural or construction equipment, with a branching hierarchy of diverse articulated parts, are specified in terms of a reference SRF associated with the part in the hierarchy to which each part is attached.

An SRF may be associated with an independent spatial object that operates within the object-space of another spatial object that acts as a reference object. Such independent object SRFs are commonly specified in terms of coordinates and parameters of a reference SRF associated with that reference object. Applications may need to relate the positions, orientations, and other properties of two or more independent objects to one another, either by using one of the independent spatial objects as the reference object, or by using a larger spatial object that acts as the reference for all the independent objects. Thus, independent object SRF relationships can include cases such as:

a) An SRF for a vehicle with its origin specified by a point corresponding to the location of the vehicle in the Geodetic WGS 1984 SRF (see 8.6.7), which serves as the reference SRF (see Figure 8.2), and its primary and secondary axis directions specified relative to the azimuth of the front of the vehicle

b) SRFs for two or more vehicles with the SRF of one chosen as the reference SRF for the others, or the geodetic SRF of the Earth (see 8.6.7) chosen as the reference SRF for all of them (see Figure 8.2).

c) Two celestiodetic SRFs (see 8.5.4) for Earth and Mars, with the SRF of one chosen as the reference for the other, or the celestiocentric SRF (see 8.5.2) of the Sun chosen as the reference for both (see Figure 8.2).

These SRF relationships can be combined in any manner that is useful to an application. Figure 8.2 illustrates several of the cases, including the nested object-spaces with which the various SRFs are associated. Vertical connections represent superior-to-subordinate SRF relationships, associating regional SRFs with reference SRFs, or associating component object SRFs with the reference SRFs for the assemblies to which they are attached. Horizontal connections represent peer-to-peer SRF relationships between independent objects. These horizontal relationships can be implemented by choosing the SRF of one of the independent objects to fill the role of reference SRF, or by choosing a reference SRF of another object that provides the overall context.

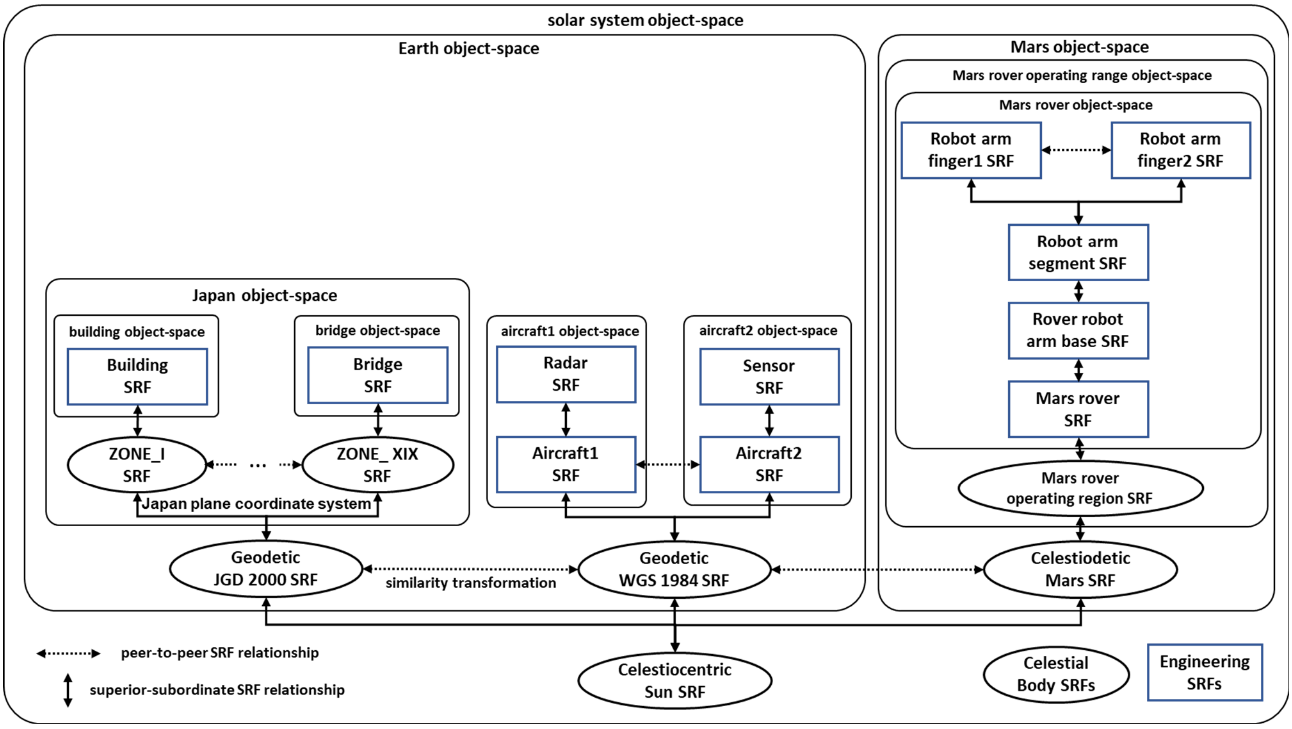

Figure 8.2 — Examples of SRF relationships

Some regional SRFs are based on map projections, or augmented map projections (see 5.3.7). Any of these SRFs can be based on spatial coordinate systems (see 5.4.2) that are translated and/or rotated with respect to a reference SRF using localization (see 5.3.6 and 8.4.4).

In many applications, some or all of the objects of interest, and their associated SRFs, can be moving. In all such dynamic cases, once a fixed time is specified, the object configuration (and the associated SRFs) can be treated as a static snapshot.

8.4.3 Induced surface SRFs

Coordinate-component surfaces of a 3D CS and corresponding induced surface CSs are defined in 5.3.4.2. Relationships between 3D CSs and corresponding induced surface CSs extend to 3D SRFs and their corresponding surface SRFs. Such a relationship is specified through the appropriate combination of a 3D ORM and a 3D CS. This can be an efficient way of using the specification of a 3D SRF to produce the specification of a surface SRF. This relationship also holds for SRF templates (see 8.5)

A 3D SRF specification may optionally identify the 3rd coordinate-component as the vertical coordinate-component for the SRF. Such an SRF can illustrate the induced surface SRF process. In such cases, the surface CS that is induced on the vertical coordinate-component surface is the CS for an induced surface SRF. The Celestiodetic and Planetodetic 3D SRF templates can be used to instantiate Surface Celestiodetic and Surface Planetodetic SRFs, respectively. The vertical coordinate-component of these 3D SRF templates is set to zero to induce the corresponding surface SRFs on the surface of the ellipsoid RD of the ORM.

EXAMPLE 1 For a directional sensor located at a fixed site on the Earth's surface, a local surface SRF with a polar coordinate system (measuring the angle counterclockwise from local east) is needed. Such a local surface SRF can be created in two steps:

1) Instantiate a local tangent space cylindrical SRF as an interim 3D SRF, using the reference SRF geodetic coordinate of the sensor for its origin. This interim 3D SRF has a lococentric cylindrical 3D CS.
2) Fix the value of the 3rd coordinate-component of the interim 3D SRF, h (height), at zero to induce a surface SRF with a lococentric surface polar CS.