7 Reference datums, embeddings, and object reference models

7.1 Overview

This document specifies *reference datums* as geometric primitives in position-space that are used to model aspects of object-space through a process termed reference datum binding. A reference datum binding is an identification of a reference datum in position-space with a corresponding constructed entity in object-space (see 7.2). Reference datums for celestial bodies of interest including Earth are specified in Annex D.

A *normal embedding* is a distance-preserving function from position-space to object-space. A normal embedding establishes a model of position-space in object-space by defining an orthonormal frame, termed the embedded frame, in object-space (see 5.2.4). The image of a bound reference datum under a normal embedding may or may not coincide with the constructed entity of the reference datum binding. If they coincide, the reference datum binding and the normal embedding are said to be compatible (see 7.3).

A set of bound reference datums with properly constrained relationships can be selected so as to be compatible with one and only one normal embedding. Such a constrained set of bound reference datums is termed an *object reference model*. Thus, an object reference model specifies a unique normal embedding. Object reference models generalize the notion of geodesy datums. Object reference models that use the same set of reference datum primitives and similar binding constraints are abstracted in the notion of an object reference model template. Object reference model templates provide a uniform method of object reference model specification (see 7.4).

Object reference models for celestial objects of interest are specified in Annex E. For these celestial objects, one object reference model is designated as the reference model for the object. The transformation from each object reference model to the reference model for the object is termed the reference transformation. A reference transformation is a type of *similarity transformation* (see 7.3.2). Similarity transformation templates are defined in 7.3.3 to facilitate the specification of reference transformations. Similarity transformations in general and reference transformations in particular may have time-dependent parameters. Thus, these transformations may be termed time-independent (static) or time-dependent (dynamic). Time-independent reference transformations for celestial object reference models are also specified in Annex E.

Object-specific rules to bind reference datums in a way that is compatible with the binding constraints of an object reference model template are defined in 7.5. These object-specific binding rules are used to provide a uniform method of specifying object reference models for specific dynamically-related celestial bodies.

7.2 Reference datums

7.2.1 Overview

A *reference datum* (RD) is a geometric primitive in position-space that is used to model an aspect of object-space through a process termed RD binding. In this document, the reference datum concept is defined for 1D, 2D, and 3D position-spaces. In the 2D and 3D cases, this document specifies a small set of reference datums for use in its own specifications. This set is not intended to be exhaustive. Additional RDs may be specified by registration in accordance with Clause 13.

7.2.2 Reference datum categories

In this document, an RD geometric primitive is expressed in terms of analytic geometry in position-space. RDs are designed to correspond to constructed entities of similar geometric type in an object-space through a process termed RD binding (see 7.2.5). These geometric types are limited to a point, a directed curve, or an oriented surface. The analytic form of the position-space representation and its corresponding object-space geometric representation are described by category and position-space dimension in Table 7.1. An RD of a given category is specified by the parameters and/or the analytic expression of its position-space representation.

Table 7.1 — RD categories

RD category	Position-space representation			Object-space representation
	1D	2D	3D	
Point	$[a]$ real a	$[a, b]$ real a, b	$[a, b, c]$ real a, b, c	a point in the object-space
Directed curve		$\boldsymbol{p} = \boldsymbol{F}(t)$, $\boldsymbol{F}$ is smooth, $\mathbb{R}^2$ valued with domain $D \subseteq \mathbb{R}^1$. Direction at $\boldsymbol{p_0} = \boldsymbol{F}(t_0)$ is $\boldsymbol{n} = \frac{d\boldsymbol{F}}{dt}(t_0)$.	$\boldsymbol{p} = \boldsymbol{F}(t)$, $\boldsymbol{F}$ is smooth, $\mathbb{R}^3$ valued with domain $D \subseteq \mathbb{R}^1$. Direction at $\boldsymbol{p_0} = \boldsymbol{F}(t_0)$ is $\boldsymbol{n} = \frac{d\boldsymbol{F}}{dt}(t_0)$.	a curve in the object-space with a designation of direction along the curve
Oriented surface			Implicit definition: $f(\boldsymbol{p}) = 0$. f is smooth, $\mathbb{R}^1$ valued for $\boldsymbol{p}$ in position-space. Positive side of surface (orientation): $f(\boldsymbol{p}) > 0$.	a surface in the object-space with a designation of one side as positive

This document specifies 2D and 3D RDs by RD category in Table 7.4 through Table 7.8. The specification elements of those tables are defined in Table 7.2. 3D RDs based on ellipsoids are described in 7.2.3 and 7.2.4 and specified in Annex D with specification elements defined in Table 7.9. Table 7.3 is a directory of RD specification tables or, in the case of 3D RDs based on ellipsoids, RD specification directories.

Table 7.2 — RD specification elements

Element	Definition
RD label	The label for the RD (see 13.2.2).
RD code	The code for the RD (see 13.2.3). Code 0 (UNSPECIFIED) is reserved.
Description	A description of the RD including any common name for the concept.
Position-space representation	The analytic formulation of the RD in position-space.

Table 7.3 — RD specification directory

Position-space dimension	RD category	Table number
2D	point	Table 7.4
3D	point	Table 7.5
2D	directed curve	Table 7.6
3D	directed curve	Table 7.7
3D	oriented surface	Table 7.8 and Table 7.10

Table 7.4 — 2D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_2D	1	Origin in 2D	$[0,0]$
X_UNIT_POINT_2D	2	x-axis unit point in 2D	$[1,0]$
Y_UNIT_POINT_2D	3	y-axis unit point in 2D	$[0,1]$

Table 7.5 — 3D RDs of category point

RD label	RD code	Description	Position-space representation
ORIGIN_3D	4	Origin in 3D	$[0,0,0]$
X_UNIT_POINT_3D	5	x-axis unit point in 3D	$[1,0,0]$
Y_UNIT_POINT_3D	6	y-axis unit point in 3D	$[0,1,0]$
Z_UNIT_POINT_3D	7	z-axis unit point in 3D	$[0,0,1]$

Table 7.6 — 2D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_2D	8	x-axis in 2D	$\boldsymbol{F}(t) \equiv [t, 0]$
Y_AXIS_2D	9	y-axis in 2D	$\boldsymbol{F}(t) \equiv [0, t]$

Table 7.7 — 3D RDs of category directed curve

RD label	RD code	Description	Position-space representation
X_AXIS_3D	10	x-axis in 3D	$\boldsymbol{F}(t) \equiv [t, 0, 0]$
Y_AXIS_3D	11	y-axis in 3D	$\boldsymbol{F}(t) \equiv [0, t, 0]$
Z_AXIS_3D	12	z-axis in 3D	$\boldsymbol{F}(t) \equiv [0, 0, t]$

Table 7.8 — 3D RDs of category oriented surface

RD label	RD code	Description	Position-space representation
XY_PLANE_3D	13	xy-plane	$0 = f(x, y, z) \equiv z$
XZ_PLANE_3D	14	xz-plane	$0 = f(x, y, z) \equiv y$
YZ_PLANE_3D	15	yz-plane	$0 = f(x, y, z) \equiv x$

7.2.3 Ellipsoidal RDs

The RDs specified in this document include RDs based on oblate ellipsoids, prolate ellipsoids, and tri-axial ellipsoids. These RDs are 3D and of RD oriented surface category. These RDs are specified based upon certain geometrically defined parameters (see A.6.2). The position-space representations of oblate and prolate ellipsoid RDs are expressed in the form:

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0 \text{ where } a \neq 0 \neq b. \quad (7.1)$$

When $a \geq b$, an RD of this form is an *oblate ellipsoid RD* with major semi-axis a and minor semi-axis b as illustrated in Figure 7.1.

Spheres shall be considered as a special case of oblate ellipsoids, where appropriate. If $a = b$, an oblate ellipsoid RD may be termed a *sphere RD*. In this case, the value $r = a = b$ is the radius of the sphere RD.

NOTE In general usage, spheres are a limiting case of oblate, prolate, and tri-axial ellipsoids.

When $a < b$, an RD of this form is a *prolate ellipsoid RD* with major semi-axis b and minor semi-axis a, as illustrated in Figure 7.1.

Instead of specifying the parameters of an oblate ellipsoid RD as the major semi-axis a and the minor semi-axis b, it is both equivalent and sometimes convenient to use the major semi-axis a and the flattening f as defined in Equation 7.2. The minor semi-axis b may be expressed in terms of the major semi-axis a and the flattening f as in Equation 7.3. The flattening of a sphere RD is zero $(f = 0)$.

$$\text{flattening definition: } f \equiv \frac{a-b}{a} \quad (7.2)$$

$$\text{minor semi-axis relationship: } b = a - af \quad (7.3)$$

The position-space representation of a tri-axial ellipsoid RD is expressed in the form:

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0. \quad (7.4)$$

The semi-axes a, b, and c shall be positive non-zero and $a \neq b \neq c \neq a$.

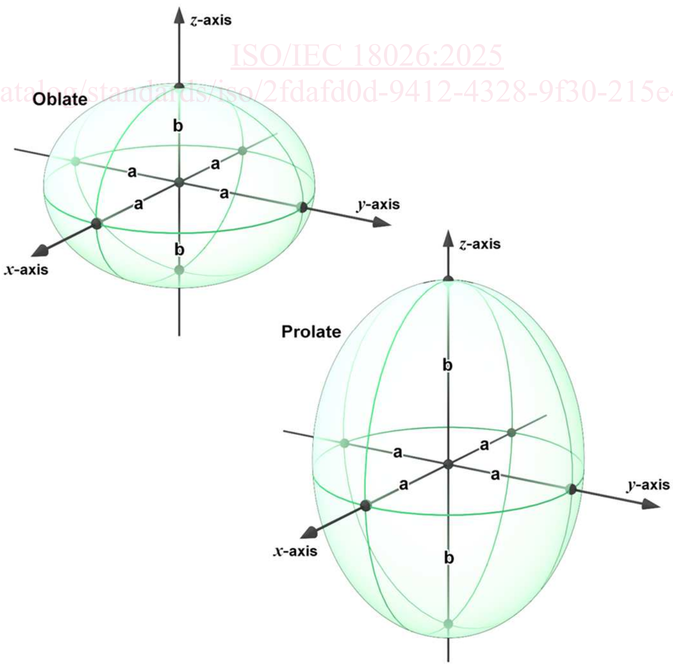

Figure 7.1 — Oblate and prolate ellipsoids

7.2.4 RDs associated with physical objects

In the case of ellipsoid RDs intended for modelling physical objects of interest, published parameter values for these RDs are used. The specification of these RDs includes the published ellipsoid parameters and the identification of the associated physical object. The specification elements for physical object RDs are defined in Table 7.9.

Table 7.9 — Physical object RD specification elements

<table>
<tr><th>Element</th><th colspan="2">Specification</th></tr>
<tr><td>RD label</td><td colspan="2">The label for the RD (see 13.2.2).</td></tr>
<tr><td>RD code</td><td colspan="2">The code for the RD (see 13.2.3).</td></tr>
<tr><td>Description</td><td colspan="2">The description including the name of the physical object as published or as commonly known.</td></tr>
<tr><td rowspan="5">Parameters</td><td>Oblate ellipsoid case</td><td>Major semi-axis, a
Flattening, f</td></tr>
<tr><td>Sphere case</td><td>Radius, r</td></tr>
<tr><td>Prolate ellipsoid case</td><td>Minor semi-axis, a
Major semi-axis, b</td></tr>
<tr><td>Tri-axial ellipsoid case</td><td>x-semi-axis, a
y-semi-axis, b
z-semi-axis, c</td></tr>
<tr><td colspan="2">RD parameters shall be specified by value or by reference (see 13.2.5).
If by value, the value(s) shall be followed by an error estimate expressed in one of the following forms:
a) error estimate: unknown
b) error estimate: assumed precise
c) error estimate (1σ): <parameter name>:<error value>
d) error interval: <parameter name> $\pm$ <error value>
EXAMPLE error estimate (1σ): a: 1 250, f^{-1}: 0,25.
If by reference, this specification element shall express the value(s) and error estimate(s) using the terminology found in the reference. These terms shall be enclosed in brackets ({}). Any parameter value that is not specified in the citation(s) shall be specified as in the “by value” case. An error estimate for b or for f^{-1} may be substituted in place of an error estimate for f.</td></tr>
<tr><td>Date</td><td colspan="2">The date the RD parameters were specified or published.</td></tr>
<tr><td>References</td><td colspan="2">The references (see 13.2.5).</td></tr>
</table>

The RDs associated with physical objects are specified in Annex D. Table 7.10 is a directory of these RDs organized by type of ellipsoid. The semi-axis and radius parameters are unitless in position-space but are bound to metre lengths when the RD is identified with the corresponding physical object-space constructed entity.

Table 7.10 — Physical RD specification table locations

Type of ellipsoid	RD table
Oblate ellipsoid	Table D.2
Sphere	Table D.3
Prolate ellipsoid	Table D.4
Tri-axial ellipsoid	Table D.5

EXAMPLE An ellipsoid reference datum with major semi-axis a and minor semi-axis b is the surface implicitly defined by:

$$f(x,y,z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} - 1 = 0$$

and is illustrated in Figure 7.2. The WGS_1984 ellipsoid reference datum has major semi-axis $a = 6\ 378\ 137$ metres and flattening $f = 1/298{,}257\ 223\ 563$, resulting in a minor semi-axis $b = a - af = 6\ 356\ 752$ metres.

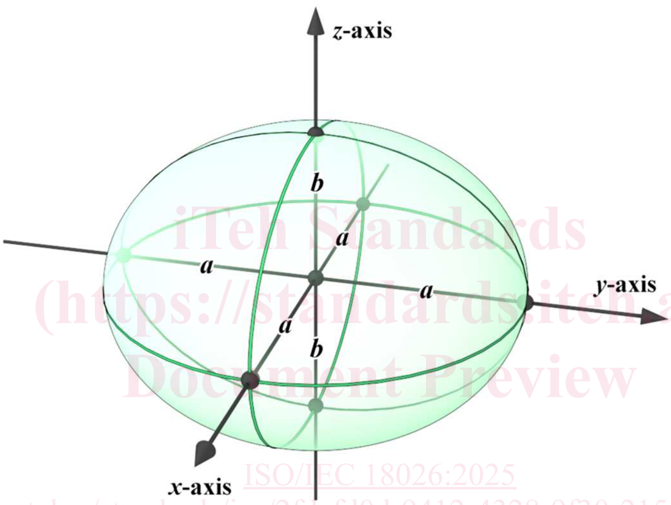

Figure 7.2 — An ellipsoid reference datum

7.2.5 RD binding

An RD is *bound* when the RD in position-space is identified with a corresponding constructed entity in object-space. In this context, a "constructed entity" is defined to mean an intrinsic, artificial, measured, or conceptual entity in object-space that is uniquely identifiable within the user's application domain. The term "corresponding" in this context means that each RD is bound to a constructed entity of the same geometric object type. That is, position-space points are bound to identified points in object-space, position-space directed lines to constructed lines or line segments in object-space, position-space directed curves to constructed curves or curve segments in object-space, position-space oriented planes to constructed planes or partial planes in object-space, and position-space oriented surfaces to constructed surfaces or partial surfaces in object-space.

When a curve or surface RD is bound, the radii of curvature on the corresponding constructed entity in object-space shall correspond to the radii of curvature in position-space. In this document, in the case of physical objects, one unit in position-space corresponds to one metre in object-space. In the case of abstract objects, one unit in position-space corresponds to the designated length scale unit in the abstract object-space. In particular, the semi-axes of an ellipsoid RD shall correspond to the semi-axes of the constructed ellipsoid to which it is bound.

If the constructed entity of an RD binding is fixed with respect to object-space, then the RD binding shall be termed an *object-fixed RD binding.* This definition assumes that the constructed entity does not move over time by an amount significant for the accuracy and time scale of an application.

Figure 7.3 illustrates two distinct bindings of a point RD. On the left, it is bound to a specific point in the abstract object-space of a CAD/CAM model. On the right, it is bound to a point in physical object-space that is on an object that has been manufactured from that CAD model.

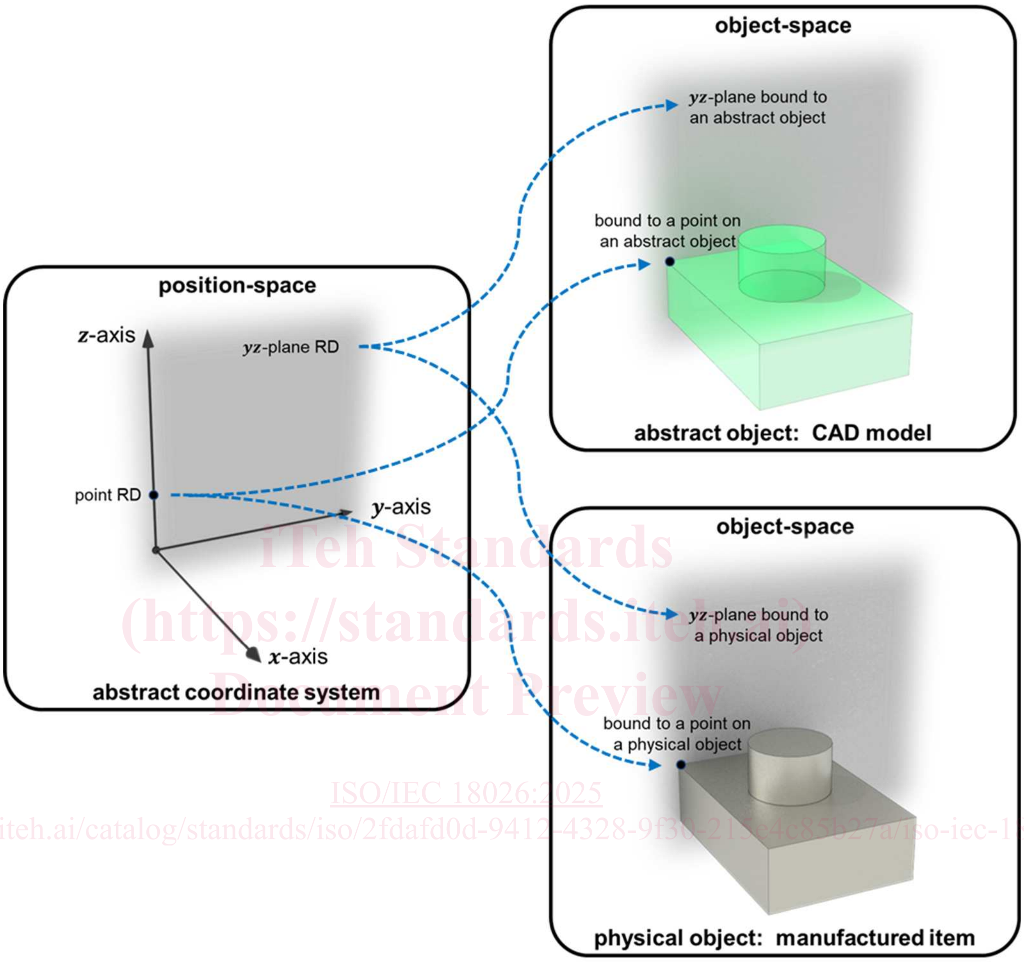

Figure 7.3 — Two RDs bound to an abstract object and to a physical object

In some application domains, bound reference datums are used to model a significant aspect of the problem domain. In geodesy, oblate ellipsoids are used to model the figure of the Earth or a subset thereof.

EXAMPLE Semi-axis values a and b, $a \geq b$, are selected to specify an oblate ellipsoid reference datum. The following steps (see Figure 7.4) illustrate one way to bind an ellipsoid reference datum specified by semi-axis values a and b to a conceptual ellipsoid that represents the figure of the Earth in a region as approximated by a geodetic survey control network:

a) A point on the surface of the reference datum is specified. This point has a computable geodetic latitude φ.

b) The specified position-space point is identified with a specific point in object-space.

c) The direction of the oblate ellipsoid rotational axis is constructed in object-space.

d) The direction of the outward surface normal at the point is constructed in object-space so that the angle it makes with respect to the oblate ellipsoid rotational axis direction is $(\pi/2 - \varphi)$.

This binding requires the specification of a point and two directions in object-space.