9 Designated spatial surfaces and vertical offsets

9.1 Overview

Many spatial applications require the specification of object-space surfaces that are more complex than a surface represented by an RD. An RD surface generating function is restricted by definition to be a multi-variate polynomial of degree 2 or less. Surfaces of interest are often more complex than this restriction allows. These surfaces are termed *designated spatial surfaces*. These surfaces often represent some conceptual or physical aspect of object-space such as a gravity equipotential surface. Some designated spatial surfaces can be analytically represented by means of a smooth surface in position-space and a normal embedding. Such a model is termed a designated spatial surface model.

For SRFs that have a vertical coordinate-component, certain designated spatial surfaces may be used to define vertical offset values. Many real-world measurement systems used in geodesy define the value of the vertical coordinate-component of an SRF to be zero at a designated spatial surface. If the point of intersection between each vertical coordinate-component curve and the designated spatial surface is unique, it specifies a vertical offset value, and the designated spatial surface is termed a *vertical offset surface* for the given SRF.

In this document, the vertical coordinate-component is always zero at an RD surface. For a given point, the difference in values of the vertical coordinate-component between the vertical offset surface and the RD surface is termed the vertical offset. If the designated spatial surface has a designated spatial surface model, then the vertical offset may be computed. In the case of SRFs that designate ellipsoidal height as the vertical coordinate-component, the API (Clause 11) provides a method for the vertical offset computation.

9.2 Designated spatial surface

A *designated spatial surface* (DSS) is a surface in object-space. A DSS may be used to represent an application-specific aspect of the object-space.

Two important cases of DSSs are:

   a) equipotential surfaces including geoids, and

   b) approximations of mean sea level surfaces based on sounding and tidal data.

EXAMPLE   The International Great Lakes Datum 1955 is associated with a DSS that conceptually represents the mean water level of certain bodies of water and extensions of the surface to inland areas. It is empirically represented by a physical network of locations with assigned values for height above the conceptual surface. Various levelling techniques are applied to extrapolate these height values to other locations. There is currently no mathematically defined surface in position-space to model the International Great Lakes Datum 1955 DSS.

A *DSS model* is comprised of a smooth surface in position-space and a normal embedding such that the normal embedding image of the position-space surface either coincides with the DSS or approximates it in an application-specific sense.

An *equipotential surface* is an implicitly defined surface given by $P(x, y, z) - c = 0$, where P is a potential function defined in (a portion of) position-space and c is a value in the range of P.

If P is a smooth function, the equipotential surface is a smooth surface. If the smooth surface is embedded into object-space with a normal embedding, it is a DSS model for the corresponding DSS in object-space.

An important special case of an equipotential surface is a mathematical model of the gravity potential of a celestial body. The *geoid* is a specific equipotential surface of the Earth's gravity field that best fits the global mean sea surface in a minimum variance sense. Global, regional, and local approximations of the geoid are developed from empirical measurements in association with specific ERMs. Gravity equipotential surfaces have also been modelled for other planets.

NOTE   The geoid cannot be measured directly. Current models of the Earth's gravity potential are usually realised as truncated power series in spherical harmonics.

9.3 Vertical offset surface

A DSS is a *vertical offset surface* (VOS) with respect to an SRF in a region of object-space if, for each point in the region, the DSS intersects the vertical coordinate-component curve containing the point exactly once. The VOS concept is restricted to SRFs that have a designated vertical coordinate-component and that are based on an object-fixed ORM (see 8.4.3).

The vertical coordinate-component zero surface is the set of points for which the vertical coordinate-component value is zero (see 5.2.1). Given a point p on the vertical coordinate-component zero surface that is in the region of a VOS, the *vertical offset* at p is the value of the vertical coordinate-component at the intersection of the VOS with the vertical coordinate-component curve that contains p. The vertical offset at p is denoted $v(p)$. If p is not in the region of a VOS or if a VOS has not been specified, the vertical offset at p shall be defined to be zero.

NOTE   All points on the same vertical coordinate-component curve have the same vertical offset value.

For a VOS with respect to an SRF based on an oblate ellipsoid (or sphere) ORM, the vertical offset at a point p on the oblate ellipsoid (or sphere) with surface geodetic coordinate (λ, φ) is denoted by $v(\lambda, \varphi)$.

In many cases, the values $v(\lambda, \varphi)$ are not known or the values are approximately known at specific locations. When a DSS has a DSS model, the $v(\lambda, \varphi)$ values may be computed. If a DSS is a VOS for two SRFs, SRF**A** and SRF**B**, and if the vertical offset function for SRF**A** $v_A(\lambda, \varphi)$ is known, then the vertical offset function for SRF**B** $v_B(\lambda, \varphi)$ may be computed from v_A as follows:

Each SRF**A** coordinate of the form $c_A = (\lambda, \varphi, v_A(\lambda, \varphi))$ lies on the VOS. If $c_B = (\lambda', \varphi', h')$ is the corresponding coordinate representation in SRF**B**, then $v_B(\lambda', \varphi') = h'$.

The API (Clause 11) provides a vertical offset computation for DSS models that are a VOS with respect to a given SRF with ellipsoidal height as the vertical coordinate-component.
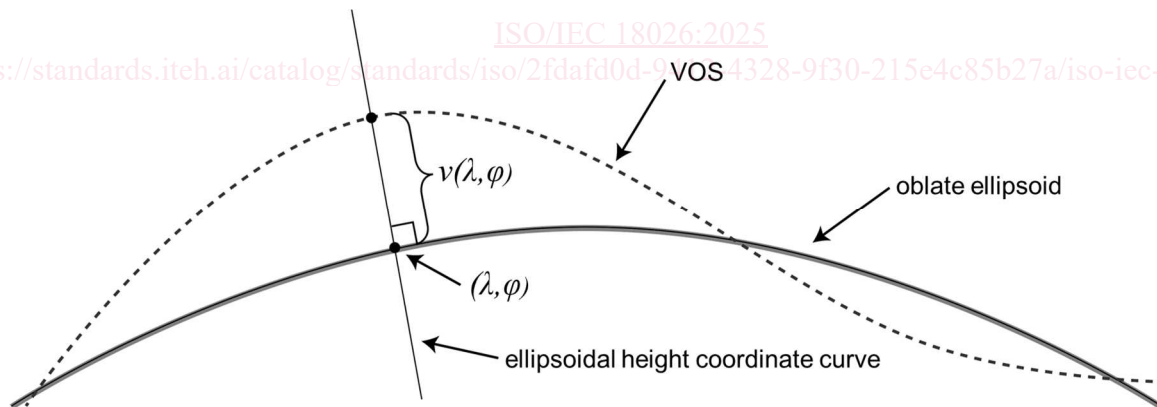

Figure 9.1 — Vertical offset surface for ellipsoidal height

EXAMPLE 1   If an SRF is derived from SRFT CELESTIODETIC or from a map projection SRFT, the ellipsoidal height coordinate-component is the designated vertical coordinate-component. Given a VOS with respect to the SRF, $v(\lambda, \varphi)$ is the distance from the ellipsoid to the VOS along the ellipsoidal height curve at (λ, φ) in the region of the VOS (see Figure 9.1).