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Part 5:
Examples

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Foreword

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This document was prepared by the Joint Committee for Guides in Metrology and was adopted by the national bodies of ISO and IEC.

A list of all parts in the ISO/IEC Guide 98 series can be found on the ISO and IEC websites.

Given that ISO/IEC Guide 98-5 is identical in content to JCGM GUM-5, the decimal symbol is a point on the line in the English version and a comma on the line in the French version.

Annex ZZ has been appended to provide a list of corresponding ISO/IEC Guides and JCGM guidance documents for which equivalents are not given in the text.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html and www.iec.ch/national-committees.

Joint Committee for Guides in Metrology

Guide to the expression of uncertainty in measurement — Part 5: Examples

Guide pour l'expression de l'incertitude de mesure — Partie 5: Exemples

JCGM GUM-5:2026

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Foreword

In 1997 a Joint Committee for Guides in Metrology (JCGM), chaired by the Director of the Bureau International des Poids et Mesures (BIPM), was created by the seven international organizations that had originally in 1993 prepared the 'Guide to the expression of uncertainty in measurement' and the 'International vocabulary of basic and general terms in metrology'. The JCGM assumed responsibility for these two documents from the Technical Advisory Group 4 of the International Organization for Standardization (ISO/TAG4).

The Joint Committee is formed by the BIPM with the International Electrotechnical Commission (IEC), the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Laboratory Accreditation Cooperation (ILAC), the International Organization for Standardization (ISO), the International Union of Pure and Applied Chemistry (IUPAC), the International Union of Pure and Applied Physics (IUPAP), and the Organisation Internationale De Métrologie Légale (OIML).

JCGM has two Working Groups. Working Group 1, 'Expression of uncertainty in measurement', has the task to promote the use of the 'Guide to the expression of uncertainty in measurement' and to prepare documents for its broad application. Working Group 2, 'Working Group on International vocabulary of basic and general terms in metrology', has the task to revise and promote the use of the International vocabulary of basic and general terms in metrology (VIM).

In 2008 the JCGM made available a slightly revised version (mainly correcting minor errors) of the 'Guide to the expression of uncertainty in measurement', labelling the document 'JCGM 100:2008'.

In 2017 the JCGM rebranded the documents in its portfolio that have been produced by Working Group 1 or are to be developed by that Group: the whole suite of documents is now known as the 'Guide to the expression of uncertainty in measurement' or 'GUM', and is concerned with the evaluation and expression of measurement uncertainty, as well as its application in science, trade, health, safety and other societal activities.

This part of the suite contains a collection of examples illustrating the methods for uncertainty evaluation described in the GUM.

This document has been prepared by Working Group 1 of the JCGM, and has benefited from detailed reviews undertaken by member organizations of the JCGM and National Metrology Institutes.

Introduction

The GUM suite of documents (parts overview in [14]) is concerned with the evaluation of uncertainty in measurement and how uncertainty is reported and used. Compact examples are given throughout the GUM suite that relate to specific areas of application and are intended to illustrate particular concepts. This document contains larger examples and case studies intended to illustrate the principles presented in the GUM suite for evaluating and reporting measurement uncertainty.

The examples are chosen to illustrate good practice in the evaluation of measurement uncertainty using the methods in the GUM, such as the JCGM 100:2008 uncertainty framework (GUF) [17, Definition 3.18] or the Monte Carlo method (MCM) [12,17] or alternative methods not (yet) covered by the guidance in the suite. In some examples, methods for uncertainty evaluation are compared to highlight aspects of their suitability for the presented case.

The context and measurement in the examples are described to the extent relevant; simplifications can have been made in the interest of presenting a concise case. The data used in the examples is, where possible, taken from real measurements. These data are not necessarily the best, representing the state-of-the-art or in any other way to be interpreted as such. Measurement procedures and practices are presented in a concise format and not intended to enable reproducing the measurement. Where appropriate, references are provided to documents describing these procedures or practices.

All examples were edited prior to inclusion in this document.

Results are generally reported in the manner described in JCGM 101:2008 [17]. However, more than the recommended one or two significant digits are often given here to facilitate comparison of the results obtained from the various approaches.

Guide to the expression of uncertainty in measurement — Part 5: Examples

0 Scope

0.1 This document gives a number of examples, which are worked out in considerable detail to illustrate the principles presented in the Guide to the expression of uncertainty in measurement (GUM) for evaluating and reporting uncertainty in measurement. Together with the (shorter) examples included in the other parts of the GUM, they should enable users of the GUM to put these principles into practice in their own work.

0.2 Because the examples are for illustrative purposes, they have by necessity been simplified (but not over-simplified). Moreover, because they and the numerical data used in them have been chosen mainly to demonstrate the principles of the GUM, neither they nor the data should necessarily be interpreted as describing real measurements, let alone the state of the art for a particular measurement. Although the data were used as provided, all calculations retained the full computational precision of the systems employed. Final and intermediate results have been rounded appropriately to suit the intended applications.

0.3 The examples are used to demonstrate, among other:

— Using knowledge about a quantity to obtain an estimate of the quantity and its associated uncertainty;

— Propagation of uncertainty using the law of propagation of uncertainty (LPU) as in JCGM 100:2008 and JCGM 102:2011;

— Propagation of distributions using analytical and numerical methods, including the Monte Carlo method (MCM), as in JCGM 101:2008 and JCGM 102:2011;

— Applying law of propagation of uncertainty (LPU) and MCM for a univariate (scalar) and multivariate (vector) measurand;

— Applying LPU and MCM for independent and dependent input quantities;

— Using MCM to validate the results provided by LPU;

— Determining a coverage interval or a coverage region;

— Using measurement uncertainty in conformity assessment as in JCGM 106:2012;

— Reporting measurement results.

0.4 An overview of the parts of the GUM suite is given in [14]. The conventions and notation applied in this document are summarised in Annex A. The symbols used throughout the document are explained in the text and in Annex B.

1 Measurement of pH: linear interpolation

1.1 Preamble

1.1.1 This example is taken from a compendium of examples [1].

1.1.2 A generic treatment of two-point and multi-point interpolation of calibration data is given with uncertainties associated with the data propagated using the law of propagation of uncertainty (LPU) and its generalization to vector measurands. The approach is applied to the measurement of hydrogen ion activity (pH). Such measurement is one of the most common in chemistry, although correlations associated with the input quantities in the measurement model are rarely taken into account. The treatment given follows common practice, which tends to give an optimistically small evaluation of the uncertainty associated with an estimated hydrogen ion activity (pH) value. A way of taking correlation into account in one typical instance is given but its implementation is problematical because of the difficulty in quantifying the correlation.

1.1.3 The VIM concept of calibration [16, Definition 2.39] as constituting two stages is used. The first stage involves fitting to measured data a function that describes the relationship of a response (dependent) variable Y to a stimulus (independent) variable X. The second stage involves using this relationship to determine the value y of the response given a value x of the stimulus. Uncertainties in both the stimulus and response variables are handled in both stages and propagated using the LPU in JCGM 100:2008 [10] and its generalization to vector measurands in JCGM 102:2011 [12].

1.1.4 The considerations of 1.1.3 are applied to pH measurement in which up to three two-point interpolations are required and uncertainties are tracked through the calculation.

1.2 pH of a test solution

1.2.1 pH, the negative logarithm to base 10 of the activity of hydrogen ion in a solution, is arguably the most measured quantity in chemistry [25]. The pH is a measure of the acidity of a solution. The pH of a solution is generally measured using a pH-sensitive glass electrode and a silver chloride (AgCl) reference electrode. The potential difference of such an electrode system is proportional to pH and forms the basis of pH measurement [28].

1.2.2 In 2002, the International Union of Pure and Applied Chemistry (IUPAC) issued a recommendation for revision of the pH scale based on the concept of a primary reference measurement procedure for pH [21]. The use of an electrochemical (Harned) cell fulfils the criteria for a primary reference measurement procedure so that a pH value thus obtained is traceable to the International System of Units (SI), here the SI measurement unit 1.

1.2.3 A solution, the pH of which is measured by such a cell at the highest metrological level, may be classified as a primary measurement standard and can be used to assign pH values to other solutions. Standard solutions are sold as certified reference materials (CRMs) to calibrate pH meters for routine use.

1.2.4 There are several approaches to pH measurement including the use of 1-point, 2-point and multi-point calibration, least-squares regression, and with or without temperature correction. Here the 2-point calibration approach, with and without temperature correction, is used. Two topics are considered: (a) when the temperature of the test solution matches that of the standard (reference) solutions and (b) when this is not the case.

1.3 Specification of the measurand

The measurand is the pH of a solution being calibrated. More generally, the measurand is the interpolated independent or dependent variable obtained from a relationship between those variables derived from data representing values of the variables. Intermediate measurands, when required, are the parameters describing the relationship.

1.4 Measurement model

1.4.1 General

There are two stages involved in calibration [16, definition 2.39]: (i) determine a calibration curve from calibration data and (ii) use that calibration curve. Because of the relative simplicity of two-point and multi-point interpolation as considered here, it may be preferable when circumstances permit to combine the stages into a single-stage model. Such a model avoids having to deal with intermediate correlation associated with the calibration curve parameters that are estimated in the first stage and used in the second. Operating in two stages corresponds to the use of a multi-stage model [15, Clause 8.4] and is necessary when the construction and use of the calibration model are carried out by different parties.

1.4.2 Generic approach to two-point calibration

1.4.2.1 Two calibration points (X_1, Y_1) and (X_2, Y_2) are given that bracket X_0, an X-value for which Y_0, the corresponding Y-value, is required under the assumption that the Y-value lies on the straight line joining the calibration points (see figure 1.1).

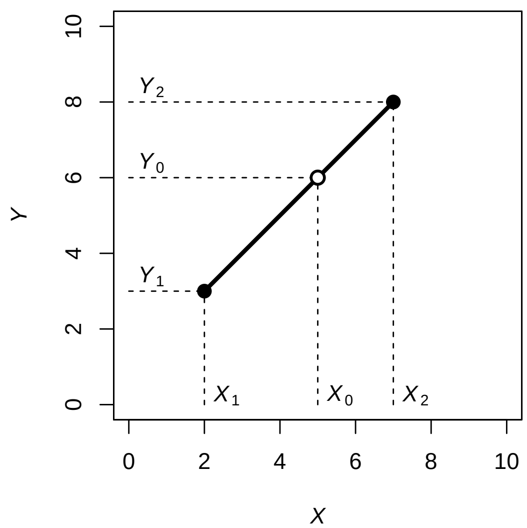

Figure 1.1: Two-point calibration using a linear interpolation function