Technical Report

ISO/TR 23509-2

Bevel and hypoid gear geometry —

Part 2: Sample calculations

Géométrie des engrenages coniques et hypoïdes —
Partie 2: Exemples de calcul

First edition
2026-08

Reference number
ISO/TR 23509-2:2026(en)

COPYRIGHT PROTECTED DOCUMENT

ISO copyright office
CP 401 • Ch. de Blandonnet 8
CH-1214 Vernier, Geneva
Phone: +41 22 749 01 11
Email: copyright@iso.org
Website: www.iso.org

Published in Switzerland

Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization. The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

ISO draws attention to the possibility that the implementation of this document may involve the use of (a) patent(s). ISO takes no position concerning the evidence, validity or applicability of any claimed patent rights in respect thereof. As of the date of publication of this document, ISO had not received notice of (a) patent(s) which may be required to implement this document. However, implementers are cautioned that this may not represent the latest information, which may be obtained from the patent database available at www.iso.org/patents. ISO shall not be held responsible for identifying any or all such patent rights.

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement. For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

A list of all parts in the ISO 23509 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

For many decades, information on bevel, and especially hypoid, gear geometry has been developed and published by the gear machine manufacturers. It is clear that the specific formulae for their respective geometries were developed for the mechanical generation methods of their particular machines and tools. In many cases, these formulae can not be used in general for all bevel gear types. This situation changed with the introduction of universal, multi-axis, computerized numerical control (CNC)-machines, which in principle are able to produce nearly all types of gearing. The manufacturers were, therefore, asked to provide CNC programs for the geometries of different bevel gear generation methods on their machines.

This document integrates straight bevel gears and the three major design generation methods for spiral bevel gears into one complete set of formulae. In only a few places, specific formulae for each method will be applied. The structure of the formulae is such that they can be programmed directly, allowing the user to compare the different designs.

The formulae of the three methods are developed for the general case of hypoid gears and to calculate the specific case of spiral bevel gears by entering zero for the hypoid offset. Additionally, the geometries correspond such that each gear set consists of a generated or non-generated wheel without offset and a pinion which is generated and provided with the toral hypoid offset.

This document was prepared with sample calculations for different bevel gear designs. They are intended for users of the ISO 23509 series to follow a whole calculation procedure formula by formula. Practical experience has shown that this way, to get into a complex subject, is very helpful.

On the other hand, this document is not intended for use by the average engineer. Rather, it is aimed at the well-versed engineer capable of selecting reasonable values for the parameters and factors in these formulae based on knowledge of similar designs and on awareness of the effects behind these formulae.

Bevel and hypoid gear geometry —

Part 2:
Sample calculations

1 Scope

This document provides sample calculations for different bevel gear designs, how the geometry is numerically determined according to the methods and formulae of ISO 23509-1:2025 .

The term “bevel gear” is used to mean straight, helical (skew), spiral bevel, zerol and hypoid gear designs. Where this document pertains to one or more, but not all, the specific forms are identified.

The manufacturing process of forming the desired tooth form is not intended to imply any specific process, but rather to be general in nature and applicable to all methods of manufacture.

The geometry for the calculation of factors used in bevel gear rating, such as ISO 10300-1 [1], ISO 10300-2 [2], and ISO 10300-3 [3], is also included.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1122-1, *Vocabulary of gear terms — Part 1: Definitions related to geometry*

ISO 23509-1:2025, *Bevel and hypoid gear geometry — Part 1: Basic methods*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 1122-1, ISO 23509-1:2025, and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at http://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Symbols and abbreviated terms

For the purposes of this document, the symbols and units given in Table 1 and Table 2 apply.

Table 1 — Symbols and units used in this document

Symbols	Description or term	Units
A	intermediate variable	-
a	hypoid offset	mm
b_1, b_2	facewidth	mm
b_{e1}, b_{e2}	facewidth from calculation point to outside	mm
b_{i1}, b_{i2}	facewidth from calculation point to inside	mm
c_1, c_2	clearance	mm
c_{be2}	facewidth factor	-
c_{ham}	mean addendum factor of wheel	-
d_{ae1}, d_{ae2}	outside diameter	mm
d_{ai1}, d_{ai2}	inner outside diameter	mm
d_{e1}, d_{e2}	outer pitch diameter	mm
d_{fe1}, d_{fe2}	outer root diameter	mm
d_{fi1}, d_{fi2}	inner root diameter	mm
d_{m1}, d_{m2}	mean pitch diameter	mm
F_{ax}	axial force	N
F_{mt1}, F_{mt2}	tangential force at mean diameter	N
F_{rad}	radial force	N
$f_{\alpha lim}$	influence factor of limit pressure angle	-
h_{ae1}, h_{ae2}	outer addendum	mm
h_{ai1}, h_{ai2}	inner addendum	mm
h_{am1}, h_{am2}	mean addendum	mm
h_{amc1}, h_{amc2}	mean chordal addendum	mm
h_{e1}, h_{e2}	outer whole depth	mm
h_{fe1}, h_{fe2}	outer dedendum	mm
h_{fi1}, h_{fi2}	inner dedendum	mm
h_{fm1}, h_{fm2}	mean dedendum	mm
$h_{i1,2}$	Inner whole depth	mm
h_m	mean whole depth	mm
h_{mw}	mean working depth	mm
h_{t1}	pinion whole depth	mm
j_{en}	outer normal backlash	mm
j_{et}	outer transverse backlash	mm
j_{mn}	mean normal backlash	mm
j_{mt}	mean transverse backlash	mm
K_{M1}, K_{M2}, K_{M3}	approximate hypoid dimension factor (subscript indicates the method used)	-
k_c	clearance factor	-
k_d	depth factor	-
k_{hap}	basic crown gear addendum factor (related to m_{mn})	-
k_{hfp}	basic crown gear dedendum factor (related to m_{mn})	-
k_t	circular thickness factor	-
m_{et}	outer transverse module	mm
m_{mn}	mean normal module	mm
n_1	pinion speed	min^{-1}
P	power	kW

Table 1 *(continued)*

Symbols	Description or term	Units
q_k	angle modification	°
$R_{e1,}$ R_{e2}	outer cone distance	mm
$R_{i1,}$ R_{i2}	inner cone distance	mm
R_{m1}, R_{m2}	mean cone distance	mm
r_{c0}	cutter radius	mm
s_{mn1}, s_{mn2}	mean normal circular tooth thickness	mm
s_{mnc1}, s_{mnc2}	mean normal chordal tooth thickness	mm
s_{mt1}, s_{mt2}	mean transverse circular tooth thickness	mm
T_1	pinion torque	Nm
t_{xi1}, t_{xi2}	inner crown to crossing point	mm
t_{xo1}, t_{xo2}	outer crown to crossing point	mm
t_{z1}, t_{z2}	pitch apex beyond crossing point	mm
t_{zF1}, t_{zF2}	face apex beyond crossing point	mm
t_{zm1}, t_{zm2}	crossing point to mean point along axis	mm
t_{zR1}, t_{zR2}	root apex beyond crossing point	mm
u	gear ratio	-
u_a	equivalent ratio	-
W_{m2}	wheel mean slot width	mm
x_{hm1}	profile shift coefficient	-
x_{sm1}, x_{sm2}	thickness modification coefficient (backlash included)	-
x_{smn}	thickness modification coefficient (theoretical)	-
z_0	number of blade groups	-
z_1, z_2	number of teeth	-
z_p	number of crown gear teeth	-
α_{dC}	nominal design pressure angle on coast side	°
α_{dD}	nominal design pressure angle on drive side	°
α_{eC}	effective pressure angle on coast side	°
α_{eD}	effective pressure angle on drive side	°
α_{nC}	generated pressure angle on coast side	°
α_{nD}	generated pressure angle on drive side	°
α_{lim}	limit pressure angle	°
β_{e1}, β_{e2}	outer spiral angle	°
β_{i1}, β_{i2}	inner spiral angle	°
β_{m1}, β_{m2}	mean spiral angle	°
Δb_{x1}	pinion face width increment	mm
Δg_{xe}	increment along pinion axis from calculation point to outside	mm
Δg_{xi}	increment along pinion axis from calculation point to inside	mm
ΔK	incrementin hypoid dimension factor	-
$\Delta\Sigma$	shaft angle departure from 90°	°
δ_{a1}, δ_{a2}	face angle	°
δ_{f1}, δ_{f2}	root angle	°
δ_1, δ_2	pitch angle	°
ε_β	face contact ratio	-
η	wheel offset angle in axial plane	°

Table 1 *(continued)*

Symbols	Description or term	Units
η_1	second auxiliary angle (see ISO 23509-1:2025 , Figure 15)	°
θ_{a1}, θ_{a2}	addendum angle	°
θ_{f1}, θ_{f2}	dedendum angle	°
ϑ	auxiliary angle	°
λ	first auxiliary angle (see ISO 23509-1:2025 , Figure 15)	°
ν	lead angle of cutter	°
ξ	auxiliary angle	°
ρ_b	epicycloid base circle radius	mm
ρ_{lim}	limit curvature radius	mm
$\rho_{m\beta}$	lengthwise tooth mean radius of curvature	mm
ρ_{P0}	crown gear to cutter centre distance	mm
Σ	shaft angle	°
$\Sigma\theta_f$	sum of dedendum angles	°
$\Sigma\theta_{fC}$	sum of dedendum angles for constant slot width taper	°
$\Sigma\theta_{fM}$	sum of dedendum angles for modified slot width taper	°
$\Sigma\theta_{fS}$	sum of dedendum angles for standard taper	°
ζ_o	pinion offset angle in face plane	°
$\zeta_{m,M1}$, $\zeta_{m,M2}$, $\zeta_{m,M3}$	pinion offset angle in axial plane (Method 1, Method 2, or Method 3)	°
$\zeta_{mi,M2}$	approximate pinion offset angle in axial plane (Method 2)	°
$\zeta_{mp,M1}$, $\zeta_{mp,M2}$, $\zeta_{mp,M3}$	pinion offset angle in pitch plane (Method 1, Method 2, or Method 3)	°
$\zeta_{mpi,M1}$	approximate pinion offset angle in pitch plane (Method 1)	°
$\zeta'_{m,M1}$, $\zeta'_{m,M2}$,	intermediate pinion offset angle in axial plane (Method 1, Method 2)	°
$\zeta'_{mp,M1}$, $\zeta'_{mp,M2}$,	intermediate pinion offset angle in pitch plane (Method 1, Method 2)	°
ζ_R	pinion offset angle in root plane	°
φ	intermediate or auxiliary angle	°

Table 2 — General subscripts

Subscript	Description
a	addendum
b	base
C	coast side
D	drive side
d	nominal design
e	effective, when used in connection with pressure angle outer, when used as a geometric reference
f	dedendum
i	inner, when used as a geometric reference approximate value, when used in connection with pinion offset angle
lim	limit
M1, M2, M3	Method 1, Method 2, Method 3
m	mean
n	normal
o	face plane
p	pitch plane

Table 2 *(continued)*

Subscript	Description
R	root plane
t	tangential, when used in connection with force transverse
1	refers to pinion
2	refers to wheel

5 Application

This document provides six sample calculations:

— Sample 1 (see Annex A) shows the geometry calculation of a spiral bevel gear pair without hypoid offset according to Method 0;

— Sample 2 (see Annex B) shows the geometry calculation of a hypoid gear set according to Method 1;

— Sample 3 (see Annex C) shows the geometry calculation of a hypoid gear set according to Method 2;

— Sample 4 (see Annex D) shows the geometry calculation of a hypoid gear set according to Method 3;

— Sample 5 (see Annex E) shows the geometry calculation of a hypoid gear set according to Method 3;

— Sample 6 (see Annex F) shows the geometry calculation of a straight bevel gear pair without hypoid offset according to Method 0.

Table 3 refers to each table of the annexes showing the input parameters and calculation results for the different sample calculation steps.

Table 3 — Content of tables of Annexes A to F

Content of Table	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6
Initial data						
Initial data for pitch cone parameters	Table A.1	Table B.1	Table C.1	Table D.1	Table E.1	Table F.1
Input data for tooth profile param	Table A.2	Table B.2	Table C.2	Table D.2	Table E.2	Table F.2
Transformation of data type II into data type I	Table A.3	Table B.3	Table C.3	Table D.3	Table E.3	Table F.3
Calculation of Samples						
Determination of pitch cone parameters	Table A.4	Table B.4	Table C.4	Table D.4	Table E.4	Table F.4
Determination of basic data	Table A.5	Table B.5	Table C.5	Table D.5	Table E.5	Table F.5
Determination of root angles and face angles	Table A.6	Table B.6	Table C.6	Table D.6	Table E.6	Table F.6
Determination of tooth depth at calculation point	Table A.7	Table B.7	Table C.7	Table D.7	Table E.7	Table F.7
Determination of root and face apexes	Table A.8	Table B.8	Table C.8	Table D.8	Table E.8	Table F.8
Determination of pinion facewidth	Table A.9	Table B.9	Table C.9	Table D.9	Table E.9	Table F.9
Determination of inner and outer spiral angles	Table A.10	Table B.10	Table C.10	Table D.10	Table E.10	Table F.10
Determination of tooth depth	Table A.11	Table B.11	Table C.11	Table D.11	Table E.11	Table F.11
Determination of tooth thickness	Table A.12	Table B.12	Table C.12	Table D.12	Table E.12	Table F.12
Determination of remaining gear dimensions	Table A.13	Table B.13	Table C.13	Table D.13	Table E.13	Table F.13

The symbols, terms and definitions used in this document are, wherever possible, consistent with other international Standards. It is known, because of certain limitations, that some symbols, their terms and definitions, as used in this document, are different from those used in similar literature pertaining to spur and helical gearing.

Annex A
(informative)

Sample 1: Calculation of geometry for a spiral bevel gear pair without hypoid offset according to ISO 23509-1:2025 , Method 0

A.1 Initial data

Sample 1 is for a spiral bevel gear pair without hypoid offset which uses Method 0 according to ISO 23509-1:2025 .

Table A.1 — Initial data for calculation of pitch cone parameters

Symbol	Description	Method 0	Method 1	Method 2	Method 3
Σ	shaft angle	90°	X	X	X
a	hypoid offset	0 mm	X	X	X
$z_{1,2}$	number of teeth	14/39	X	X	X
d_{m2}	mean pitch diameter of wheel	-	-	X	-
d_{e2}	outer pitch diameter of wheel	176,893 mm	X	-	X
b_2	wheel facewidth	25,4 mm	X	X	X
β_{m1}	mean spiral angle of pinion	-	X	-	-
β_{m2}	mean spiral angle of wheel	35°	-	X	X
r_{c0}	cutter radius	114,3 mm	X	X	X
z_0	number of blade groups (only face hobbing)	-	-	X	X

Table A.2 — Additional data for calculation of gear dimensions

Data type I		Data type II	
Symbol	Value	Symbol	Value
α_{dD}	20°		
α_{dC}	20°		
$f_{\alpha lim}$	0		
x_{hm1}	-	c_{ham}	0,247 37
k_{hap}	-	k_d	2,000
k_{hfp}	-	k_c	0,125
x_{smn}	-	k_t	0,091 5
		W_{m2}	-
j_{en}	0,127 mm		
θ_{a2}	2,134 2°		
θ_{f2}	6,493 4°		
q_k	0°		

Table A.3 — Transformation of data type II into data type I

Formula	Result
$x_{\mathrm{hm1}} = k_{\mathrm{d}} \cdot \left(\frac{1}{2} - c_{\mathrm{ham}}\right)$	0,505
$k_{\mathrm{hap}} = \frac{k_{\mathrm{d}}}{2}$	1
$k_{\mathrm{hfp}} = k_{\mathrm{d}} \cdot \left(k_{\mathrm{c}} + \frac{1}{2}\right)$	1,25
$x_{\mathrm{smn}} = \frac{k_{\mathrm{t}}}{2}$	0,046

A.2 Calculation of Sample 1 according to Method 0